Mathematical Reviews

Published monthly by The American Mathematical Society

TABLE OF CONTENTS

Volume 22, Number 8B August 1961 Reviews 7153–7912

MATHEMATICAL REVIEWS

Published by

THE AMERICAN MATHEMATICAL SOCIETY, 190 Hope St., Providence 6, R.I.

Sponsored by

THE AMERICAN MATHEMATICAL SOCIETY
THE MATHEMATICAL ASSOCIATION OF AMERICA
THE INSTITUTE OF MATHEMATICAL STATISTICS
THE EDINBURGH MATHEMATICAL SOCIETY
SOCIÉTÉ MATHÉMATIQUE DE FRANCE
DANSK MATEMATISK FORENING
HET WISKUNDIG GENOOTSCHAP TE AMSTERDAM
THE LONDON MATHEMATICAL SOCIETY
POLSKIE TOWARZYSTWO MATEMATYCZNE
UNIÓN MATEMÁTICA ARGENTINA
INDIAN MATHEMATICAL SOCIETY
UNIONE MATEMATICA ITALIANA
THE SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS

Edited by

E. Hille W. S. Massey J. V. Wehausen

S. H. Gould, *Executive Editor* A. J. Lohwater, *Assistant Executive Editor*

I. Barsotti, Chandler Davis, W. Freiberger, W. J. LeVeque and J. A. Zilber, *Associate Editors*

H. A. Pogorzelski, *Copy Editor*

Editorial Office

MATHEMATICAL REVIEWS, 190 Hope St., Providence 6, R.I.

Subscription: Price $50 per year ($25 per year to individual members of sponsoring societies). Checks should be made payable to MATHEMATICAL REVIEWS. Subscriptions should be addressed to the American Mathematical Society, 190 Hope St., Providence 6, R.I.

The preparation of the reviews appearing in this publication is made possible by support provided by a grant from the National Science Foundation. The publication was initiated with funds granted by the Carnegie Corporation of New York, the Rockefeller Foundation, and the American Philosophical Society held at Philadelphia for Promoting Useful Knowledge. These organizations are not, however, the authors, owners, publishers or proprietors of the publication, and are not to be understood as approving by virtue of their grants any of the statements made or views expressed therein.

Mathematical Reviews is published in 1961 in twelve monthly issues, each in two parts, A and B; and a single index issue covering both parts. Reviews and pages are numbered consecutively with respect to the issue order 1A, 1B, . . ., 12A, 12B. When the letter A or B is prefixed to a review number, it indicates in which part the review appears.

Journal references in Mathematical Reviews are now given in the following form: J. Brobdingnag. Acad. Sci. (7) 4 (82) (1952/53), no. 3, 17–42 (1954), where after the abbreviated title one has: (series number) volume number (volume number in first series if given) (nominal date), issue number if necessary, first page–last page (imprint date). In case only one date is given, this will usually be interpreted as the nominal date and printed immediately after the volume number (this is a change from past practice in Mathematical Reviews where a single date has been interpreted as the imprint date). If no volume number is given, the year will be used in its place. The symbol ★ precedes the title of a book or other non-periodical which is being reviewed as a whole.

References to reviews in Mathematical Reviews before volume 20 (1959) are by volume and page number, as MR 19, 532: from volume 20 on, by volume and review number, as MR 20 #4387. Reviews reprinted from Applied Mechanics Reviews, Referativnyĭ Žurnal, or Zentralblatt für Mathematik are identified in parentheses following the reviewer's name by AMR, RŽMat (or RŽMeh, RŽAstr. Geod.), Zbl, respectively.

Printed in Great Britain by William Clowes and Sons, Limited, London and Beccles

Mathematical Reviews

Vol. 22, No. 8B August, 1961 Reviews 7153–7912

PROBABILITY

See also A6744, 7195, 7299, 7300, 7316, 7906.

7153:

Goldberg, Samuel. ★Probability: An introduction. Prentice-Hall Mathematics Series. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1960. xiv + 322 pp. $7.95.

Any review of this book must, in fairness, take cognizance of the fact that it is directed toward students lacking both calculus and even a semblance of mathematical maturity. Such a target population may grow apace if recent proposals for revamping college and high school curricula are adopted or transcended.

The probabilistic setting imposed is that of a finite sample space; the definition of probability is general, in contradistinction to that of college algebra textbooks which requires probabilities (although not their users) to be rational. The treatment of sets (chapter 1) and of the basic notions of probability theory (chapter 2) is painstaking and nice. The author is sensitive to semantic difficulties likely to plague a non-mathematical reader and has a word to say on the knotty problem of choosing a sample space for an experiment. A brief chapter 3 presents elementary combinatorics and the ensuing one deals with random variables, (discrete) probability distributions (univariate, joint and conditional), means, variances and correlations, culminating in a weak law of large numbers. The final chapter 5 discusses Bernoulli trials, testing a (binomial) hypothesis and provides an example of decision-making.

Unfortunately, no glimmer of the scope (even under the imposed restrictions) and beauty of probability theory emerges from this book. This might have been provided in part by some dynamic examples, possibly employing the reflection principle. The reviewer would gladly have jettisoned the decision-making example (which seems more counter-point than harmony) to this end. But the book is studded with problems and, in the hands of a competent, imaginative instructor, should provide a pleasant and salutary introduction to the subject for members of the population in question. *H. Teicher* (New York)

7154:

★Selected Translations in Mathematical Statistics and Probability. Vol. I. Printed for Institute of Mathematical Statistics by American Mathematical Society, Providence, R.I., 1961. v + 306 pp. $4.80.

25 articles translated from the Russian; see below: #7155–7168, 7188, 7190, 7195, 7206–7212, 7227.

7155:

Čulanovskiĭ, I. V. On cycles in Markov chains. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 1–5. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl. Akad. Nauk SSSR **69** (1949), 301–304 [MR **11**, 256].

7156:

Rozenknop, I. Z. On some properties of the totality of closed paths in a system of n states and given transitions among them. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 7–12. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Izv. Akad. Nauk SSSR. Ser. Mat. **14** (1950), 95–100 [MR **11**, 445].

7157:

Sapogov, N. A. The stability problem for a theorem of Cramér. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 41–53. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Izv. Akad. Nauk SSSR. Ser. Mat. **15** (1951), 205–218 [MR **13**, 51].

7158:

Prohorov, Yu. V. Asymptotic behavior of the binomial distribution. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 87–95. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Uspehi Mat. Nauk **8** (1953), no. 3 (35), 135–142 [MR **15**, 138].

7159:

Dobrušin, R. L. Limit theorems for a Markov chain of two states. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 97–134. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Izv. Akad. Nauk SSSR. Ser. Mat. **17** (1953), 291–330 [MR **15**, 329].

7160:

Gnedenko, B. V. On the role of the maximal summand in the summation of independent random variables. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 135–143. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Ukrain Mat. Ž. **5** (1953), 291–298 [MR **15**, 238].

7161:

Skorohod, A. V. Asymptotic formulas for stable distribution laws. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 157–161. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl. Akad. Nauk SSSR **98** (1954), 731–734 [MR **16**, 493].

7162:

Zolotarev, V. M. Expression of the density of a stable distribution with exponent α greater than one by means of a frequency with exponent $1/\alpha$. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 163–167. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl. Akad. Nauk SSSR **98** (1954), 735–738 [MR **16**, 493].

7163:

Skorohod, A. V. On a theorem concerning stable distributions. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 169–170. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Uspehi Mat. Nauk **9** (1954), no. 2 (60), 189–190 [MR **16**, 52].

7164:

Dynkin, E. B. Some limit theorems for sums of independent random variables with infinite mathematical expectations. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 171–189. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Izv. Akad. Nauk SSSR. Ser. Mat. **19** (1955), 247–266 [MR **17**, 865].

7165:

Zolotarev, V. M. On analytic properties of stable distribution laws. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 207–211. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Vestnik Leningrad. Univ. **11** (1956), no. 1, 49–52 [MR **17**, 1096].

7166:

Sanov, I. N. On the probability of large deviations of random variables. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 213–244. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Mat. Sb. (N.S.) **42 (84)**, no. 1 (1957), 11–44 [MR **19**, 466].

7167:

Hájek, Jaroslav. On a property of normal distributions of any stochastic process. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 245–252. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Czechoslovak Math. J. **8 (83)** (1958), 610–617 [MR **21** #3045].

7168:

Rozanov, Yu. A. Spectral theory of multi-dimensional stationary random processes with discrete time. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 253–306. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Uspehi Mat. Nauk **13** (1958), no. 2 (80), 93–142 [MR **22** #5076].

7169:

Wolk, E. S. Torpedo hit probabilities. SIAM Rev. **2** (1960), 292–296.

Assume a dispatcher launches a torpedo in straight line motion in a plane on a desired path toward a target also moving with uniform straight line motion in a plane. The dispatcher has available estimates of the direction and speed of the target. This paper assumes that all the estimates and the actual course of the torpedo are subject to error. It is assumed in each case that the true parameter has a distribution, the mean value of which is the estimate used in the calculation. After making a number of simplifying assumptions, the paper gives a formula for computing the probability that the torpedo will come within a distance δ of the target, since it is equipped with a homing device which results in a hit if this occurs. The approximation to the hit probability is expressed as a bivariate normal integral and it is stated that in most applications further approximation can be made which leads to a univariate normal integral. The author states that the calculation has been programmed and run on a 704 computer, but no computational results are given.

D. Teichroew (Stanford, Calif.)

7170:

Ibragimov, I. A.; Černin, K. E. On the unimodality of stable laws. Teor. Veroyatnost. i Primenen. **4** (1959), 453–456. (Russian. English summary)

It is proved that each stable distribution function is unimodal. This is achieved by explicit computation of derivatives of the density function

$$p(x, \alpha, \beta) = \frac{1}{\pi} \operatorname{Re} \int_0^\infty \exp\{itx - t^\alpha e^{-\frac{1}{2}i\pi\alpha(1-|1-\alpha|)\beta}\}\, dt$$

obtained by Zolotarev [Vestnik Leningrad. Univ. **11** (1956), no. 1, 49–52; MR **17**, 1096]. An argument first reduces the problem to the case where $\beta = 1$.

K. L. Chung (Syracuse, N.Y.)

7171:

Higuti, Zyunsiro. Remarque sur la répartition faible dans un espace localement convexe. I. Ann. Inst. Statist. Math. Tokyo **12** (1960), 63–67.

Dans cette note on donne une condition nécessaire et suffisante pour qu'une répartition faible [au sense de I. E. Segal, Amer. J. Math. **76** (1954), 721–732; MR **16**, 149] d'un espace dénombrablement normé complet et parfait soit réalisable dans son bidual $X^{**} = X$.

R. Theodorescu (Bucharest)

7172:

Theodorescu, Radu. Some remarks on abstract random variables and functions. Bull. Math. Soc. Sci. Math. Phys. R. P. Roumaine (N.S.) **2 (50)** (1958), 343–351.

Random variables X with values in a Banach space $\mathfrak{X}$

are studied (i.e., weakly measurable functions from a probability space into $\mathfrak{X}$). For some purposes, $\mathfrak{X}$ is assumed further to be a commutative C^* algebra. First and second moments and various sorts of convergence are examined. For $\mathfrak{X}$-valued stochastic processes on an interval, the author considers various sorts of continuity, differentiability, integrability, and polynomial approximation.

J. Feldman (Princeton, N.J.)

7173:

Gnedenko, B. V. Limit theorems of probability theory. Proc. Internat. Congress Math. 1958, pp. 518–528. (Russian) Cambridge Univ. Press, New York, 1960.

The author presents a brief survey of probability limit theorems—their assumptions, conclusions, and interrelations—in a general mathematical context but without proofs. He also provides a reference list of 23 papers published in Russian and English which covers both early and recent results.

H. P. Edmundson (Pacific Palisades, Calif.)

7174:

Obretenov, Apostol. Eine Bemerkung über das starke Gesetz der grossen Zahlen. Math. Nachr. **17**, 151–155 (1959).

The author rediscovers a result of Marcinkiewicz and Zygmund; a more general form of it can be found in a paper by the reviewer [Amer. J. Math. **69** (1947), 189–192; MR 8, 471]. *K. L. Chung* (Syracuse, N.Y.)

7175:

Rosenblatt, M. An aggregation problem for Markov chains. Information and decision processes, pp. 87–92. McGraw-Hill, New York, 1960.

Expository paper. *K. L. Chung* (Syracuse, N.Y.)

7176:

Chung, Kai Lai. ★Markov chains with stationary transition probabilities. Die Grundlehren der mathematischen Wissenschaften, Bd. 104. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1960. x+278 pp. DM 65.60.

This monograph is concerned with Markov chains, that is, Markov processes with a denumerable state space. Further, the chains are assumed to have a stationary transition mechanism. There are two parts, the first concerned with discrete parameter Markov chains and the second with continuous parameter chains. The primary concern of the author is the recent development of a general theory of infinite state Markov chains. There is no concern with detailed analysis of very special types of chains such as the birth and death processes or reversible chains. Notes at the end of the different sections comment on the material presented and add a historical perspective.

As the author notes, many of the basic ideas and results in the literature on infinite state discrete parameter Markov chains are due to the pioneering work of Doeblin and Kolmogorov [see Doeblin's papers in Bull. Math. Soc. Roumaine Sci. **39** (1937), no. 1, 57–115; no. 2, 3–61; Bull. Soc. Math. France **66** (1938), 210–220; and Kolmogorov's papers in Math. Sb. (N.S.) **1** (1936), 607–610; Byull. Univ. Moskov. (A) **1** (1937), no. 3, 1–16]. Basic definitions and concepts are introduced in the first two sections of part one. The classification of states and a discussion of recurrence and nonrecurrence follow in sections 3 and 4. Much of the development and terminology here is that of Kolmogorov rather than Feller. Conditions for recurrence of states and illustrative examples are given in the next section. The main theorem of Kolmogorov on the limit behavior as $n\to\infty$ of the transition probabilities

$$P[x_n(w) = j | x_0(w) = i] = p_{ij}^{(n)}$$

and related results are presented in section 6. The proof follows that of Erdös, Feller and Pollard [Bull. Amer. Math. Soc. **52** (1949), 201–204; MR **10**, 367]. Renewal processes are discussed at some length. In section 9 the author introduces the concept of a taboo probability, the conditional probability ${}_H p_{ij}^{(n)}$ of going from i to j in n steps without passing through the set of states H in between. The asymptotic properties of these taboo probabilities are studied. Generating functions are considered as a tool useful in studying the limit behavior of transition probabilities. The existence or nonexistence of moments of first entrance time probabilities of a given order is shown to hold uniformly throughout a class of states (a maximal collection of mutually communicating states). System theorems are discussed for a Markov chain starting with an optional random variable. This is useful in discussing functionals of the chain. Ergodic theorems and further limit theorems like the central limit theorem and the law of the iterated logarithm occupy sections 15 and 16. The first part of the book concludes with a brief discussion of some of the recent results on transient Markov chains.

Part two is largely based on the work of Doob [Trans. Amer. Math. Soc. **58** (1945), 455–473; **63** (1948), 422–438; MR **7**, 210; **9**, 598], Lévy [Ann. Sci. École Norm. Sup. (3) **68** (1951), 327–381; **69** (1952), 203–212; MR **13**, 959; **14**, 663] and the author. A brief discussion of various sections in this second part follows. The first three sections deal with regularity properties of the transition probability matrices $P(t)=(p_{ij}(t))$, $0\le t<\infty$. It is shown that measurability of the transition probability matrix elements implies continuity of the elements in $(0,\infty)$. A result of Ornstein stating that measurability of the transition matrix implies that each $p_{ij}(t)$ is either identically zero or never zero in $(0,\infty)$ is given. Standard transition matrices, that is, matrices for which $\lim_{t\to 0} p_{ij}(t)=\delta_{ij}$, are then discussed at length. Such transition matrices are automatically continuous in $[0,\infty)$. Chains with such matrices are of principal interest and this assumption is held to throughout the rest of the book. The differentiability properties of these matrices are examined. The derivative $p_{ii}'(0)=q_i$ is shown to always exist though it may be infinite. Further, $p_{ij}'(0)$, $i\neq j$, always exists and is finite. If q_j is finite, $p_{ij}(t)$ has a continuous derivative in $[0,\infty)$ for every i and $p_{ij}'(t+s)=\sum_k p_{ik}(t)p_{kj}'(s)$ is satisfied for $t\ge 0$, $s>0$. Much of the remainder of the book is concerned with the discussion of regularity properties of the sample functions of the Markov chain. Definitions and measure-theoretic foundations are given in section 4 in preparation for this discussion. The concept of separability of a chain is introduced and some of its consequences are investigated. In section 5 the sets of constancy of the sample functions of the chain are examined. The minimal state space of the chain is assumed to be discrete and it is compactified by adjoining the fictitious state ∞. The states are divided into two classes, the stable states i for which $q_i<\infty$ and the instantaneous states for which $q_i=\infty$.

Under the assumption of separability it is shown that for almost all sample functions w, the closure of the set $S_i(w)=\{t|x_t(w)=i\}$ for each stable state i is the union of disjoint closed intervals whose number is finite in every finite t-interval. The local regularity properties of the sample functions are examined in the next section. Let $t\geq 0$ be fixed. Then for almost all sample functions it is shown that (i) if $x_t(w)=i$ with i stable, then $x_s(w)\to i$ as $s\to t$; (ii) if $x_t(w)=i$ with i instantaneous, $x_s(w)$ has precisely the two limit values i and ∞ as $s\downarrow t$ or $s\uparrow t$. Even under the conditions of both separability and measurability the sample functions can be taken as lower semicontinuous, lower right or lower left semicontinuous. Consequences of these possible specifications are considered in section 7. Optional random variables and the strong Markov property are treated in sections 8 and 9 respectively. The discussion of these concepts is much more delicate in the continuous parameter case than in the discrete parameter case. However, the concepts are required in the author's development of results in the continuous parameter case which parallel those already obtained in the discrete parameter situation. The classification of states, taboo probability functions and ratio limit theorems are dealt with in the next three sections. Section 13 considers discrete approximations $\{x_{n\delta},\ n=0, 1, 2, \cdots\}$ to the continuous chain $\{x_t,\ t\geq 0\}$. In section 14 analogues of the results in part one on functionals are obtained. The post-exit process, that is, the process starting with departure from a stable state, is examined in section 15. The next section deals with the transition to a stable state. Conditions under which the backwards or forwards differential equations (the Kolmogorov differential equations) are satisfied are considered in terms of the infinitesimal coefficients q_{ij} in section 17. The minimal solution satisfying both sets of equations is constructed in section 18. Section 19 is concerned with the first infinity (explosion) of the process. Examples illustrating the content of the results are discussed in section 20. The book closes with addenda on last exit times and transition probability functions.

The monograph is a detailed and careful exposition. It will be invaluable to everyone who wishes to do research on infinite state Markov chains. The current state of knowledge in the field is well represented and remarks are made on many of the problems that still remain open. One is inclined to agree with the author that it is unfortunate that intuitively plausible analogues of many of the results for discrete time parameter chains seem to require such elaborate machinery when extended to continuous time parameter chains. *M. Rosenblatt* (Providence, R.I.)

7177:

Kendall, David G. Unitary dilations of Markov transition operators, and the corresponding integral representations for transition-probability matrices. Probability and statistics: The Harald Cramér volume (edited by Ulf Grenander), pp. 139–161. Almqvist & Wiksell, Stockholm; John Wiley & Sons, New York; 1959. 434 pp. $12.50.

Let (p_{jk}) be the transition matrix of a discrete parameter homogeneous Markov chain which is irreducible, i.e., all states form one (essential) class (of mutually communicating states). A set of positive finite numbers $\{m_j\}$ satisfying $\sum_a m_a p_{aj}\leq m_j$ for all j is called a positive subinvariant measure (psim). It is shown that at least one psim exists. Consider the complex Hilbert space H with the typical element $x=\{x_1, x_2, \cdots\}$ and let T be a linear transformation such that $(Tx)_k=\sum_a x_a(m_a/m_k)^{1/2}p_{ak}$, where $\{m_j\}$ is a fixed psim. Then T does not increase the l_2-norm, i.e., it is a contraction. A theorem of Sz.-Nagy states that H is imbeddable in a Hilbert space H^+ so that $T^n x=JU^n x$ for all x in H and $n\geq 0$, where U is a unitary operator in H^+ and J is the projection from H^+ onto H. U is the unitary dilation of T. A known theorem on spectral resolution in Hilbert space then yields the representation:

$$p_{jk}^{(n)} = (m_k/m_j)^{1/2}\oint e^{in\theta}\mu_{jk}(d\theta),$$

where μ_{jk} is a complex-valued Borel measure supported by the unit circumference and $\mu_{jk}=\bar{\mu}_{kj}$. Using the known limit theorems concerning the asymptotic behavior of $p_{jk}^{(n)}$ it is shown that apart from atoms at the dth roots of unity (d=period of the chain) the measure μ_{jk} is absolutely continuous and that μ_{jj} does not depend on the choice of the psim. The operator is self-adjoint if and only if $m_j p_{jk}=m_k p_{kj}$ for every j and k. There exists such a psim if and only if the chain is reversible in the sense of Kolmogorov. The representation is extended to an inessential class by adjoining a new state. Finally, the state j is called geometrically ergodic if and only if $p_{jj}^{(n)}$ converges to its limit (supposed to exist) geometrically fast. It is shown that this property holds for all or none of the states in an aperiodic class. Such properties, now called "class properties", have been studied by the reviewer [#7176 above]. *K. L. Chung* (Syracuse, N.Y.)

7178:

Kendall, David G. Unitary dilations of one-parameter semigroups of Markov transition operators, and the corresponding integral representations for Markov processes with a countable infinity of states. Proc. London Math. Soc. (3) **9** (1959), 417–431.

This is an extension of the paper reviewed above to the continuous parameter case, based on the same ideas. The existence of a psim satisfying $\sum_a m_a p_{ak}(t)\leq m_k$ for all k has also been proved more explicitly in the reviewer's book [#7176], p. 202. The case of the Feller (or minimal) process corresponding to a given conservative Q-matrix (q_{jk}) is specially discussed. The results regarding irreducibility and reversibility of the minimal process, and the characterization of the psim's for the latter may be summed up in the following one proved in the paper: $\{m_j\}$ is a psim for the minimal process if and only if $\{m_j q_j\}$ is a psim for the associated jump process (see p. 236 of the cited book) with one-step transition matrix $\{r_{jk}\}$, where $r_{jk}=(1-\delta_{jk})q_{jk}/q_j$. For, as far as the transition of the chain is concerned without regard to the lapse of time, the minimal process and the jump process may be "confounded". This remark goes back to P. Lévy, and can be amply justified by a special case of the strong Markov property. *K. L. Chung* (Syracuse, N.Y.)

7179:

Haight, F. A. Queueing with reneging. Metrika **2** (1959), 186–197. (German summary)

Reneging is a refusal to wait exercised at any point of a waiting period and not solely on arrival as in "balking". For the simplest single server system (exponential distribution functions for inter-arrival and service time intervals), the author first examines the formulation of a

reneging decision function, based solely on $n(t)$, the number of departures at time t after a given arrival. The result arrived at is of sequential nature, too elaborate for a detailed résumé. Next, a utility function for joining the queue is formulated for a known distribution function of reneging. Finally, stationary state probabilities are found for a queue with both balking and reneging.

J. Riordan (New York)

7180:

Finch, P. D. On the transient behaviour of a simple queue. J. Roy. Statist. Soc. Ser. B **22** (1960), 277–284.

The author considers a single server queue with Poisson arrivals and general distribution of service times. He is particularly interested in the size of the queue upon completion of service to individual customers. A set of rather complex generating functions are developed which offer a means for solution for any particular service time distribution. For exponential service, the explicit generating functions and, from these, the desired probabilities are determined. The author also obtains for the general service time distribution case the limiting probabilities (i.e., steady-state rather than transient conditions) that j (≥ 1) customers remain in the queue at the completion of each service.

H. M. Gurk (Princeton, N.J.)

STATISTICS

See also 7154, 7179, 7278.

7181:

Neyman, Jerzy. Indeterminism in science and new demands on statisticians. J. Amer. Statist. Assoc. **55** (1960), 625–639.

Historically, and by examples, the author brings out the increasingly important and complex role of stochastic models in science and the challenge such models pose for statistical inference.

L. J. Savage (Ann Arbor, Mich.)

7182:

Good, I. J. Weight of evidence, corroboration, explanatory power, information and the utility of experiments. J. Roy. Statist. Soc. Ser. B **22** (1960), 319–331.

The paper sharpens some work of Popper [*The logic of scientific discovery*, Hutchinson, London, 1959; MR **21** #6318]. Conditions that corroboration and explanatory power of a hypothesis should satisfy are formulated; it is then shewn that essentially there is only one explicatum for each of these, and that they involve the author's concepts of 'information' and 'weight of evidence'. It is suggested that when utilities are not available it may be useful to base the choice of experiments on the expectation of the corroboration.

D. V. Lindley (Aberystwyth)

7183:

Fisher, R. A. On some extensions of Bayesian inference proposed by Mr Lindley. J. Roy. Statist. Soc. Ser. B **22** (1960), 299–301.

It is remarked that if the joint fiducial distribution of the mean and variance from a random sample from a normal distribution is used as the prior distribution for a second independent sample from the same distribution, the resulting posterior distribution, obtained using Bayes' theorem, is the same as if the fiducial argument had been applied to the combined sample.

D. V. Lindley (Aberystwyth)

7184:

Savage, Leonard J. Recent tendencies in the foundations of statistics. Proc. Internat. Congress Math. 1958, pp. 540–544. Cambridge Univ. Press, New York, 1960.

A very brief lucid expository lecture, containing some historical material, on the relevance to statistics of theories of utility and of personal (=subjective) probability.

I. J. Good (Teddington)

7185:

Churchman, C. West; Ratcosh, Philburn. (Editors). ★Measurement: Definitions and theories. John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London; 1959. vii+274 pp. $7.95.

This book reproduces papers delivered at a symposium in December, 1956. The editors point out in the preface that this "is a book of contrasts. This volume does not constitute a series of connected discourses on measurement. It is not a textbook on the subject of measurement. But it may present as good a picture as possible of what the workers in the foundations of measurement are concerned about in the middle of the twentieth century."

In view of the difficulty in summarizing thirteen articles each on a different aspect of the subject, only the titles of the articles and the authors are given here. Peter Caws: Definition and measurement in physics; S. S. Stevens: Measurement, psychophysics, and utility; Paul Kircher: Measurements and managerial decisions; C. West Churchman: Why measure?; Karl Menger: Mensuration and other mathematical connections of observable material; Patrick Suppes: Measurement, empirical meaningfulness, and three-valued logic; R. Duncan Luce: A probabilistic theory of utility and its relationship to Fechnerian scaling; Henry Margenau: Philosophical problems concerning the meaning of measurement in physics; Arthur Pap: Are physical magnitudes operationally definable?; John L. McKnight: The quantum theoretical concept of measurement; E. J. Gumbel: Measurement of rare events; Clyde H. Coombs: Inconsistency of preferences as a measure of psychological distance; Donald Davidson and Jacob Marschak: Experimental tests of a stochastic decision theory.

It is fortunate that Galileo did not see the difficulties in the concept of measurement pointed out in the above articles, or modern science would never have gotten started.

M. Kline (New York)

7186:

Salvemini, Tommaso. Distribution of the mean difference taken from an equally distributed mass. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 415–417. (French summary)

7187:

Hostelet, G. Le concours des mathématiques dans les mesures des grandeurs caractéristiques des faits statistiques et dans leur interprétation. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 375–384. (English summary)

7188:

Linnik, Yu. V. On polynomial statistics in connection with the analytical theory of differential equations. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 191–206. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Vestnik Leningrad. Univ. **11** (1956), no. 1, 35–48 [MR **17**, 983].

7189:

Ito, Koichi. Asymptotic formulae for the distribution of Hotelling's generalized T_0^2 statistic. II. Ann. Math. Statist. **31** (1960), 1148–1153.

Using characteristic functions the author derives an asymptotic formula for the cumulative distribution function of Hotelling's generalized T_0^2 statistic for general values of the number p of dimensions in the non-null case ($\xi \neq 0$). This result includes as a special case the previous result of the author [same Ann., **27** (1956), 1091–1105; MR **18**, 958] for the null distribution of T_0^2, and shows certain properties of T_0^2 which help determine the power of the test based on the statistic for moderately large samples. Both of the author's results provide an approximate complete analysis of the T_0^2 test, although the exact null and non-null distributions of T_0^2 are not known at present.

H. P. Edmundson (Pacific Palisades, Calif.)

7190:

Linnik, Yu. V. Linear statistics and the normal law. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 59–61. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl. Akad. Nauk SSSR **83** (1952), 353–355 [MR **14**, 60].

7191:

Gumbel, E. J. Bivariate exponential distributions. J. Amer. Statist. Assoc. **55** (1960), 698–707.

Properties of two bivariate distribution functions, each with marginal exponential density functions, are examined in detail and contracted with each other and with the bivariate normal. The functions are

$$F(x, y) = 1 - e^{-x} - e^{-y} + e^{-x-y-\alpha xy},$$
$$F(x, y) = (1 - e^{-x})(1 - e^{-y})(1 + \alpha e^{-x-y}).$$

In both, the variables are non-negative, $0 \leq \delta \leq 1$, $-1 \leq \alpha \leq 1$. It is hoped that the discussion may serve as a "warning against the indiscriminate use of normal correlation and regression analysis". *J. Riordan* (New York)

7192:

Crawford, J. R.; Walsh, John E. Empirical examination of Edgeworth series. Ann. Inst. Statist. Math. Tokyo **12** (1960), 13–26.

The authors examine five types of probability approximations, which involve the first seven terms of the Edgeworth series expansion for the distribution of a continuous random variable T. For each approximation the probability expressions considered in the investigation are $\Pr(T \leq t)$, $\Pr(-t \leq T \leq t)$ and $\Pr(-t+1 \leq T \leq t)$, where T has zero mean, unit variance and specified central moments μ_3, μ_4, μ_5.

Tables are given for meaningful limits for the three types of probability expressions.

The results indicate that the types of approximations investigated are of doubtful usefulness for cases where the third and higher order moments of T differ substantially from those for the normal variable having the same mean and variance. *Hian Liang Ang* (Bandung)

7193:

Freimer, Marshall; Gold, Bernard. Note on the distribution of locally maximal elements in a random sample. Ann. Math. Statist. **31** (1960), 518.

7194:

Philipson, Carl. A note on linear transforms of two sets of random variables. Skand. Aktuarietidskr. **1959**, 132–143 (1960).

The author studies the asymptotic behavior of the distribution and cumulants of $Y = \sum_{i=0}^{n} h_i(v_{i+1} - v_i)/(n+1)$, where $0 = v_0 \leq v_1 \leq \cdots \leq v_{n+1} = n+1$ are the order statistics of n independent random variables uniformly distributed on the interval 0 to $n+1$, and $h_0, h_1, \cdots, h_n$ are given numbers. His aim is to clarify a point in the work of G. Blom [*Statistical estimates and transformed beta-variables*, Almqvist & Wiksells, Stockholm, 1958; MR **20** #2055], not imposing the condition that $\sum_{i=0}^{n} h_i = 0$.

E. Parzen (Stanford, Calif.)

7195:

Dynkin, E. B. Necessary and sufficient statistics for a family of probability distributions. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 17–40. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Uspehi Mat. Nauk **6** (1951), no. 1 (41), 68–90 [MR **12**, 839].

7196:

Schmetterer, L. On unbiased estimation. Ann. Math. Statist. **31** (1960), 1154–1163.

Much of the theory of unbiased estimators appraised in terms of mean squared error is extended to appraisals in terms of mean pth power errors.

L. J. Savage (Ann Arbor, Mich.)

7197:

Roy, J.; Chakravarti, I. M. Estimating the mean of a finite population. Ann. Math. Statist. **31** (1960), 392–398.

This paper considers a general scheme of sampling from a finite population of N elements, which includes ordinary random sampling with replacement and also without replacement. Each population element has a real number assigned to it, and these are denoted $y_1, \cdots, y_N$. The population mean and variance are defined in the conventional way. Unbiased linear estimators are defined in a natural way. Such an estimator is called regular if the ratio of its variance over the population variance is a constant, identically in the y_i's, and is called linearly invariant if transforming each y_i to $ay_i + b$ subjects the estimator to the same transformation. It is proved that a regular estimator is necessarily linearly invariant. Considering variance as a measure of loss, a characterization is given of a complete class of estimators. A lower bound is given for the

variance of a regular estimator. Necessary and sufficient conditions for this lower bound to be achieved are given in terms of the type of sampling scheme.

M. Dwass (Evanston, Ill.)

7198:

Hájek, Jaroslav. On the theory of ratio estimates. Bull. Inst. Internat. Statist. **37** (1960), no. 2, 219–226. (French summary)

7199:

Jílek, Miloš; Líkař, Otakar. Tolerance limits of the normal distribution with known variance and unknown mean. Austral. J. Statist. **2** (1960), 78–83.

The author gives, together with extensive tables, an expository survey of the results obtained on the tolerance limits in the form $\bar{x} \pm k\sigma$, $\bar{x}$ being sample mean, σ^2 population variance and k a constant, for the case in which mean is unknown but variance σ^2 known in normal population.

C. Hayashi (Tokyo)

7200:

Sprott, D. A. Necessary restrictions for distributions a posteriori. J. Roy. Statist. Soc. Ser. B **22** (1960), 312–318.

The method whereby a fiducial distribution from one sample is used as a prior distribution in Bayes' theorem for a second sample is called Bayes' method. It is shewn that Bayes' method is only invariant under a change in order of the samples from a member of the exponential family of distributions depending on a single parameter when these distributions are either normal or gamma. These cases are separately investigated. The results generalize those of Lindley [same J. **20** (1958), 102–107; MR **20** #2052]. Condition (ii) of the paper would be clearer if it read "they must remain invariant under all permutations of the observations leaving the likelihood unchanged".

D. V. Lindley (Aberystwyth)

7201:

Bahadur, R. R. Stochastic comparison of tests. Ann. Math. Statist. **31** (1960), 276–295.

Given a sample space S of points s, let $\{T_n\}$ be a sequence of tests defined on S for testing a hypothesis H involving a parameter θ. Assume that large values of T_n are significant. If, under H, $\lim_{n\to\infty} P(T_n < x) = F(x)$, for any given sample point s, $L(s) = 1 - F(T_n(s))$ is called the level attained by T_n. Further, let $K_n(s) = -2 \log L(s)$.

Given two sequences of tests, $\{T_n^{(1)}\}$ and $\{T_n^{(2)}\}$, the arguments of the paper depend on the practical principle that if H does not hold, for a given s, test 2 is superior to test 1 if and only if $L^{(2)}(s) < L^{(1)}(s)$. $\varphi_{1,2}(\theta) = p \lim_{n\to\infty} K_n^{(1)}/K_n^{(2)}$ is called the asymptotic efficiency of sequence 1 relative to sequence 2. The paper discusses the relationship of the new concepts to methods of comparison based on power function considerations. In particular, it is shown that, subject to regularity conditions, $\lim_{\theta\to\theta_0} \varphi_{1,2}(\theta)$ is equal to the Pitman efficiency for the hypothesis $\theta = \theta_0$. Several examples illustrate the new concepts.

G. E. Noether (Boston, Mass.)

7202:

Dunnett, C. W. On selecting the largest of k normal population means. J. Roy. Statist. Soc. Ser. B **22** (1960), 1–40.

The author considers the problem of selecting the largest of k normal population means with a priori normal distributions of the means. It is assumed that the data come from a k-variate normal population with equal variances and equal covariances, and that the a priori distribution likewise has this property. The problem is immediately reduced to the case of independence. Bayes and minimax risk and minimax regret procedures are obtained for the choice of sample size for two loss functions—the loss a linear function of the deviation between the maximum and the selected mean, or the loss constant if the deviation exceeds a fixed quantity. The procedures obtained are compared with several others in the literature.

H. Rubin (E. Lansing, Mich.)

7203:

Colton, Theodore. A test procedure with a sample from a normal population when an upper bound to the standard deviation is known. J. Amer. Statist. Assoc. **55** (1960), 94–104.

The tests suggested use the upper bound to the standard deviation in place of the sample estimate. Tests are based on the normal distribution, when dealing with one population, and on the chi-square distribution, when dealing with several populations. It is shown that the power of the modified normal test can under certain conditions exceed the power of the t-test. The true first kind of error is smaller than that of the t-test. The analogue of the t-test in case of samples from more than one population is the F-test and is compared to a modified chi-square test. Tables are constructed, which for degrees of freedom, first kind and second kind of error give the range within which the upper bound for the standard deviation must lie if these tests are to be more powerful than the t-test resp. F-test.

J. Janko (Prague)

7204:

Törnqvist, Leo. Variances and covariances of least square estimates of regression coefficients. Arkhimedes **1959**, no. 1, 17–20. (Finnish)

An article in Finnish which pertains to the paper in Skand. Aktuarietidskr. **1957**, 219–226 [MR **20** #4906] by the same author.

O. Lehto (Helsinki)

7205:

Witting, H. A generalized Pitman efficiency for non-parametric tests. Ann. Math. Statist. **31** (1960), 405–414.

Edgeworth expansions of the distribution functions of the test statistics are made which allow matching of power functions (computing Pitman efficiency) up to terms in the inverse squares of the sample sizes. Primary emphasis is on the Wilcoxon two-sample test compared to the t-test. The Edgeworth expansion is intuitively reasonable and gives remarkably accurate results when comparisons to exact values for normal alternatives (very small samples) and rectangular alternatives are made.

I. R. Savage (Cambridge, Mass.)

7206:

Gnedenko, B. V.; Korolyuk, V. S. On the maximum discrepancy between two empirical distributions. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 13–16.

Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl. Akad. Nauk SSSR **80** (1951), 525–528 [MR **13**, 570].

7207:

Gnedenko, B. V.; Mihalevič, V. S. Two theorems on the behavior of empirical distribution functions. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 55–57. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl. Akad. Nauk SSSR **75** (1952), 25–27 [MR **14**, 60].

7208:

Mihalevič, V. S. On the mutual disposition of two empirical distribution functions. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 63–67. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl Akad. Nauk SSSR **85** (1952), 485–488 [MR **14**, 297].

7209:

Gnedenko, B. V.; Rvačeva, E. L. On a problem of the comparison of two empirical distributions. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 69–72. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl. Akad. Nauk SSSR **82** (1952), 513–516 [MR **13**, 760].

7210:

Gnedenko, B. V.; Mihalevič, V. S. On the distribution of the number of excesses of one empirical distribution function over another. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 83–85. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl. Akad. Nauk SSSR **82** (1952), 841–843 [MR **13**, 760].

7211:

Gihman, I. I. On the empirical distribution function in the case of grouping of the data. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 77–81. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl. Akad. Nauk SSSR **82** (1952), 837–840 [MR **13**, 666].

7212:

Gnedenko, B. V. Some results on the maximum discrepancy between two empirical distributions. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 73–75. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Dokl. Akad. Nauk SSSR **82** (1952), 661–663 [MR **13**, 760].

7213:

Cohn, Richard; Mosteller, Frederick; Pratt, John W.; Tatsuoka, Maurice. Maximizing the probability that adjacent order statistics of samples from several populations form overlapping intervals. Ann. Math. Statist. **31** (1960), 1095–1104.

From the authors' summary: "Let samples of size n be drawn from each of k univariate continuous cumulative distribution functions on the same real line, and consider the intersection of the k intervals between the rth and $(r+1)$st order statistics in the several samples. Then, to maximize the probability that that intersection be nonempty the distributions should be identical. Furthermore, for each sample, consider two intervals, that between the rth and $(r+1)$st and that between the sth and $(s+1)$st order statistics; then to maximize the probability that both the intersection of the "r" intervals and the intersection of the "s" intervals be nonempty, the distributions again should be identical and the value of the maximum probability is

$$\frac{\binom{n}{r}^k\binom{n-r}{s-r}^k}{\binom{kn}{kr}\binom{k[n-r]}{k[s-r]}}."$$

I. R. Savage (Cambridge, Mass.)

7214:

Bradley, R. A.; Pendergrass, R. N. Ranking in triple comparisons. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 229–241. (French summary)

An experiment with t treatments is to be run with n replications of balanced incomplete blocks of size 3. If treatments i, j, k, are used in a block, the observable will be the ranking $x_{r_1} > x_{r_2} > x_{r_3}$ where r_1, r_2, r_3 is a permutation of i, j, k. Under the discussed model:

$$P(x_{r_1} > x_{r_2} > x_{r_3}) =$$
$$\pi_{r_1}{}^2\pi_{r_2}/[\pi_{r_1}{}^2(\pi_{r_2}+\pi_{r_3})+\pi_{r_2}{}^2(\pi_{r_1}+\pi_{r_3})+\pi_{r_3}{}^2(\pi_{r_1}+\pi_{r_2})]$$

where $\pi_i \geq 0$ and $\sum \pi_i = 1$. For this model methods for computing the maximum likelihood estimates of the π_i, the large sample (n) distribution of these estimates, likelihood ratio tests of all $\pi_i = t^{-1}$ versus some $\pi_i \neq t^{-1}$, likelihood ratio tests of the model, and an example are given.

I. R. Savage (Cambridge, Mass.)

7215:

Bell, C. B.; Blackwell, David; Breiman, Leo. On the completeness of order statistics. Ann. Math. Statist. **31** (1960), 794–797.

Roughly put, a statistic T is called complete with respect to a class of probability laws, if $E_P h(T) = 0$ for all P in that class implies that $h(T) = 0$ except for P sets of measure 0 for all P. This paper considers the case of T the set of order statistics, that is, the ordered values of independent and identically distributed random variables each with distribution F. P in this case is the product measure induced by F. A class of probability distributions with respect to which the order statistics T is complete is called symmetrically complete. When the class of probability distributions is determined by letting F range over all continuous cumulative distribution functions, its symmetric completeness is known. This paper gives a new proof of this result, whose ideas are then extended from the real

line to a more general measure-theoretic version in which the role of F is replaced by an arbitrary nonatomic measure. *M. Dwass* (Evanston, Ill.)

7216:

Hill, Bruce M. A relationship between Hodges' bivariate sign test and a non-parametric test of Daniels. Ann. Math. Statist. **31** (1960), 1190–1192.

This paper proves that the null distribution of a statistic proposed by Hodges [same Ann. **26** (1955), 523–527; MR **17**, 56] for a bivariate sign test is the same as the null distribution of a statistic proposed by Daniels [ibid. **25** (1954) 499–513; MR **16**, 273] for a distribution-free test for regression parameters. *M. Dwass* (Evanston, Ill.)

7217:

Scheffé, Henry. ★The analysis of variance. John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London; 1959. xvi+477 pp. $14.00.

One can hardly say succinctly just what the analysis of variance is. But there is unquestionably a nexus of topics, each connected to some of the others, that goes by this name and covers a large portion of the theory of statistics that is of frequent applicability. Scheffé's interpretation of the term will be conveyed, and other ends well served, by giving the titles of the two "parts" and the ten chapters of the book which are in turn divided into sections.

Part I. The analysis of variance in the case of models with fixed effects and independent observations of equal variance: Point estimation; Construction of confidence ellipsoids and tests in the general case under normal theory; The one-way layout. Multiple comparison; The complete two-, three-, and higher-way layouts. Partitioning a sum of squares; Some incomplete layouts: Latin squares, incomplete blocks, and nested designs; The analysis of covariance.

Part II. The analysis of variance in the case of other models: Random-effects models; Mixed models; Randomization models; The effects of departures from the underlying assumptions.

The coverage is unusually complete and penetrating, and for many such minor topics as are not treated in detail here, there are adequate references to the literature. There are topics to be found here that are in no other book, and of course there are some altogether new developments. For example, the comprehensive treatment of simultaneous confidence intervals advances the subject beyond the previous literature. Again, derivation, as opposed to mere exemplification, of the mathematics of the analysis of covariance seems not to have been published heretofore. Two topics that have been overlooked are the analysis of designs slightly unbalanced by the omission of a few observations (or by the inclusion of a few extra ones) and the treatment of split plots. The book is particularly well provided with tables, charts, and exercises, and the bibliography is very valuable.

The author's theoretical viewpoint, like that of practically all modern literature on statistics to date, tends strongly to be objectivistic. For example, he comes out in favor of retaining certain unbiased estimates of variances, though these are capable of assuming negative values (page 229). Yet, he takes steps toward the more reasonable and progressive Bayesian position, for example, in footnote 4, page 150. The conflicting attitudes are illustrated by two contrasting footnotes (pp. 41 and 65) about an ingenious, but now unreasonable, two-stage procedure. Throughout the book, theory is mingled with insightful practical comments and sidelights. Whatever new developments may be in store for statistical theory, the lasting value of this book as a whole is indubitable.

While occasionally avoiding certain technical refinements of little practical importance, the many mathematical passages of the book are general, complete and rigorous. But some of the mathematical arguments are unnecessarily long and difficult to follow, for want of systematic application of a few simple geometric ideas and images. The mathematical tone of the whole is to some extent set by appendices that provide a short course on vectors and matrices. These seem to the reviewer to emphasize mere formalism at the expense of both geometry and algebra. Symptomatic is the introduction of the diagonal form of a symmetric operator in Euclidean space, which the reviewer believes finds no essential use throughout the book.

There are very few errors and relatively few misprints. The "only if" part of Exercise 4.2 on page 145 is somewhat too narrow. A putative short cut (in the middle of page 417) seems to be based on the misapprehension that it is more difficult to solve one n-dimensional system of linear equations than to evaluate two $n \times n$ determinants.

No book that covers such a great variety of technical ideas as this one can be really light reading. But, as has already been implied, a different choice of mathematical methods might have helped. Also, in spite of evident thought and ingenuity, the technical notations remain dazzling and hard to remember; some eye strain might have been relieved by little things like omitting chapter and section numbers from the individual equations, for these already appear at the top of each page. Finally, the great care that has evidently been expended on keeping the book elementary and sufficiently self-contained for self study has made it somewhat slow moving for advanced readers.

Whatever may be said in criticism is overshadowed by the merits of this book which is unique in its field and will be indispensible to all who seriously do, study, or teach, statistics in connection with experimentation.

L. J. Savage (Ann Arbor, Mich.)

7218:

Feinleib, Manning. A method of analyzing log-normally distributed survival data with incomplete follow-up. J. Amer. Statist. Assoc. **55** (1960), 534–545.

The author gives a method of estimation of three parameters in log-normal distribution by using survival data with incomplete follow-up (all elements of sample are not dead). Let $t=\{\log(x-a)/b\}/c$ be normally distributed, x being the duration of survival measured from an initial point, a, b, and c, being three parameters. Hitherto, the method of estimation by using a straight line drawn on log-normal graph paper and the maximum likelihood method of estimation for complete survival data have been discussed. In this paper the author uses life table method by grouping the incomplete data, i.e., calculating d_i, the number of deaths that occur during the ith time interval, and gives concretely the method of estimation (he calls modified maximum likelihood method) of three parameters by substituting y_i^* with d_i instead of x into the

equations with respect to x obtained by the maximum likelihood mentioned above, where y_i^* is given by $t_i^* = \{\log(y_i^* - a)/b\}/c$ and $t_i^* = \frac{1}{2}(t_i + t_{i+1})$ except the first $(i=1)$ and the last $(i=p)$ points which are estimated straightforwardly by the definitions, and where t_i, t_{i+1} are the end points determining the ith interval. As an example a comparison of several methods is given and the author's method is shown to be the best. But, here, the variances of the estimates and the best way of grouping the data are not described. *C. Hayashi* (Tokyo)

7219:

Box, George E. P. Fitting empirical data. Ann. New York Acad. Sci. **86**, 792–816 (1960).

A mature exposition of an important branch of statistics, to which the author has made great contributions. One feature of particular interest is practical discussion of genuinely nonlinear fitting problems and their solution with the help of tact and a special, publicly available, IBM-704 program. Another is insightful comments on the role of prior distributions in statistics.

L. J. Savage (Ann Arbor, Mich.)

7220:

Cornfield, Jerome; Steinfeld, Jesse; Greenhouse, Samuel W. Models for the interpretation of experiments using tracer compounds. Biometrics **16** (1960), 212–234.

In the authors' words: "It is the purpose of the present paper to (a) sketch a kind of theoretical model required for estimation [in experiments involving the use of radioactive tracers], paying particular attention to the biological assumptions, (b) to illustrate the essential dependence of the value of an apparently simple and well-defined magnitude on the model assumed and (c) to consider some statistical issues involved in the application of the models to theoretical data." The authors are led to a system of simultaneous linear differential equations with constant coefficients which in turn yield linear combinations of exponentials to be fitted to observed data. The paper gives many penetrating observations on the difficulties of fitting sums of exponentials. Perhaps the most important sentence in the article is: "It has been an interesting, and, for those engaged in the biological sciences, a somewhat novel experience to discover that the realization of this program requires the formulation of explicit quantitative models of the phenomena being studied."

C. J. Maloney (Frederick, Md.)

7221:

Eisenhart, Churchill. Some canons of sound experimentation. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 339–350. (French summary)

7222:

Taguchi, Genichi. Linear graphs for orthogonal arrays and their applications to experimental designs with the aid of various techniques. Rep. Statist. Appl. Res. Un. Jap. Sci. Engrs. **6** (1959), 133–175.

A graphical representation for experimental designs is provided by assigning a point or node to each main effect, the segment joining these nodes representing their first order interaction whenever the design is such that interaction is unconfounded. The extension to higher level designs is given. The graphs may serve in design in a fashion analogous to structural formulas in chemistry. A list of twelve techniques for adapting orthogonal designs to actual experimental requirements are briefly sketched.

C. J. Maloney (Frederick, Md.)

7223:

Tukey, John W. Conclusions vs decisions. Technometrics **2** (1960), 423–433.

An after-dinner speech in which the author argues that decision theory should not be allowed to conquer the whole of statistics, and that it needs to be supplemented by a 'conclusion theory'. If by 'decision theory' is meant a theory in which an act must be decided upon in advance, corresponding to all future contingencies, then the reviewer would also resist the conquest. On the other hand, a conclusion seems to be a particular kind of decision, namely a decision to accept a hypothesis and to risk acting on it for the time being. Therefore any theory of rational conclusions could be regarded as a part of a theory of rational decisions. The essential difficulty is that the judgments of the utilities of hypotheses are often 'wide', so that the use of these utilities in coming to conclusions tends to be only implicit.

A conclusion is a decision to adopt a rule with the help of which the rationality of other decisions (including, in particular, some conclusions) may be inferred. The distinction which the author wishes to draw between conclusion theory and decision theory may perhaps be logically justified if, by convention, the latter is restricted to those decisions which are not conclusions.

I. J. Good (Teddington)

7224:

Machol, Robert E. (Editor). ★Information and decision processes. McGraw-Hill Book Co., Inc., New York-Toronto-London, 1960. xi+185 pp. $5.95.

A Symposium on Information and Decision Processes was held at Purdue University in August 1959. This book is an outgrowth of that conference, in that each of the ten speakers has contributed a chapter; some of these chapters are identical to the conference presentations, while in other cases the authors have chosen to modify their papers. In addition, two papers from the 1958 Purdue Conference on the same subject are included.

All these papers will be reviewed separately in these pages.

7225:

Flood, Merrill M. Sequential decisioning. Information and decision processes, pp. 34–52. McGraw-Hill, New York, 1960.

Expository paper. *R. D. Luce* (Philadelphia, Pa.)

7226:

Anscombe, F. J. Notes on sequential sampling plans. J. Roy. Statist. Soc. Ser. A **123** (1960), 297–306.

In an earlier paper [same J. **112** (1949), 180–206; MR **11**, 449] the author gave tables on the operating characteristics and average sample size of various sequential sampling plans to control fraction defective. These tables were calculated using the Poisson approximation to the binomial distribution. Subsequently Dvoretzky, Kiefer

and Wolfowitz [Ann. Math. Statist **24** (1953), 254–264; MR **14**, 997] and Kiefer and Wolfowitz [Naval Res. Logist. Quart. **3** (1956), 205–219; MR **18**, 833] studied and gave tabulations for sequential tests of the mean of a Poisson process. In the present paper these tables are compared and extended. The author also notes an improved approximation of the Poisson distribution function.

D. G. Chapman (Seattle, Wash.)

7227:

Jiřina, M. Sequential estimation of distribution-free tolerance limits. Select. Transl. Math. Statist. and Probability, Vol. 1, pp. 145–155. Inst. Math. Statist. and Amer. Math. Soc., Providence, R.I., 1961.

Translation of Czechoslovak Math. J. **2 (77)** (1952), 221–232 [MR **15**, 637].

7228:

Вальд, А. [Wald, Abraham]. ★Последовательный анализ. [Sequential analysis]. Translated from the English by P. A. Bakut, B. M. Gerasimov, I. N. Kuznecov, A. A. Kurikša; edited by B. A. Sevast'yanov. Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1960. 328 pp. 10.15 r.

The original [Wiley, New York, 1947] was reviewed in MR 8, 593. Appended to the new edition as supplements are translations into Russian of the author's papers in Proc. 2nd Berkeley Symposium Math. Statist. and Probability (1950), pp. 1–11, Univ. of California Press, Berkeley, Calif., 1951; Ann. Math. Statist. **17** (1946), 466–474, 493–497; **19** (1948), 326–339 (with J. Wolfowitz); and Proc. Internat. Congr. Math. 1950, vol. 1, pp. 231–243, Amer. Math. Soc., Providence, R.I., 1952 [MR **13**, 367; **8**, 283, 284; **10**, 201; **13**, 480].

7229:

Anderson, T. W. A modification of the sequential probability ratio test to reduce the sample size. Ann. Math. Statist. **31** (1960), 165–197.

$X_1, X_2, \cdots$ are independent, identically distributed random variables, each with a normal distribution with known variance and unknown mean θ. The problem is to test the simple hypothesis that $\theta = -\mu$ against the simple alternative that $\theta = \mu$ with preassigned probabilities of error, where μ is a preassigned positive value. The Wald sequential probability ratio test takes m observations, where m is the smallest positive integer for which it is not true that $b < X_1 + X_2 + \cdots + X_m < a$, b, a being preassigned constants, chosen to give the desired probabilities of error. If $X_1 + X_2 + \cdots + X_m \leq b$, the hypothesis that $\theta = -\mu$ is accepted, if $X_1 + X_2 + \cdots + X_m \geq a$, the hypothesis that $\theta = \mu$ is accepted.

The author investigates a more general class of tests, where at the nth stage of sampling, b is replaced by $c_1 + d_1 n$ and a is replaced by $c_2 + d_2 n$ if $n < N$, and at the Nth stage both b and a are replaced by k. Here c_1, c_2, d_1, d_2, N, k are preassigned constants, and N may be infinite. The author approximates the probabilities of error and the expected sample size of such tests by replacing the sequence $X_1 + X_2 + \cdots + X_m$ $(m = 1, 2, \cdots)$ by a Gaussian stochastic process $X(t)$ with continuous time parameter t, and finding exact expressions for the probabilities of error and the expected sampling time for this continuous time parameter case. It is shown that by an appropriate choice of c_1, c_2, d_1, d_2, N, k, the resulting test substantially reduces the maximum expected number of observations (the maximum being taken with respect to θ) compared to the Wald test with the same probabilities of error.

L. Weiss (Ithaca, N.Y.)

7230a:

Vogel, Walter. A sequential design for the two armed bandit. Ann. Math. Statist. **31** (1960), 430–443.

7230b:

Vogel, Walter. An asymptotic minimax theorem for the two armed bandit problem. Ann. Math. Statist. **31** (1960), 444–451.

These papers concern symmetric strategies in the n-step two armed bandit problem for large n. Nature chooses two probabilities $\sigma_n > \tau_n$, and the experimenter goes through the n-step problem with probabilities (σ_n, τ_n) or (with equal likelihood) (τ_n, σ_n), using an arbitrary strategy s_n. The loss L_n is the difference between the expected score $n\sigma_n$ with clairvoyance and the expected score with s_n.

There exist sequences of strategies s_n making $L_n = O(n^{1/2})$, against any strategies of nature; in fact, one can secure $\limsup (L_n/n^{1/2}) \leq c$, with $c \sim 0.376$. For certain strategies of nature the experimenter has no strategies making $L_n = o(n^{1/2})$; indeed, nature can secure $\liminf (L_n/n^{1/2}) \geq c'$, with $c' \sim 0.1876$.

The good strategies s_n are developed in the first paper, with much more detail on their behavior against certain strategies of nature for moderate n.

J. R. Isbell (Seattle, Wash.)

7231:

de Ghellinck, Guy. Les problèmes de décisions séquentielles. Cahiers Centre Études Rech. Oper. **2**, 161–179 (1960).

Howard [*Notes on operations research*, Technology Press, Cambridge, Mass., 1959; MR **22** #626; Chapt. 7] has presented an algorithm to obtain in a finite number of steps the optimum decision rule associated with a sequential decision process. In this paper the author shows that each step of Howard's process corresponds to a vertex of the convex polyhedron determined by the solution of a linear program. Moreover, the maximization criterion having the same structure as that underlying the change of base in the simplex method makes possible the attainment at any step of a new vertex, not necessarily adjacent to the vertex attained in the preceding step.

D. G. Chapman (Seattle, Wash.)

7232:

Chorafas, Dimitris N. ★Statistical processes and reliability engineering. D. Van Nostrand Co., Inc., Princeton, N.J.-Toronto-London-New York, 1960. xiv + 438 pp. $12.75.

The author superficially introduces reliability engineers to diverse topics from factor analysis to ergodic theorems.

I. R. Savage (Cambridge, Mass.)

7233:

Freudenthal, A. M. Prediction of fatigue life. J. Appl. Phys. **31** (1960), 2196–2198.

Dynamic fatigue results from the application of N

repetitions of a fixed stress with constant frequency, static fatigue from the use of a fixed stress (ultimate load) during time t. In both cases time is designated by x. The probability of survival $R(x)$ beyond age N in cycles or t in time is called the reliability function; the derivative of its logarithm is the risk function $r(x)$. In the usual assumption of an exponential (or logarithmic-normal) distribution the risk function is independent of x (or decreases with increasing x). This contradiction to the physical facts does not exist in the asymptotic distribution of smallest values. If the minimum life ε is zero the model life $\tilde{x}$ in the dynamic case is $\tilde{x}=\nu(1-1/\alpha)^{1/\alpha}$, where ν, called the characteristic life, is such that $R(x)=1/e$. If the dynamic risk is combined with that of an ultimate load with the return period T and $\varepsilon=0$ the risk function is

$$r(x) = 1/T+(\alpha/\nu)(\nu/x)^{\alpha-1}$$

where T has to be estimated separately. Minor misprints have been corrected in this review.

E. J. Gumbel (New York)

7234:

Epstein, Benjamin. Estimation from life test data. Technometrics **2** (1960), 447–454.

Author's summary: "In this paper four estimation procedures are discussed which are useful if one wishes to make point or confidence interval estimates from life test data. The first two procedures can be applied when the underlying density function of life is exponential. The third and fourth procedures are non-parametric. Each of the procedures is illustrated by means of numerical examples." *L. A. Aroian* (Los Angeles, Calif.)

7235:

Epstein, Benjamin. Statistical life test acceptance procedures. Technometrics **2** (1960), 435–446.

Author's summary: "This paper describes statistical methods for testing hypotheses about the mean of an exponential distribution of life. Advantage is taken of the time-ordered nature of life test data to shorten substantially the time required to reach a decision. Replacement, non-replacement, sequential, non-sequential, and truncated procedures are described. Some useful tables are given at the end of the paper."

Some excellent examples are included.

L. A. Aroian (Los Angeles, Calif.)

7236:

Thionet, P. L'ajustement des résultats des sondages sur ceux des dénombrements. Rev. Inst. Internat. Statist. **27** (1959), 8–25. (English summary)

This paper discusses a number of methods of "reweighting" the results of a census if one has a virtually complete enumeration of the population with respect to two characteristics. In particular, the author considers the estimation of the numbers N_{ij} of elements in each cell of a (grouped) bivariate population, given a sample and knowing both marginal distributions.

The author's novel contribution is the suggestion that, when estimating the overall average of some quantity $\bar{z}=\sum_i\sum_j N_{ij}\bar{z}_{ij}$, one might act as if the N_{ij} were such as to minimize the variance of $\bar{z}$, i.e., one might choose estimates $\hat{N}_{ij}$, having the correct marginal distributions, so as to minimize $\sum\sum a_{ij}\hat{N}_{ij}^2$, where a_{ij} is the variance of $\bar{z}_{ij}$, assumed known. *E. M. L. Beale* (Teddington)

NUMERICAL METHODS

See also A6968, 7185, 7498, 7844, 7891.

7237:

Laasonen, Pentti. Some present-day problems of numerical mathematics. Arkhimedes **1958**, no. 2, 36–40. (Finnish)

A brief expository account is given of the present state of the following topics of numerical analysis: solution of a linear equation system, relaxation methods, truncation error of discrete approximations to the solution of a Dirichlet problem, eigenvalues of matrices, and sequence transformations. Special consideration is devoted to the suitability of the described methods to electronic computers. *O. Lehto* (Helsinki)

7238:

Lehmer, D. H. Discrete variable methods in numerical analysis. Proc. Internat. Congress Math. 1958, pp. 545–552. Cambridge Univ. Press, New York, 1960.

The author gives a short discussion of the deliberate use of discrete variables on a digital computer as contrasted with the more common view of regarding the quantities in the computer as approximations to real numbers. He describes in detail a program which resulted in a disproof of a conjecture of Hardy and Littlewood that if

$$1 \geqq a_1 \geqq a_2 \geqq \cdots \geqq 0$$

be the coefficients of the convergent series

$$f^*(\theta) = 1+a_1\cos\theta+a_2\cos 2\theta+\cdots$$

and if the numbers c_r are such that the $|c_r|$ are a rearrangement of the a_r, then the function

$$f(\theta) = 1+c_1\cos\theta+c_2\cos 2\theta+\cdots$$

satisfies

$$\int_0^\pi |f^*(\theta)|\,d\theta \leq \int_0^\pi |f(\theta)|\,d\theta \tag{1}$$

The computer suggested the fact that the functions

$$f(\theta) = 1-\tfrac{3}{4}\cos\theta+\tfrac{3}{4}\cos 2\theta-\tfrac{1}{2}\cos 3\theta+\tfrac{3}{4}\cos 4\theta,$$
$$f^*(\theta) = 1+\tfrac{3}{4}\cos\theta+\tfrac{3}{4}\cos 2\theta+\tfrac{3}{4}\cos 3\theta+\tfrac{1}{2}\cos 4\theta$$

do not satisfy the inequality (1). This fact may be verified very simply, although it was necessary for the computer to search through a large number of possible cases.

[See also Lehmer, J. London Math. Soc. **34** (1959), 395–396, 485; MR **22** #2843.] *C. B. Haselgrove* (Manchester)

7239:

Dummer, K.-F. Zum Rechnen mit dekadischer Ergänzung. Wiss. Z. Hochschule Elektrotechn. Ilmenau **5** (1959), 175. (Russian, English and French summaries, unbound insert)

7240:

Hamilton, Walter C. On the least-squares plane through a set of points. Acta Cryst. **14** (1961), 185–189.

Author's summary: "A recent discussion by Schomaker, Waser, Marsh and Bergman of the eigenvalue equation

determining the best least-squares plane through a set of points is extended. A non-diagonal weight matrix is introduced, the errors associated with the coefficients of the plane are discussed, a criterion for the rejection of a given set as co-planar is given, and a more general form of the equations, valid for the case where certain restrictions are placed on the plane, is presented."

7241:

Martensen, E. Trigonometrische Glättung periodischer Funktionswerte. Z. Angew. Math. Mech. **40** (1960), 425–426.

Given the values $f_\mu = f(\mu\pi/N)$ $(\mu = 0, \cdots, 2N-1)$ and $f_\mu = f_{\mu-2N}$ $(\mu \geqq 2N)$ of a function $f(\varphi)$ with period 2π, the author derives the smoothing formula

$$(1)\quad f_\mu = \tfrac{1}{2}f_\mu + \sum_{\nu=1,\ \nu \text{ odd}}^{2N-1} f_{\mu+\nu}[\tfrac{1}{4}N^{-1}(-1)^{(\nu-1)/2}(\operatorname{ctg}(\tfrac{1}{4}\varphi_\nu) - (-1)^N \operatorname{tg}(\tfrac{1}{4}\varphi_\nu))].$$

For N odd, (1) is obtained by least-square-fitting a trigonometric polynomial of degree $\frac{1}{2}(N-1)$ to the f_μ. In the case of N even, the arithmetic mean of such polynomials with degree $\frac{1}{2}N$ and $\frac{1}{2}(N-2)$ is used.

W. C. Rheinboldt (Syracuse, N.Y.)

7242:

Maehly, Hans J. Methods for fitting rational approximations. I. Telescoping procedures for continued fractions. J. Assoc. Comput. Mach. **7** (1960), 150–162.

The author sets out to generalize the Lanczos method for telescoping power series over finite intervals to the domain of continued fractions. If

$$f(x) = \frac{a_1 x|}{|b_1}$$

denotes a convergent continued fraction, whose termination after n terms leads to a rational fraction

$$R_n(x) = \frac{P_n(x)}{Q_n(x)},$$

the author develops a practicable procedure for determining the coefficients of the polynomials P and Q in such a way that the error $|f(x) - R_n(x)|$ will possess a minimum upper bound.

Z. Kopal (Manchester)

7243:

Thacher, Henry C., Jr. Derivation of interpolation formulas in several independent variables. Ann. New York Acad. Sci. **86**, 758–775 (1960).

We quote from the author's introduction: "In one dimension . . . it is possible to write down not only explicit formulas, such as the Lagrangian and Hermite formulas, but even tables of interpolation coefficients. In several variables, however, so much more information must be given, and so many more combinations may arise, that it seems almost impossible to anticipate the formulas that will be needed. It is the purpose of this paper to give, in as great generality as possible, a method of deriving interpolation formulas for any particular set of data and of estimating the error associated with the formula."

The main point of the paper is the theorem: Let $\{\phi_j(\mathbf{x})\}$ be a set of W linearly independent basis functions; let $\{\mathscr{L}_i\}$ be a set of W linear operators, not necessarily all different, and let $\{\mathbf{x}_i\}$ be a set of points, not necessarily all different. Then a necessary and sufficient condition that a linear combination $\Phi(\mathbf{x})$ of the $\phi_j(\mathbf{x})$ exist satisfying W conditions of the form

$$\mathscr{L}_i\Phi(\mathbf{x}_i) = \mathscr{L}_i f(\mathbf{x}_i)$$

is that the determinant

$$|\mathscr{L}_i\phi_j(\mathbf{x}_i)| \neq 0 \quad (i, j = 1, 2, \cdots, W).$$

The function $\Phi(\mathbf{x})$ is given explicitly in terms of a determinant.

Approximately one-half of the paper is devoted to five detailed examples on the following subjects: (1) Interpolation proper, (2) Osculatory interpolation, (3) Interpolation with moments, (4) Solution of the one-dimensional wave equation, and (5) Estimation of the error in the first example.

A. H. Stroud (Madison, Wis.)

7244:

Wynn, P. The rational approximation of functions which are formally defined by a power series expansion. Math. Comput. **14** (1960), 147–186.

The author investigates the Padé table of the formal power series $\sum_{k=0}^{\infty} c_k z^{-k-1}$ (and likewise also $\sum_{k=0}^{\infty} c_k x^k$). After reporting on earlier results concerning the orthogonal polynomials associated with the continued fraction

$$\sum_{k=0}^{m-1} c_k z^{-k-1} + z^{-m}\,\frac{c_m}{|z-\alpha_0^{(m)}|} - \frac{\beta_0^{(m)}}{|z-\alpha_1^{(m)}|} - \cdots,$$

on Stieltjes' expansion theorem, and on the quotient difference algorithm, he refers to the important fact that the upper half of the Padé table can be constructed also with the aid of the ε-algorithm [see Wynn, Math. Tables Aids Comput. **10** (1956), 91–96; MR **18**, 801]. To this end he applies the rules of the ε-algorithm to the sequence of polynomials $S_m(z) = \sum_{k=0}^{m-1} c_k z^{-k-1}$ and executes all operations in the domain of rational functions.

The entries $E_s^{(m)}(z)$ in the even-numbered columns of the ε-table form the transposed upper half of the Padé table (subsequently called the "E-array"), whereas the entries $E_{s+1}^{(m)}(z)$ in the odd-numbered columns are auxiliary functions (it turns out that they are polynomials in z).

The paper is accompanied by various examples (also for the series $\sum_0^\infty c_k x^k$) and a survey of the various algorithms which allow computation of the E-array (either as a whole or single entries of it). Furthermore the author discusses the computing effort involved. It turns out that the quotient difference algorithm together with the recurrence formulae for the orthogonal polynomials is most economical for computing a single entry of the E-array, whereas the ε-algorithm is preferable for computing a two-dimensional section of the E-array.

H. Rutishauser (Zürich)

7245:

Salzer, Herbert E. Alternative formulas for osculatory and hyperosculatory inverse interpolation. Math. Comput. **14** (1960), 257–261.

L'interpolation proposée revient à l'interpolation osculatrice ou hyperosculatrice de la fonction inverse à l'aide des dérivées première ou seconde de la fonction inverse. Ceci constitue une simplification par rapport aux méthodes

antérieures. L'auteur suggère une présentation des tables numériques qui faciliterait l'interpolation osculatrice inverse. *J. Kuntzmann* (Grenoble)

7246:

Lambe, C. G. Approximate values of Lamé functions. Quart. J. Math. Oxford Ser. (2) **10** (1959), 206–213.

Es handelt sich um Näherungslösungen der Laméschen Differentialgleichung, wenn die Differenz zwischen zwei der Singularitäten klein ist. Verf. geht von der Differentialgleichung in algebraischer Form aus und schliesst sich für die Lösungen der Bezeichnung von E. L. Ince an. Für den Eigenwertparameter wird eine asymptotische Näherungsformel abgeleitet auf Grund der Polynomlösung und der Rekursionsformel für die Koeffizienten dieser Lösung. Mit Hilfe der Näherungswerte der Eigenwertparameter werden Näherungslösungen erster Art der Differentialgleichung aufgestellt. Diese werden für einige Spezialfälle betrachtet. Zum Schluss werden die Laméschen Funktionen zweiter Art, welche den obigen Näherungslösungen erster Art entsprechen, aufgestellt. *M. J. O. Strutt* (Zürich)

7247:

Lubkin, S. A note on approximating e^x. Comm. ACM **3** (1960), 649.

Rational approximations to e^x which are approximants of a continued fraction for $e^{x/n}$ raised to the nth power where $n=2^m$. The number of multiplications needed to achieve a prescribed accuracy is so large that this approximation is not useful in computers.

E. Kogbetliantz (New York)

7248:

Berin, A. Rapidly convergent expressions for evaluating e^x. Comm. ACM **3** (1960), 500.

Approximation to e^x based on the expression $(1+x/n)^{n+x/2}$, where n is an integer. Even when improved as in the Editor's note, this approximation cannot be useful in computers: the accuracy of six correct digits for instance necessitates nine multiplications while the very well known rational approximations need only three.

E. Kogbetliantz (New York)

7249:

Schmittroth, Louis A. Numerical inversion of Laplace transforms. Comm. ACM **3** (1960), 171–173.

Real-valued functions $f(t)$ satisfying the following conditions are first considered. The inversion integral of $F(s)$, the Laplace transform of $f(t)$, whose path of integration is the imaginary axis in the s plane, converges to $f(t)$ for all real t, where $f(t)=0$ when $t<0$; $F(s)$ is analytic in the half plane $x \geqq 0$, where $s=x+i\omega$. Write $F(i\omega)=\phi(\omega)+i\chi(\omega)$. The author points out that f is then represented by either two Fourier integrals:

$$f(t) = \frac{\pi}{2}\int_0^\infty \chi(\omega)\sin \omega t\, dt = \frac{\pi}{2}\int_0^\infty \phi(\omega)\cos \omega t\, dt.$$

The numerical quadrature of Fourier integrals presented by H. Hurwitz and P. F. Zweifel [Math. Tables Aids Comput. **10** (1956), 140–149; MR **18**, 337] is then applied to compute $f(t)$. Modifications of the procedure are discussed for cases in which $F(s)$ has poles in the half plane $x>0$ or very near the imaginary axis, and in case t has small values. Results of sample computations on a digital computer are tabulated.

R. V. Churchill (Ann Arbor, Mich.)

7250:

Sagastume Berra, Alberto E.; Fernández, German. ★Álgebra y cálculo numérico [Algebra and numerical computation]. Editorial Kapelusz, Buenos Aires, 1960. xviii+726 pp. $8.50.

The only unusual aspect of this volume is that it combines a fairly standard and conventional course in theory of equations with a fair amount on numerical techniques, including polynomial interpolation as well as the solution of linear and nonlinear equations, and also nomograms. The methods are quite standard, and directed toward hand rather than machine computation. Also they are quite elementary: Horner's method, Newton's method, and Graeffe's method, for example, for the solution of polynomial equations, Gaussian elimination and the Gauss-Seidel method for systems of linear equations. Explanations are quite ample, and illustrative examples abundant.

A. S. Householder (Oak Ridge, Tenn.)

7251:

Householder, A. S.; Bauer, F. L. On certain iterative methods for solving linear systems. Numer. Math. **2** (1960), 55–59.

The authors consider a class of iterative methods for solving the real or complex system $Ax=h$ where A is a non-singular matrix of order n, h is a given column matrix, and x is an unknown column matrix. The iterative methods considered are of the form $x_{\nu+1}=x_\nu+Y_\nu u_\nu$, where the $n\times\rho_\nu$ matrix Y_ν with linearly independent columns and the $\rho_\nu\times 1$ matrix u_ν are chosen in various ways so that $\|s_{\nu+1}\|<\|s_\nu\|$. Here $s_\nu=x-x_\nu$, and the norm of s is defined by $\|s\|^2=s^HGs$ where G is a suitable positive definite matrix. It is shown that for given x_ν, Y_ν, and G one should let $u_\nu=(Y_\nu^HGY_\nu)^{-1}Y_\nu^HGs_\nu$ in order to minimize $\|s_{\nu+1}\|$. However, the choice of Y_ν is restricted since s_ν is not known; however, use is made of the fact that $As_\nu=h-Ax_\nu$ is known.

It is shown that suitable choices of G, Y_ν, and u_ν lead to a number of iterative methods including the Gauss-Seidel method, the method of relaxation, the method of Gastinel [C. R., Acad. Sci. Paris **246** (1958), 2571–2574; MR **20** #1404], the method of steepest descent, and block relaxation. For all of these methods except for the Gauss-Seidel method, bounds are derived for $\|s_{\nu+1}\|^2/\|s_\nu\|^2$ in terms of the spectral condition number of A [Householder, J. Soc. Indust. Appl. Math. **6** 1958, 189–195; MR **20** #2835] and the order of the matrix. In particular, it is shown that the method of steepest descent converges at worst approximately $4n$ times as fast as the method of relaxation and the method of Gastinel. *D. M. Young, Jr.* (Austin, Tex.)

7252:

Hochstadt, Harry. On the factorization of certain polynomials. SIAM Rev. **2** (1960), 140–147.

The author illustrates with 4th and 6th degree polynomials a method of successive approximation for finding a quadratic factor of certain given polynomials.

M. Marden (Milwaukee, Wis.)

7253:

Hsu, L. C. A few useful modifications of Newton's approximation method of solving real equations. Math. Student **26** (1958), 145–153.

The paper is an exposition of some modifications of Newton's method for the solution of equations. These modifications consist essentially in replacing the derivative which appears in that formula by a difference quotient.

H. H. Goldstine (Yorktown Heights, N.Y.)

7254:

Cutteridge, O. P. D. Some tests for the number of positive zeros and for the numbers of real and complex zeros of a real polynomial. Proc. Inst. Elec. Engrs. C **107** (1960), 105–110.

Let Δ_i be the leading principal minor of order $i \times i$ of the bigradient determinant

$$\begin{vmatrix} b_{n-1} & b_{n-2} & b_{n-3} & b_{n-4} & \cdots \\ a_n & a_{n-1} & a_{n-2} & a_{n-3} & \cdots \\ 0 & b_{n-1} & b_{n-2} & b_{n-3} & \cdots \\ 0 & a_n & a_{n-1} & a_{n-2} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

where $b_i = (i+1)a_i$. The author proves that the polynomial

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0,$$

with real a_i, has $k-2N$ distinct real zeros and N distinct pairs of complex zeros, where Δ_{2k-1} is the highest non-vanishing member of the sequence $\Delta_1, \Delta_3, \Delta_5, \cdots, \Delta_{2n-1}$, and N is the number of changes of sign in this sequence. He proves a similar result concerning the number of positive zeros of $f(x)$, and also expresses both results in terms of the number of negative coefficients in the expansion of $f'(x)/f(x)$ in continued fractions. The results are derived from Sturm's theorem, and the author claims that they are more amenable to computation than Sturm's result. Numerical examples are included.

F. W. J. Olver (Teddington)

7255:

Michel, A. Étude de l'erreur dans la représentation approchée des intégrales doubles. Chiffres **3** (1960), 79–84. (English, German and Russian summaries)

L'auteur donne l'expression exacte du reste d'une formule de cubature approchée sous forme d'une somme finie d'intégrales simples et doubles partant sur des dérivées partielles de la fonction à intégrer. (Cette formule généralise une formule analogue pour les intégrales simples.)

J. Kuntzmann (Grenoble)

7256:

Stroud, A. H. A bibliography on approximate integration. Math. Comput. **15** (1961), 52–80.

This bibliography initiates a new section "Surveys and Expository Papers" in Math. Comput. In preparing this bibliography the author has performed a task of great value to research workers in numerical analysis and to those who use approximate integration in their work. While it is difficult to make such a work complete, it does contain over 400 entries. Reference is frequently given to appropriate review articles but the bibliography is otherwise not annotated. Of particular value are the numerous references to the recent work of Russian mathematicians.

P. C. Hammer (Madison, Wis.)

7257:

Walton, Thomas S.; Polachek, Harry. Calculation of transient motion of submerged cables. Math. Comput. **14** (1960), 27–46.

The authors describe a numerical procedure for determining the transient motion of submerged cables. In the mathematical model the cable is divided into segments, and for each segment a pair of simultaneous second-order differential equations is obtained, with x and y, the horizontal and vertical coordinates, respectively, of the segment as dependent variables and with time, t, as the independent variable. A procedure is described for proceeding from one time step to the next. Limitations on the size of the time increment Δt are given to insure numerical stability. Some numerical results and graphs are given.

D. M. Young, Jr. (Austin, Tex.)

7258:

Lapeyre, Renée; Laudet, Michel. Intégration numérique des trajectoires marginales en Optique électronique. C. R. Acad. Sci. Paris **251** (1960), 863–865.

Intégration d'une équation:

$$y'' + f(x)y = \varphi(x, y, y')$$

par une méthode de Runge-Kutta implicite. Exemple numérique.

J. Kuntzmann (Grenoble)

7259:

Wendroff, Burton. On centered difference equations for hyperbolic systems. J. Soc. Indust. Appl. Math. **8** (1960), 549–555.

In an n-dimensional rectangle let the m-dimensional vector u (i) satisfy the system $\sum_{i=1}^{n} a_i \partial u/\partial x_i = f(x, u)$, where a_i are $m \times m$ positive definite matrices, and (ii) assume prescribed values on n faces, assumed to lie in the coordinate planes. The author describes two closely related centered difference methods for solving this problem numerically. Their behavior is analyzed by a difference analog of Friedrichs' energy method. In particular, if $\hat{L}u = f$ are the difference equations, and $\hat{N}u = \bar{u}$ is an average of u over a mesh hypercube, then $\hat{N}u \circ \hat{L}u = \sum \partial_i \hat{Q}_i + \hat{P}$, where ∂_i is a partial difference operator with respect to x_i, and $\hat{Q}_i$ are non-negative quadratic forms for one method, positive definite for the other. By summation over part of the volume of the rectangle one can manipulate this expression to find bounds for L_2 norms of errors (i.e., differences of two solutions). This establishes convergence of both methods, stability of one. By a different choice of $\hat{N}u$ stability is established for the second method for $n = 2$.

J. H. Giese (Aberdeen, Md.)

7260:

Pouzet, Pierre. Méthode d'intégration numérique de l'équation intégrale de Volterra de seconde espèce. C. R. Acad. Sci. Paris **250** (1960), 3101–3102.

La méthode de Runge-Kutta pour les problèmes différentiels de conditions initiales s'étend sans changement essentiel aux équations intégrales de Volterra

$$\varphi(x) = f(x) + \int_{x_0}^{x} G[x, s, \varphi(s)]\, ds.$$

J. Kuntzmann (Grenoble)

7261:

Pouzet, Pierre. Intégration numérique des équations intégrodifférentielles du type Volterra. C. R. Acad. Sci. Paris **250** (1960), 3269–3270.

La généralisation de la méthode de Runge-Kutta proposée par l'auteur pour les équations intégrales du type de Volterra de 2ème espèce s'étend aux équations intégrodifférentielles du type

$$\varphi^{(p)}(x) = F(x, \varphi, \varphi', \cdots, \varphi^{(p-1)}) + \int_{x_0}^{x} G(x, 0, \varphi, \cdots, \varphi^{(p)})\, ds.$$

J. Kuntzmann (Grenoble)

7262:

Oules, Hubert. Résolution numérique des équations intégrodifférentielles linéaires du second ordre à coefficients constants. C. R. Acad. Sci. Paris **251** (1960), 504–506.

Méthode de résolution de l'équation

$$\int_{t_0}^{t} \varphi(t, \tau) y(\tau)\, d\tau + y(t) + ay'(t) + by''(t) = f(t)$$

pour prédiction et correction. Exemple numérique.

J. Kuntzmann (Grenoble)

7263:

Oules, Hubert. Une méthode de résolution numérique de l'équation intégrodifférentielle linéaire de Volterra. C. R. Acad. Sci. Paris **250** (1960), 3937–3939.

L'auteur propose pour l'équation intégrodifférentielle

$$\int_{t_0}^{t} \varphi(t, \tau) y(\tau)\, d\tau + y(t) + ay'(t) = f(t)$$

une méthode analogue aux méthodes à pas liés avec prédiction et correction pour les problèmes différentiels de conditions initiales. Un exemple est donné.

J. Kuntzmann (Grenoble)

7264:

Wilkinson, J. H. Error analysis of floating-point computation. Numer. Math. **2** (1960), 319–340.

The author considers how the rounding errors in floating-point arithmetic affect the solution of polynomial equations and the calculation of the eigenvalues of a matrix. The results are expressed in a new way. When an equation $f(z) \equiv \sum_0^n a_r z^r = 0$ is solved numerically the results obtained are the exact zeros of some other polynomial $g(z) \equiv \sum_0^n (a_r + \delta_r) z^r$. Bounds are given for δ_r / a_r.

Similar error bounds are given for the elements of a symmetric triple diagonal matrix whose eigenvalues are the calculated eigenvalues for a given similar matrix. The more difficult problem of the calculation of the eigenvalues of a lower Hessenburg matrix, that is a matrix such that $a_{ij} = 0, j \geq i + 2$, is considered. A large number of numerical examples is given.

Unfortunately there is a considerable number of misprints. *C. B. Haselgrove* (Manchester)

7265:

Stammberger, A. Nomographische Auflösung von Gleichungen 4. Grades. Wiss. Z. Hochschule Elektrotechn. Ilmenau **5** (1959), 121–124. (2 plates) (Russian, English and French summaries, unbound insert)

Two nomograms have been developed for the real solutions of equations of fourth order. The result represents an improvement of the method given by J. Vurcfeld [Apl. Math. **3** (1958), 223–232; MR **19**, 1200]. The paper also includes one nomogram for determining the positive root of a particular type of fourth order equation.

S. Kulik (Long Beach, Calif.)

7266:

Dolby, J. L. Graphical procedure for fitting the best line to a set of points. Technometrics **2** (1960), 477–481.

The author gives a graphical procedure for obtaining the slope, b, and the intercept, a, of the straight line of best fit to a set of points in two dimensions. The solution is derived in the dual coordinate system (a, b) and (x, y) by use of mapping. *S. Kulik* (Long Beach, Calif.)

7267:

Smirnov, S. V.; Potapov, M. K. Best construction of the curvilinear scale of an approximate Cauchy nomogram. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 5, 165–170. (Russian)

The authors apply the theory of approximations to the development of a method of constructing approximate Cauchy nomograms. The method can be applied to either graphical or numerical calibration of scales.

S. Kulik (Long Beach, Calif.)

7268:

Karst, Edgar. New factors of Mersenne numbers. Math. Comput. **15** (1961), 51.

Author's summary: "I have tested for prime factors all the Mersenne numbers $2^p - 1$ corresponding to prime exponents p in the interval $3000 < p < 3500$. The limit of the search for factors was $9p^2$ when no new factor was previously known; otherwise the limit was $3p^2$."

The first new factor is 145,777 for $p = 3037$. For $p = 3329$, the new factors are 665,801; 1,005,359; 26,225,863.

H. Cohn (Tucson, Ariz.)

7269:

Chistova, E. A. ★Tables of Bessel functions of the true argument and of integrals derived from them. Pergamon Press, New York-London-Paris-Los Angeles, 1959. 523 pp. (1 insert) $15.00.

The Russian original [Vyčislitel'nyĭ Centr, Izdat. Akad. Nauk. SSSR, Moscow, 1958] is reviewed in MR **21** #2366. The title in the present version is inaccurately translated; it should read "... functions with real argument ..."

7270:

Ferraro, Alfredo. ★Piccolo dizionario di metrologia generale: Con particolare riferimento al sistema Giorgi. Nicola Zanichelli Editore, Bologna, 1959. xv + 293 pp. (1 plate) 3000 Lire.

Dieses Diktionär der Metrik, unter besonderer Berücksichtigung des Systems von Giorgi, gibt präzise und durch Tabellen, numerische Daten und Formeln erläuterte Definitionen der einschlägigen Begriffe. Beispiele: Accelerazione (Beschleunigung): Tabelle für m/s², cm/s², km/Stunde Sek, Fuss/s², Yard/s², Meile/Stunde Sek, Meile/Stunde Min., sowie Dimensionsformel. Accelerazione di gravità (Beschleunigung der Schwerkraft): Wertetäbelle für den Pol, Greenwich, Paris, Potsdam, Milano, Roma,

Aden, Equator. Ammettenza (Admittanz): Definitionsformeln für akustische Admittanzen sowie elektrische Admittanzen. Permeabilità: absolute, relative Permeabilität, Formeln, Tabelle für verschiedene Werkstoffe. Valore: Absolutwert, Effektivwert, Maximalwert, arithmetischer Mittelwert, quadratischer Mittelwert. Das Buch enthält eine Fülle von Angaben, die sonst nur zerstreut aufzufinden sind, und ist gewissenhaft zusammengestellt.

M. J. O. Strutt (Zürich)

COMPUTING MACHINES

See also 7850, 7892.

7271:

Ivall, T. E. ★Electronic computers: Principles and applications. Iliffe & Sons Ltd., London; Philosophical Library, New York; 1960. viii + 263 pp. $15.00.

The first edition of this book was a collection of papers by other people which was put together as a whole by the present author. This collection has now been re-written as a coherent account of the subject. Unfortunately the author does not seem to be an active worker in the field so that most of the opinions which are expressed lack authority.

The chapters are: (1) Historical; (2) General principles; (3–4) Analog computing circuits (principles); (5) Circuit details of analog computers; (6) Analog computer applications; (7–8) Digital computer circuits; (9) Storage systems; (10) Digital computer details; (11) Programming; (12) Digital computer applications; (13) Recent developments; (14) Computers of the future; Index.

There are no references or bibliography, no mathematical discussions or examples and the book is suitable only as an elementary introduction for 'radio hams'.

A. D. Booth (London)

7272:

Lauriola, Luca. Matematica e calcolatrici elettroniche. Civiltà delle Macchine **8** (1960), 16–19. (French, German, English, and Spanish summaries)

Popular exposition.

7273:

Samelson, K. Der Stand der Technik des Maschinenrechnens. Math.-Tech.-Wirtschaft **7** (1960), 1–4.

7274:

Bennett, J. M.; Dakin, R. J. Computers as an aid in computer design assessment. Comput. J. **3** (1960/61), 253–255.

Authors' summary: "Simulation routines provide a useful tool for assessing proposed computer designs. An example involving the simulation of a small magnetic-drum computer with various possible modifications is described."

7275:

Appleby, J. S.; Blake, D. V.; Newman, E. A. Techniques for producing school timetables on a computer and their application to other scheduling problems. Comput. J. **3** (1960/61), 237–245.

Authors' summary: "This paper describes a method by which a timetable for a medium-sized school can be produced on a general-purpose digital computer in about 1½ hours. The techniques used and time taken by human beings and the computer program are compared. The use of this work in other scheduling problems is briefly described."

7276:

Flores, Ivan. Computer time for address calculation sorting. J. Assoc. Comput. Mach. **7** (1960), 389–409.

When sorting internally by address calculation, an approximate location is computed for each item being sorted. If this location is available the item is placed there, otherwise items already sorted are moved, allowing the new item to be inserted into the sequence. A computer flow chart for this process is given. Assuming that the item keys are randomly distributed over a given range, so that a linear formula for estimating the approximate location of an item can be used, the expected number of comparisons and shifts is found. Then, by introducing parameters for the operation times of a class of computers, formulas are derived for the average sorting time.

In a representative situation the sorting time per item is computed as a function of $R=S/F$ where S is the number of items being sorted, and F is the number of storage locations assigned to receive the items during the sort. It is found that there is a shallow minimum for $1/R$ approximately $=2.5$, a result in close agreement with some empirical results previously observed.

C. C. Gotlieb (Toronto)

7277:

Pope, David A.; Stein, Marvin L. Multiple precision arithmetic. Comm. ACM **3** (1960), 652–654.

This note outlines the principles which underlie the use of multiple length numbers in electronic computers. The method is independent of any actual computer and it is illustrated by application to a division algorithm. The paper concludes with an interesting discussion of the numerical efficiency of the method and a valuable bibliography of other work on similar lines.

L. J. Slater (Cambridge, England)

7278:

Leone, Fred C. (Editor). Statistical programs for high speed computers. Technometrics **3** (1961), 123–126.

7279:

Sauer, R. Numerische Mathematik beim Einsatz von Rechenautomaten. Bul. Inst. Politehn. Iaşi (N.S.) **5 (9)** (1959), no. 3–4, 65–68. (Russian and Romanian summaries)

A brief expository article touching on recent trends in automatic programming and the rôle of numerical stability in high-speed computing.

Walter Gautschi (Oak Ridge, Tenn.)

7280:

Newell, A.; Tonge, F. M. An introduction to information processing language V. Comm. ACM **3** (1960), 205–211.

The paper is an informal introduction to Information

Processing Language V (IPL-V), a language for manipulating symbols and list structures, presently implemented on the IBM 650, 704, and 709. The authors discuss the types of programming problems that list languages are designed to solve, and describe the methods of data representation, the set of basic instructions, the interpretive cycle, and the method for allocating and accounting for computer memory (available space list) in IPL-V. Two examples of IPL-V code are given.

The processes of IPL-V operate on single symbols, on lists of symbols, and on list structures (recursively defined as lists whose elements are symbols, lists, or list structures). The language contains processes for manipulating particular classes of lists: description lists and push-down lists. The way in which sets of computer words are used to represent the several kinds of lists is described and illustrated.

The authors discuss briefly the advantages and disadvantages of information processing languages as compared with other kinds of programming systems.

H. A. Simon (Pittsburgh, Pa.)

7281:

Raymond, F.-H. Méthode d'intégration fractionnée. Ann. Assoc. Internat. Calcul. Anal. **2** (1960), 127–131.

Suppose the integrator of an electronic differential analyser is evaluating the function

$$\varphi(t) = \int_0^t \dot{\varphi}(t)\, dt.$$

If $\dot{\varphi}(t)$ is represented on such a scale that its range of variation does not exceed the limits of the apparatus, improved accuracy may be obtained by dividing the range of the integral $\varphi(t)$ into a number of segments $\varphi(t) = n_i\theta + \varphi_i(t)$ (n_i = integer) such that $-\theta \leq \varphi_i(t) \leq \theta$. The paper describes relay circuits associated with electronic integrators for achieving this splitting of the range of integration, together with the means of correcting the errors arising from the finite time of operation of relays.

J. G. L. Michel (Teddington)

7282:

Kuntze, Karlheinz. Zur Verfahrenstechnik für die Lösung von Schwingungsproblemen mittels eines elektronischen Analogrechners. Ann. Assoc. Internat. Calcul Anal. **2** (1960), 170–180.

Describes the application of a repetitive electronic differential analyser to the study of oscillation problems. The examples illustrated were obtained on a Telefunken Type RA.463/2 instrument. The determination of period, phase, subharmonic oscillations, limit cycles, stable and unstable oscillations, etc., is discussed.

J. G. L. Michel (Teddington)

7283:

Eterman, I. I. ★Analogue computers. Translated from the Russian by G. Segal; translation edited by B. H. Venning. Pergamon Press, New York-Oxford-London-Paris, 1960. ix + 264 pp. $8.50.

Most of the existing books, in English, deal with analog computers either from the purely engineering viewpoint, or as devices which have to be set up by various tricks in order to solve a given problem.

This book looks at the matter from the mathematical standpoint and, although it contains two chapters on circuits and elements and on the structure of some Soviet machines, the major portion of the work is devoted to a detailed mathematical study of general methods of problem solving and an analysis of the error structure which arises.

There are very many numerical examples taken from actual practice and a long appendix which contains all of the finite difference formulae which are likely to be of utility when analog techniques are used to solve two dimensional problems. The appendix also contains numerous tables of values obtained on Soviet machines.

The standard of rigour is acceptable and although proofs are not given, suitable references are provided. As a refreshingly different approach to analog computation this book can be strongly recommended.

A. D. Booth (London)

7284:

Meissl, P. Behandlung von Wasserschlossaufgaben mit Hilfe eines elektronischen Analogrechners. I, II. Math.-Tech.-Wirtschaft **7** (1960), 9–13, 74–77.

MECHANICS OF PARTICLES AND SYSTEMS

See also A6670, 7339, 7711, 7843, 7847.

7285:

Manžeron, D. [Mangeron, D.]; Yasyulënis, A. I. Sopra alcune proprietà estremali delle accelerazioni ridotte spaziali d'ordine qualunque concernenti la teoria dei meccanismi. Bul. Inst. Politehn. Iaşi (N.S.) **5 (9)** (1959), no. 1–2, 307–310. (Russian. Italian and Romanian summaries)

7286:

Tavhelidze, D. S.; Manžeron, D. [Mangeron, D.] A new class of reduced accelerations of any order applied in the theory of space mechanisms. Bul. Inst. Politehn. Iaşi (N.S.) **5 (9)** (1959), no. 1–2, 303–306. (Russian. English and Romanian summaries)

7287:

Bottema, O. Zur Kinematik der Schlittenbewegung. Monatsh. Math. **64** (1960), 226–232.

After recalling Carathéodory's results on the sliding motion of a sled without sideways slip [C. Carathéodory, Z. Angew. Math. Mech. **13** (1933), 71–76; G. Hamel, *Theoretische Mechanik*, Springer, Berlin, 1949; MR **11**, 548; pp. 465–469] the author discusses the sledding motion more profoundly.

The sled S is attached by the orthogonal coordinate axes ξ, η, where ξ is the axis of symmetry of S and η is the axis connecting points of S in contact with the resting plane.

In the motion of S, the Ball's curve becomes a circle K having its center on the ξ-axis and passing through the center of gravity of S. Denoting by F a half (ξ, η)-plane outside of K, the locus of any point outside of F is convex; that inside of F has two points of inflexion; that on the η-axis has a cusp, and that on K has an undulation point.

K. Yamada (Nagaoka)

7288:
Surber, T. E. On the use of quaternions to describe the angular orientation of space vehicles. J. Aerospace Sci. **28** (1961), 79–80.

7289:
Freudenstein, Ferdinand. Trends in the kinematics of mechanisms. Appl. Mech. Rev. **12** (1959), 587–590.
Survey article.

7290:
Ganesha Rao, H. M. An analytical method for the determination of velocities and accelerations and its application to the synthesis of mechanisms. Proc. 4th Congress Theoret. Appl. Mech. 1958, pp. 195–206. Indian Soc. Theoret. Appl. Mech., Kharagpur.
Starting from the geometrical properties of the four bar mechanism the author deduces formulae for the angular velocities and accelerations of the bars. For the more special case of the slidercrank motion they are used to derive the geometry of the mechanism when the velocities and accelerations are given. Some numerical examples are worked out. *O. Bottema* (Delft)

7291:
Ripianu, A. Une propriété du mouvement cardanique. Inst. Politehn. Cluj. Lucrări Sti. **1** (1958), 133–146. (Romanian. Russian and French summaries)
Wenn ein Endpunkt des bewegten Stabes beim Kardanbewegung konstante Beschleunigung hat, dann ist der Ort des Beschleunigungspoles in der festen Ebene eine Kurve 10. Ordnung. *O. Bottema* (Delft)

7292:
Wunderlich, Walter. Über Gleitkurvenpaare aus Radlinien. Math. Nachr. **20** (1959), 373–380.
Die im Inneren einer Steinerschen dreispitzigen Hypozykloide liegenden Tangentenstrecken haben bekanntlich sämtlich dieselbe Länge. Eine erste Verallgemeinerung gab Morley 1894: eine m-spitzige Hypozykloide läszt sich in einer $(m+1)$-spitzigen unter ständiger Berührung umwenden. Fréchet zeigte 1902 dass sich eine Ellipse in einer dreispitzigen Hypozykloide, dieselbe an drei Stellen berührend umwenden läszt (wobei wohl der ganze Ellipsenumfang, hingegen nur gewisse Teilbogen der Zykloide in reeller Weise zur Berührung kommen.) Die jetzt vom Verf. gegebene dritte Verallgemeinerung enthält die Fréchetsche als Einzelfall. Sei k_1 eine Radlinie die als Bahnkurve von P beim Rollen eines Kreises c in einem Kreis c_1 entsteht. Nun läszt man c_1 unter Mitnahme von k_1 in einem Kreis c_2 rollen. Verf. wendet für das Studium dieser Bewegungen die Gauszsche Zahlenebene an und zeigt dass die Hüllbahn von k_1 aus zwei Komponenten besteht. Sind a, ma und na die Radien der Kreise c, c_1 und c_2, dann ist die eine Komponente eine Gruppe von untereinander kongruenten Radlinien und zwar für $m=1$ eine einzige Trochoide, für rationales m eine endliche, für irrationales m eine abzählbar unendliche Anzahl von Trochoiden. Verf. zeigt dass die zweite Komponente dann und nur dann eine Zykloide ist wenn $n=2m-1$. Für $n=3$, $m=2$ hat man den Fréchetschen Fall; es werden noch $n=5$, $m=3$ und $n=2$, $m=3/2$ näher betrachtet und durch schöne Figuren erläutert. Alle solche Paare von Gleitkurven von denen die eine in der anderen zwangläufig unwendbar ist bilden im Sinne der Getriebelehre ein höheres kinematisches Elementenpaar.
O. Bottema (Delft)

7293:
Pelzer, Werner. Über die Kinematik affin-veränderlicher ebener Systeme. Collect. Math. **11** (1959), 13–47.
Mittels Matrizenrechnung wird eine affine ebene Kinematik entwickelt, wobei eine affin-veränderliche Gangebene E_2 vorausgesetzt wird, während die Rastebene E_1 euklidisch ist. Man kann dann offenbar die Geschwindigkeit und die Beschleunigung eines Punktes von E_2 definieren. Verf. findet von Burmester herrührende Sätze zurück und fügt neue Resultate hinzu. Es werden dabei z.B. behandelt: die Geschwindigkeitsverteilung, die Örter von Punkten mit gegebenem Geschwindigkeitsquadrat, der Wendekegelschnitt, die Ballschen Punkte, die Bresseschen Kegelschnitte (wobei ein Satz von Burmester durch ein Gegenbeispiel widerlegt wird), die selbstentsprechenden Geraden, der Ort der Punkte deren Bahntangenten durch einen gegebenen Punkt gehen, und die Geradenhüllbahnen.
O. Bottema (Delft)

7294:
Egesoy, E. Analytische Behandlung und Berechnung von Hypoidrädern. Comm. Fac. Sci. Univ. Ankara Sér. A **10** (1959), 46–55. (Turkish summary)
Allgemeine Betrachtungen über raümliche Kinematik und eine Anwendung, mittels des Studyschen Übertragungsprinzip, auf die Bewegung von zwei Drehhyperboloïden, die auf einander schroten. *O. Bottema* (Delft)

7295:
Jentsch, Lothar. Die Volumen- und Oberflächenträgheitsmomente von Dodekaeder und Ikosaeder. Wiss. Z. Hochsch. Bauwesen Leipzig No. **5** (1959), 9–13.

7296:
Sandström, Bengt. A note on deceleration processes. Kungl. Tekn. Högsk. Handl. Stockholm No. 164 (1960), 11 pp.
Author's summary: "Some relations are given in the form of inequalities between the distance, initial velocity, deceleration and rate of deceleration for the braking of a body to standstill."

7297:
Bentsik, E. Una classe di moti del corpo rigido soggetto a forze di potenza nulla. Rend. Sem. Mat. Univ. Padova **29** (1959), 318–327.
L'A., seguendo l'ordine di idee del G. Grioli [cf. gli stessi Rend. **27** (1957), 90–102; MR **19**, 899] concernente studi del moto di un corpo rigido soggetto a forze di potenza nulla, determina, in assenza di attrito, tutti i moti rotatori che soddisfano alla condizione $\mathbf{K}_0 = a\mathbf{H} + b\mathbf{k}$ con a e b costanti e con $\mathbf{H}$ e $\mathbf{k}$ vettori invariabili rispettivamente nello spazio e nel corpo, nel caso di una sollecitazione del

tipo delle forze centrifughe composte. Mostra, infine, che possono aversi anche moti richiesti coincidenti con delle precessioni regolari se il corpo ha struttura giroscopica.

D. Mangeron (Iaşi)

7298:

Golubev, V. V. ★Lectures on integration of the equations of motion of a rigid body about a fixed point. Translated from the Russian by J. Shorr-Kon. Published for the National Science Foundation by the Israel Program for Scientific Translations, 1960; available from the Office of Technical Services, U.S. Department of Commerce, Washington 25, D.C. vii + 229 pp. $2.50.

Translation of *Lekcii po integrirovaniyu uravnenii dviženiya tyaželogo tverdogo tela okolo nepodvižnoĭ točki* [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1953; MR **15**, 904].

7299:

Crandall, Stephen H. Random vibration. Appl. Mech. Rev. **12** (1959), 739–742.

Survey article.

7300:

Bolie, Victor W.; Long, Francis M. A theoretical study of a randomly excited mechanical oscillator. Proc. Iowa Acad. Sci. **67** (1960), 382–388.

Authors' summary: "A mathematical investigation is made of the feasibility of extracting usable mechanoelectric power from a critically damped and randomly excited mechanical oscillator. Two different analytic functions are used to approximate practical squared-velocity spectra for the motion of the vehicle supporting the oscillator, and the corresponding expressions for extractable power are developed. Numerically computed results are presented graphically to illustrate the effects of varying the oscillator parameters."

7301:

Lyon, Richard H. On the vibration statistics of a randomly excited hard-spring oscillator. J. Acoust. Soc. Amer. **32** (1960), 716–721.

The equation of motion of the system under study is $d^2y/dt^2 = -2\alpha dy/dt - g(y) + f(t)$, where $f(t)$ is white noise with spectral density D, $g(y) = \omega_0^2(y + y^3/4\kappa^2)$, and α, ω_0, κ are positive constants. In the steady state the joint density of y and $u = dy/dt$ is

$$p(y, u) = C \exp\left[-2\alpha D^{-1}\{u^2 + \omega_0^2(y^2 + y^4/8\kappa^2)\}\right].$$

The author also finds an approximate expression for the distribution of the extrema of $y(t)$. In a discussion that follows, the so-called carrier component is (very heuristically) defined, and the problem of finding the density of the amplitudes of the carrier reduced to an integral equation which is solved approximately. *E. Reich* (Aarhus)

7302:

Čžan, Sy-in. On stability of motion of systems with variable coefficients. Acta Mech. Sinica **4** (1960), 46–54. (Chinese. Russian summary)

In two previous papers in Prikl. Mat. Meh. **23** (1959), 230–238, 640–649 [MR **22** #1128, 2760] the author discussed the stability of motion during a finite time interval $t_0 \leq t \leq T$ by means of the function $V = e^{-\alpha(t)}\sum_s x_s^2$ which had been used by Liapunov. In this paper the discussion is extended to the problem of stability for the whole time interval $t \geq t_0$, in the sense of Liapunov. The function $\alpha(t)$ is found to satisfy the relation

$$-\alpha(t) \leq \int \psi_k(t)\, dt + C,$$

where ψ_k is the smallest root of a characteristic equation formulated from the given equation of motion. The conclusion is that, if the integral $\int_{t_0}^{t} \psi_k(t)dt$ has a definite lower limit for $t \geq t_0$, the unperturbed motion is stable in the sense of Liapunov, and, if the integral approaches $+\infty$, as $t \to \infty$, the unperturbed motion approaches stability. Examples show that the method leads to better results than other methods. The author further extends Liapunov's concept of a characteristic number to that of a characteristic function. *Y. Y. Yu* (Brooklyn, N.Y.)

7303:

Jones, R. P. N. A modified energy method for determining natural frequencies. J. Appl. Mech. **28** (1961), 146–147.

7304:

Dei Poli, Sandro. Vibrazioni e instabilità dinamica del sistema di due aste articolate agli estremi. Ist. Lombardo Accad. Sci. Lett. Rend. A **93** (1959), 29–41.

Das obige Problem führt ohne äussere Kräfte auf eine Differentialgleichung der Form

$$\ddot{q} - \alpha^2(q - q^3) = 0.$$

Wenn eine sinusförmig von der Zeit abhängige äussere Kraft auf das mittlere Scharnier wirkt, entsteht die Differentialgleichung von G. Duffing. Bei Annahme einer zusätzlichen Reibung kommt zu obiger Gleichung links ein Glied $B\dot{q}$ hinzu. Eine Stabilitätsbetrachtung der Lösungen der Gleichung von G. Duffing führt auf eine Differentialgleichung von Mathieu. Es werden Lösungen dieser Differentialgleichung und Parameterwerte betrachtet. In bestimmten Bereichen der Parameterebene sind Lösungen vorhanden, die mit der Zeit anwachsen (instabile Bereiche). In einem Diagramm werden einige solcher Bereiche numerisch angegeben. *M. J. O. Strutt* (Zürich)

7305:

Pirogov, I. Z. On the stability of a gyroscopic system. Prikl. Mat. Meh. **23** (1959), 1134–1136 (Russian); translated as J. Appl. Math. Mech. **23**, 1623–1626.

The system considered satisfies a set of linear differential equations with time-dependent coefficients. Liapunov's function is determined in terms of transformed variables, using Roitenberg's method [Prikl. Mat. Meh. **22** (1958), 167–172; MR **20** #4687]. Stability conditions are found for the given system and for an equivalent system with constant coefficients. *A. W. Babister* (Glasgow)

7306:

Klotter, K.; Kreyszig, E. Über die Grenzamplitude selbsterregter Schwingungen bei kleinem Energieaustausch. Z. Angew. Math. Mech. **40** (1960), 55–61.

Gli AA., seguendo l'ordine delle idee di quella via di studio delle oscillazioni autoeccitati che consta nella considerazione delle classi speciali di equazioni differenziali che ne possono essere considerati come generalizzazioni dell'equazione differenziale ormai classica del van der Pol, proseguono la serie dei Loro risultati già raggiunti [cf. Klotter e Kreyszig, Ing.-Arch. **25** (1957), 389–403; MR **19**, 899] e ne studiano, tramite il metodo della variazione lenta dell'amplitudine e della fase come pure con quello del Kryloff e Bogoliuboff [cf. N. Minorsky, David Taylor Model Basin Rep. No. 534 (1944); 546 (1945); 558 (1946); *Introduction to non-linear mechanics*, J. W. Edwards, Ann Arbor, Mich., 1947; MR 8, 207, 583], il problema di determinazione dell'amplitudine limite per le oscillazioni autoeccitate in presenza del piccolo cambio di energia, cristallizzato nell'equazione differenziale di forma

$$\ddot{q} - (\operatorname{sign} \dot{q})\varepsilon\kappa^2 h(q, \dot{q}) + \kappa^2 q = 0 \quad (\varepsilon > 0).$$

D. Mangeron (Iaşi)

7307:

Ojalvo, I. U.; Bleckman, G. L. On Duffing's equation. J. Appl. Mech. **28** (1961), 139–140.

7308:

Girardin, P.; Tesson, F. ★Les effets secondaires de jet d'un engin autopropulsé. Publ. Sci. Tech. Ministère de l'Air, No. 373, Paris, 1960. x+61 pp. 16.00 NF.

The authors consider the secondary effects of a jet of an auto-propulsive motor. In the first chapter they discuss the simplifying hypotheses of the proposed model of the missile, the coordinate systems and the fundamental equations of motion. A considerable part of this chapter is devoted to the fundamental relations, mostly in their integral formulations, expressing such quantities as the torque vectors of the inertia forces, of the magnitudes of the relative accelerations, etc. As a particular case the authors consider the equations of a steady motion of a rocket. In all these equations the effects of the jet upon all the possible aspects of the motion are very strongly emphasized. The assumed simplifications and restrictions are of such a nature that the essential characteristics of the jet effect are clearly demonstrated. The second chapter deals with the general theory of a material system having a variable mass. The technique constructed is such that again various details of the secondary jet effects are clearly seen. The particular items in question are as follows: fundamental mathematical relations, partial time derivative of the integral of the vector functions, partial time derivatives of the torques, application to a moving rocket, moments, movement around the center of gravity, etc. In the Appendix the authors briefly discuss some previous works on the subject in question (2 American, 2 French, 1 English).

M. Z. v. Krzywoblocki (E. Lansing, Mich.)

7309:

Saltzer, Charles. A note on the canonical form of the thrust-programming problem for power-limited flight. J. Aerospace Sci. **28** (1961), 172–173.

STATISTICAL THERMODYNAMICS AND MECHANICS

See also A6824, 7531, 7623.

7310:

Gibbs, J. Willard. ★Elementary principles in statistical mechanics: developed with especial reference to the rational foundation of thermodynamics. Dover publications, Inc., New York, 1960. xviii+207 pp. $1.45.

Gibbs' well-known classic has for a long time been unavailable, except as part of his Collected Works, and the publication of a cheap edition will be welcomed by all who are interested in statistical mechanics. Although at times relatively dry, it still presents one of the best accounts of classical ensemble theory, including the theory of grand ensembles. It is a pity that the publishers have not taken the trouble to add an index. *D. ter Haar* (Oxford)

7311:

★Handbuch der Physik (herausgegeben von S. Flügge), Bd. 3/2. Prinzipien der Thermodynamik und Statistik. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1959. vii +678 pp. DM 160.00.

This volume contains the five articles [#7312–7316] reviewed below.

7312:

Guggenheim, E. A. Thermodynamics, classical and statistical. Handbuch der Physik, Bd. 3/2, pp. 1–118. Springer, Berlin-Göttingen-Heidelberg, 1959.

In this substantial article of the new *Handbuch der Physik* the author gives a clear exposition of the principles of equilibrium thermodynamics and supplements it with a number of applications which illustrate the general theory. The presentation is modern and largely disregards the historical development which is summarized at the end of the article in a series of historical notes and quotations.

Part A is devoted to the fundamentals of classical thermodynamics: the basic laws, reversible processes, the various thermodynamic variables and functions, equilibrium conditions, etc. Part B presents the application of the general principles in a number of concrete examples.

Part C treats briefly the fundamentals of statistical mechanics and their relation to thermodynamics. As mentioned explicitly by the author, the presentation follows that of Tolman in his classical book on *The principles of statistical mechanics*. The basic principles are formulated for quantum systems; classical statistics is taken as a limiting case. Contour integrations and the method of steepest descent are frequently used. Application of the general equations to systems of non-interacting molecules is dealt with in part D.

Part E is devoted to statistical thermodynamics, which the author defines as "the combined application of classical thermodynamics and statistical mechanics to obtain results more detailed than can be obtained from the former alone and to obtain them more briefly than is possible from the latter alone". A number of special problems are treated: classical ideal gases, rotation and vibration of molecules, non-linear molecules, ideal gaseous mixtures, crystals, etc. The degenerate ideal gases and the photon gas (radiation) are treated in more detail in a separate part F. Part G

considers very briefly systems in external fields, gravitational, electric and magnetic.

In view of its size the number of items discussed in this Handbook article is quite large. This is achieved through concise presentation and great clarity of exposition.

L. Van Hove (Utrecht)

7313:

Falk, G.; Jung, H. Axiomatik der Thermodynamik. Handbuch der Physik, Bd. 3/2, pp. 119–175. Springer, Berlin-Göttingen-Heidelberg, 1959.

Very few physical theories have been the subject of an axiomatic treatment. A notable exception is thermodynamics for which Carathéodory proposed an axiomatic construction in 1909 in a famous paper entitled "Untersuchungen über die Grundlagen der Thermodynamik" [Math. Ann. 67 (1909), 355–386]. Whereas Carathéodory had a special motivation for his work, namely to set up thermodynamical theory without using the then unfamiliar and often ill-defined concept of heat, the axiomatics of thermodynamics appears to have drawn attention also in later times despite the fact that the great development of statistical mechanics has removed the mysteries surrounding the heat concept.

Instead of reviewing the subject, as one would expect in an article written for the *Handbuch der Physik*, the authors have presented a new axiomatic formulation of thermodynamics. The main difference with Carathéodory's is that, in addition to heat, temperature has also been eliminated from the axiomatic construction. The latter is carried out in detail and described with great clarity, extensive preliminary considerations preceding and motivating the choice of each axiom. As compared to Carathéodory's work the spirit is much more set-theoretic and less analytic. The basic operations are union of systems, used to define extensive variables, and reduction of systems, providing a definition of intensive variables. Two forms of "insulation" play a fundamental role. They are adiabatic and energetic insulation, allowing a definition of entropy and energy respectively.

In an appendix the authors give a brief account of Carathéodory's axiomatics of 1909, in a terminology and notation suited for comparison with their own formulation, and discuss some of the differences. *L. Van Hove* (Utrecht)

7314:

Münster, A. Prinzipien der statistischen Mechanik. Handbuch der Physik, Bd. 3/2, pp. 176–412. Springer, Berlin-Göttingen-Heidelberg, 1959.

This extensive contribution to the new edition of the *Handbuch der Physik* presents a thorough review of the principles of statistical mechanics. While adopting a modern standpoint in his selection and presentation of topics, the author succeeds in giving a broad historical perspective of this beautiful part of theoretical physics. Two chapters of comparable length deal with classical and quantum statistics; they show considerable parallelism. The third chapter is devoted to the foundations of thermodynamics, fluctuation theory and phase changes.

After a few considerations on phase space and ensembles, the discussion of classical statistical mechanics opens with a survey of the axiomatic foundations. The two prevalent approaches, based on ergodicity and representative ensembles respectively, are described in their historical development and their present status. Despite some weaknesses (like the statement without proper proof on p. 207 that quasi-ergodicity implies metric transitivity) this analysis of a most difficult question is very instructive. The approach to equilibrium is then discussed, starting with the Boltzmann equation and the problems related to its statistical interpretation. Tolman's treatment of nonstationary ensembles and Kirkwood's theory of irreversible processes are described at some length. The first chapter ends with the ensemble theory of equilibrium systems and the statistical analogues of thermodynamic functions. The whole formalism is based on the microcanonical ensemble, the canonical theory being given only brief attention.

Quantum statistics is then treated. Generalities on the density matrix and the quantum mechanical definition of macroscopic variables are followed by a review of the axiomatic foundations which, as in the classical case, discusses the ergodic problem and the method of representative ensembles. The next topic is the approach to equilibrium. Following Pauli the author considers in succession the Boltzmann equation for ideal quantum gases and the master equation for more general systems; the H-theorem is discussed in both cases. The balance of the second chapter concerns equilibrium theory, which is treated at greater length than for classical systems. The three conventional ensembles are discussed and a short account is given of various approximation methods for calculating the sum of states, in particular high and low temperature expansions as well as Feynman's method for solving the Bloch equation.

The last chapter starts with an extension of the statistical foundation of thermodynamics to the case of general extensive or intensive variables. The fluctuations of such variables are also discussed. A critical analysis is then given of the theory of phase changes. After a brief historical survey the author gives a very sound and clear exposition of the present state of knowledge on this difficult and fascinating subject. The problem being as yet essentially unsolved, he presents the few general theoretical results which can be considered well established and discusses the prevalent views on the open questions. As an example Bose-Einstein condensation in absence of forces is treated in detail. The chapter ends with the theory of one-dimensional systems.

In conclusion we have here a well written and well thought out review of the present status of statistical mechanics. Certain topics, often taken among the most interesting ones, are given a detailed treatment reminiscent of a monograph. Others are mentioned more briefly. The guiding principle has always been to stress the main achievements and problems, a goal which in the reviewer's opinion has been successfully reached.

L. Van Hove (Utrecht)

7315:

Meixner, J.; Reik, H. G. Thermodynamik der irreversiblen Prozesse. Handbuch der Physik, Bd. 3/2, pp. 413–523. Springer, Berlin-Göttingen-Heidelberg, 1959.

In their contribution to the new *Handbuch der Physik* Meixner and Reik survey the principles and applications of the thermodynamics of irreversible processes. This chapter of thermodynamics, the theoretical foundations of which were laid by Onsager in 1931, has been systematically developed in the last twenty years. Many

applications have been treated and efforts have been made to clarify further the statistical foundations of the theory.

After a historical introduction the authors present the general thermodynamical theory of irreversible phenomena in continuous media, based on the conservation laws, the entropy production formula, and the phenomenological relations between forces and fluxes. A large number of applications are then given, including media with internal variables, electromagnetic effects and relativistic systems. The last chapter is devoted to the statistical foundations. The very important reciprocity relations of Onsager, extended by Casimir, are discussed: they are known to be a manifestation of microscopic reversibility. The relation between fluctuations and irreversible processes is reviewed. A few remarks are made on the problem of understanding irreversibility in terms of mechanical properties.

L. Van Hove (Utrecht)

7316:

Ramakrishnan, Alladi. Probability and stochastic processes. Handbuch der Physik, Bd. 3/2, pp. 524–651. Springer, Berlin-Göttingen-Heidelberg, 1959.

Volume 3/2 of the new *Handbuch der Physik*, devoted to the principles of thermodynamics and statistics, ends with an article on probability and stochastic processes by A. Ramakrishnan. In the author's own words, "it is an attempt to present to the physicist a physical approach to the theory of stochastic processes, a field till recently the close preserve of the mathematician". A deliberate and successful effort is made to adapt the language and presentation to the physicist's needs. In addition physics has been the source of most of the numerous examples.

Part A deals with probability theory. Classical concepts and distribution laws are reviewed, as well as some notions directly related to statistical mechanics such as for example the Maxwell-Boltzmann, Fermi-Dirac and Bose-Einstein distributions. Stochastic processes are considered in part B. The general theory is mentioned very briefly, the definition being presented first in physical terms. Many interesting physical examples are then analyzed. Thus cascade problems of the type encountered in cosmic ray physics and stochastic problems in astrophysics are considered at some length. A number of mathematical considerations are presented in relation to these examples.

Although the student of modern statistical physics will not find in this article all the mathematical information he may need—the selection of topics is a little too eclectic for this purpose—he will get a clear picture of the concepts and methods used in the theory of stochastic processes. In addition physicist and mathematician alike will find a sampling of the very great variety of physical situations which are usefully analyzed in terms of this theory.

L. Van Hove (Utrecht)

7317:

O'Toole, J. T.; Dahler, J. S. Molecular friction in dilute gases. J. Chem. Phys. **33** (1960), 1496–1504.

From the authors' summary: "The velocity and temperature dependencies of the molecular friction coefficients of a dilute gas are examined for several choices of the pair interaction potential. Also considered are the frictional loss of kinetic energy and, for a special case, the dissipation of molecular internal angular momentum."

R. H. Kraichnan (New York)

7318:

Snider, R. F.; Curtiss, C. F. Kinetic theory of moderately dense gases: rigid sphere limit. Phys. Fluids **3** (1960), 903–904.

This is a short paper which corrects a subsidiary calculation of the authors' previous paper [Phys. Fluids **1** (1958), 122–138; MR **22** #2077] in which the coefficients of viscosity and thermal conduction of a gas of rigid spherical molecules were obtained, correct to terms quadratic in the density. The corrected results are in agreement with those obtained by Enskog for dense gases of rigid spheres. With a more realistic molecular model, Enskog's method would not apply.

H. S. Green (Adelaide)

7319:

Gross, E. P.; Ziering, S. Kinetic theory of linear shear flow. Phys. Fluids **1** (1958), 215–224.

Les auteurs déterminent la fonction de distribution des vitesses d'un gaz monoatomique s'écoulant entre deux parois parallèles maintenues à la même température qui se déplacent avec la même vitesse dans des directions opposées. On suppose que sur les parois les molécules sont en partie réfléchies, et en partie redistribuées suivant une distribution Maxwellienne (correspondant à la température des parois, et à une vitesse moyenne égale à celle de la paroi). Le problème consiste à résoudre l'équation de Boltzmann linéarisée (la vitelle des parois est faible par rapport à la vitesse du son) avec les conditions limites énoncées ci-dessus. Posant

$$f(\mathbf{c}, x) = (1+\phi(\mathbf{c}, x))f_0(\mathbf{c})$$

(f_0: distribution Maxwellienne à la température des parois et pour une vitesse moyenne nulle; Ox: axe normal aux parois) et

$$\phi = \begin{cases} \phi^+(\mathbf{c}, x) & c_x > 0 \\ \phi^-(\mathbf{c}, x) & c_x < 0 \end{cases}$$

la méthode utilisée consiste à rechercher la solution sous la forme

$$\phi^\pm = a_0^\pm(x)c_z + a_1^\pm(x)c_x c_z,$$

la première approximation correspondant à $a_1^\pm = 0$.

La vitesse moyenne et le tenseur des pressions peuvent être explicité en fonction de $a_{0,1}^\pm$; des résultats numèriques sont présentes pour le gaz de sphères dures et pour des nombres de Knudsen variables. Les résultats sont également applicables dans la région des gaz de Knudsen et pour les hautes pressions.

J. Naze (Marseille)

7320:

Simon, Albert; Harris, E. G. Kinetic equations for plasma and radiation. Phys. Fluids **3** (1960), 245–254.

Les auteurs étudient le système formé par N particules et une infinité dénombrable d'oscillateurs, contenus dans un volume V; à partir de l'équation de Liouville du système, ils obtiennent rigoureusement la chaîne d'équations liant les fonctions de distribution dans l'espace-phase de s particules et t oscillateurs. Généralisant la méthode de Rosenbluth-Rostoker, ils développent les fonctions de distribution successives suivant les puissances de la charge e d'une particule, en se plaçant dans le cas limite $e, m \to 0$, $N, V \to \infty$, les rapports e/m, Ne/V restant constants (m masse d'une particule).

Les équations correspondant à l'ordre zéro ne faisant pas intervenir les interactions entre particules et/ou oscillateurs, on suppose que les fonctions de distribution réduites successives peuvent se mettre sous forme de produits de fonctions de distribution d'une particule et d'un oscillateur. On obtient la suite des équations de Vlasov, et une équation pour le champ des oscillateurs, qui conduit à la forme statistique des équations de Maxwell.

Les équations correspondant au premier ordre contiennent les interactions entre particules et/ou oscillateurs; elles conduisent à l'équation de Fokker-Planck pour les particules et une "équation de Fokker-Planck" pour le rayonnement, en supposant que toute fonction de distribution peut s'exprimer à cet ordre en termes des fonctions de corrélation particule-particule ou particule-oscillateur.

J. Naze (Marseille)

7321:

Harris, E. G.; Simon, Albert. Coherent and incoherent radiation from a plasma. Phys. Fluids **3** (1960), 255–258.

Les équations de Vlasov sont ici obtenues directement à partir de l'équation de Liouville. Considérant la fonction de distribution f_λ d'un oscillateur, on définit l'entropie du champ électromagnétique

$$S_{\text{rad}} = \sum_\lambda S_\lambda = -\sum_\lambda K \int f^\lambda \log f^\lambda \, dq_\lambda \, dp_\lambda,$$

et on montre que S_λ et S_{rad} sont des constantes du mouvement.

En particulier l'entropie du champ est infinie négative si les phases et les amplitudes du champ de rayonnement sont bien définies en fonction du temps (champ cohérent), tandis qu'elle est accrue par toute incohérence. Il en résulte que le rayonnement incohérent ne peut être décrit au moyen des équations de Vlasov; il sera pour cela nécessaire de considérer les termes d'ordre supérieur à zéro du développement introduit dans l'article précédent [voir l'analyse précédente]. *J. Naze* (Marseille)

7322:

Hollinger, Henry B.; Curtiss, C. F. Kinetic theory of dense gases. J. Chem. Phys. **33** (1960), 1386–1402.

A Boltzmann equation for dense gases is derived from the Liouville equation. The relation between this approach and previous ones is discussed. *S. Simons* (London)

7323:

Jacobsohn, B. A. Relations involving the forward scattering amplitude in a Fermi gas. Physica **26** (1960), supplement, S 78–S 80.

Author's summary: "Several exact relationships are given for the ground state of a Fermi gas with interaction. These involve the two-particle forward scattering amplitude and the effective mass at the Fermi surface."

7324:

Chapman, Sydney; Cowling, T. G. ★The mathematical theory of non-uniform gases: An account of the kinetic theory of viscosity, thermal conduction, and diffusion in gases. Cambridge University Press, New York, 1960. xxiii + 431 pp. $2.95.

The first edition of this work [1939] was reviewed in MR **1**, 187. The second edition [1952], of which this is a reprint, contained a series of Notes added to indicate some of the advances made in the subject since the first edition appeared. Minor alteration in the text, including some corrections, were also made.

7325:

Simonin, Raymond F. Cinématique moléculaire de surface. C. R. Acad. Sci. Paris **250** (1960), 1798–1800.

Résumé de l'auteur: "On étudie la cinématique des molécules superficielles d'une veine d'eau en écoulement uniforme, percutant normalement le plan d'eau d'un bassin. Le calcul montre que la percussion déprime la surface libre du bassin, qui reste de révolution avec une courbure totale constante, c'est-à-dire d'aire minimale, et que la dépression se couvre de molécules superficielles de la veine avec une vitesse aréolaire constante."

J. Naze (*Marseille*)

7326:

Guiraud, Jean-Pierre. Sur l'interprétation statistique de la théorie phénoménologique des processus irréversibles. C. R. Acad. Sci. Paris **251** (1960), 213–215.

Le problème que se pose l'auteur est d'obtenir un calcul simple des coefficients de viscosité et de conductibilité. Effectivement les résultats de première approximation de Chapman et Cowling [*The mathematical theory of non-uniform gases*, Univ. Press, Cambridge, 1939, MR **1**, 187; also see #7324] sont retrouvés à la suite d'une théorie nouvelle féconde et beaucoup plus élémentaire. La théorie cinétique des gaz n'est utilisée que pour fournir l'expression de la fonction de dissipation que l'on évalue en supposant que la fonction de distribution diffère de la fonction de Maxwell par des termes proportionnels aux gradients des vitesses et des températures. La comparaison avec l'expression classique en mécanique des milieux continus fournit les valeurs des coefficients de dissipation.

P. Germain (Paris)

7327:

Moreau, E.; Salmon, J. Action de l'opérateur de collision élastique de Boltzmann sur une fonction isotrope des vitesses, dans un gaz de Lorentz imparfait. J. Phys. Radium **21** (1960), 217–222. (English summary)

Cet article contient divers compléments à l'étude présentée par Bayet, Delcroix, et Denisse [same J. (8) **15** (1954), 795–803; **16** (1955), 274–280; **17** (1956), 923–930, 1005–1009; MR **16**, 550, 890; **18**, 611; **19**, 785]. On y donne d'une part une démonstration de la formule de Chapman-Cowling permettant de calculer l'action des collisions élastiques sur la partie isotrope de la fonction de distribution des vitesses des particules légères d'un gaz Lorentzien imparfait. D'autre part, les auteurs montrent que cette formule n'a pas de sens pour les faibles vitesses (la démonstration suppose en effet que la vitesse d'une particule légère est très supérieure à la vitesse quadratique moyenne des particules lourdes). Dans le domaine des faibles vitesses l'opérateur calculé ci-dessus doit donc être remplacé par un opérateur intégral, dont les auteurs proposent une expression. *J. Naze* (Marseille)

7328:

Green, Melville S. Topological derivation of the Mayer density series for the pressure of an imperfect gas. J. Mathematical Phys. **1** (1960), 391–394.

Author's summary: "A new derivation of Mayer's classical density expansion for the pressure of an imperfect gas based on a classification of cluster graphs according to topological criteria is presented. The classification is a generalization of the classification of simple trees into trees with centers and trees with bicenters."

E. A. Jackson (Princeton, N.J.)

7329:

Morita, Tohru; Hiroike, Kazuo. A new approach to the theory of classical fluids. I. Progr. Theoret. Phys. **23** (1960), 1003–1027.

A derivation is given of an exact but formal integral equation to determine the pair distribution function in fluids, equivalent to a set of equations derived previously by Van Leeuwen, Groeneveld and De Boer [Physica **25** (1959), 792–808; MR **22** #1146]. To solve the integral equation, approximations must be made, the simplest of which is the 'hypernetted chain approximation' discussed previously by Morita [Letters to the Editor, Prog. Theoret. Phys. **23** (1960), 175–177, 385–387]. The integral equation is derived independently by a variational method, and the whole discussion is illustrated by the use of bond-figures.

H. S. Green (Adelaide)

7330:

Hiroike, Kazuo. A new approach to the theory of classical fluids. II. Multicomponent systems. Progr. Theoret. Phys. **24** (1960), 317–330.

This is a generalization for multicomponent systems of the theory elaborated in Part I [see review, #7329]. The fundamental integral equation is derived in a somewhat more systematic way than in the earlier paper, and a general expression is obtained for the free energy. The method adopted lends itself to the application of Morita's 'hypernetted chain approximation'. *H. S. Green* (Adelaide)

7331:

Tani, Shô-ichiro. Effect of lattice-electron interaction on the Landau diamagnetism. Progr. Theoret. Phys. **23** (1960), 1157–1162.

The author derives the correction to the Landau diamagnetism for electrons interacting (weakly) with the lattice vibrations in a crystal. Using second quantization of the phonon field to describe the lattice, the calculation is made up to second order in the electron-phonon coupling constant. As anticipated by a somewhat different calculation due to Peierls, one gets only a small correction to the Landau diamagnetism (0.06% for Na).

R. M. May (Cambridge, Mass.)

7332:

Young, Russell D. Theoretical total-energy distribution of field-emitted electrons. Phys. Rev. (2) **113** (1959), 110–114.

Author's summary: "The Fowler-Nordheim equation is derived in terms of total electron energy so as to obtain the total-energy distribution of field-emitted electrons. At 0°K the new distribution width is less than $\frac{1}{3}$ of that obtained from the previous 'normal-energy' theory. A surprising mirror-image symmetry is observed between the zero-temperature total-energy field emission and the zero-field normal-energy thermionic emission distributions. The total-energy distribution is also derived for the zero-field thermionic case and this is found to be the mirror image of the normal-energy zero-temperature field emission distribution.

"The new distribution applies to problems involving the total energy of electrons before and after emission."

7333:

Hurst, C. A.; Green, H. S. New solution of the Ising problem for a rectangular lattice. J. Chem. Phys. **33** (1960), 1059–1062.

The algebra of emission and absorption operators is used to compute the partition function of the 2-dimensional, 0 external field, nearest neighbor interaction, Ising model. The level of simplifying assumptions is the same as that in Kac-Ward and the result is the same. The approach is novel but the presentation is Procrustean, e.g., the key deduction from equation (3) to equation (5) is not adequately justified. Unfortunately, not all of the authors' remarks on the state of the literature are dependable.

S. Sherman (Detroit, Mich.)

7334:

Tamor, S. Structure of transport equations. J. Chem. Phys. **33** (1960), 1420–1425.

By considering a modified Liouville equation, a new approach is given to the derivation of transport equations. The connection between this approach and the time-smoothing method of Kirkwood is discussed.

S. Simons (London)

7335:

Auer, P. L.; Tamor, S. Statistical mechanics of the nonuniform gas. J. Chem. Phys. **33** (1960), 1426–1431.

The approach developed by S. Tamor [see preceding review] is applied here to the derivation of a transport equation for a non-uniform dense gas. In the limit of infinite dilution, the Boltzmann equation is obtained.

S. Simons (London)

7336a:

Kanazawa, Hideo; Watabe, Mitsuo. Green function method for electron gas. I. General formulation. Progr. Theoret. Phys. **23** (1960), 408–425.

7336b:

Kanazawa, Hideo; Misawa, Setsuo; Fujita, Emiko. Green function method for electron gas. II. Dispersion relation of plasmons. Progr. Theoret. Phys. **23** (1960), 426–432.

7336c:

Kanazawa, Hideo; Matsudaira, Noboru. Green function method for electron gas. III. Diamagnetism. Progr. Theoret. Phys. **23** (1960), 433–446.

In the above three papers (I, II, and III) a Green's function formalism is developed for and applied to a non-relativistic electron gas interacting via the Coulomb potential. Time-dependent two-particle Green's functions are

used to find: (1) the Bohm-Pines plasmon dispersion relation in I, and an improved version in II, which includes exchange; (2) the eigenvalue equations in I of an exciton and a plasmon in insulators; and (3) the exact high density correction to the Landau diamagnetism in III. Temperature-dependent two-particle Green's functions are applied in I to the grand potential, and to the screening of the Coulomb potential in the low temperature limit and in the classical limit, which leads to the Debye shielding length. Time- and temperature-dependent Green's functions are introduced in I and their utility for the study of transport properties is discussed. The formal similarity of the three different Green's function types is stressed, and a terse presentation generally prevails.

J. R. Klauder (Murray Hill, N.J.)

7337:

Tittle, C. W. Theory of neutron logging. I. Geophysics **26** (1961), 27–39.

Author's summary: "An analytical theory of epithermal neutron logging is presented. One-group diffusion theory is applied to the slowing down of neutrons from a point fast neutron source in infinite continuous media, in a single cylinder, and in concentric cylinders representing a fluid-filled borehole and the surrounding formation. Numerical results are given for the epithermal neutron flux in a water-filled hole six inches in diameter, passing through limestone of 10 percent or 30 percent porosity. Preliminary semiquantitative agreement is obtained with the relative response of a commercial logging instrument in the range of 10 to 100 percent porosity."

7338:

Adams, Edward N. Irreversible processes in isolated systems. Phys. Rev. (2) **120** (1960), 675–681.

It is suggested that for any physical system there exist two transport equations, one causal and the other anti-causal, both being consistent with the fundamental equations of mechanics. The problem is discussed of how experimental procedures always yield a situation obeying the causal transport equation. *S. Simons* (London)

7339:

Bellman, Richard; Kalaba, Robert; Wing, G. Milton. Invariant imbedding and mathematical physics. I. Particle processes. J. Mathematical Phys. **1** (1960), 280–308.

The authors make use of "invariant imbedding", i.e., the imbedding of certain physical processes within a family of processes of a similar nature, in order to arrive at an analytic formulation of classical particle processes, e.g., diffusion theory, radiative transfer, transport theory, multiple scattering and random walk. The standard method for treating such processes eventually comes to reducing them to the form of a system of linear equations. The authors propose the alternate scheme of reducing such problems to an iteration of certain nonlinear transformations. In particular the invariant imbedding method provides a way to reduce systems of linear differential equations subject to boundary conditions at a pair of points to initial value problems (relative to some parameter, not necessarily time) for nonlinear systems.

The paper contains 55 references to the literature. For further reading on invariant imbedding by the same authors as the paper under review one may consult, e.g., Proc. Nat. Acad. Sci. U.S.A. **46** (1960), 1646–1649.

A. A. Mullin (Urbana, Ill.)

7340:

Green, Melville S. Comment on a paper of Mori on time-correlation expressions for transport properties. Phys. Rev. (2) **119** (1960), 829–830.

Kinetic coefficients such as heat conductivity are expressed by time-correlation functions of flux. The expressions given by H. Mori [Phys. Rev. (2) **112** (1958), 1829–1842; MR **20** #6823] and by the author [J. Chem. Phys. **22** (1954), 398–413; MR **15**, 921] are discussed and are shown to be equivalent with only some apparent differences in the statistical ensembles used and in the definition of heat flux.

R. Kubo (Tokyo)

7341:

Mathews, P. M.; Shapiro, I. I.; Falkoff, D. L. Stochastic equations for nonequilibrium processes. Phys. Rev. (2) **120** (1960), 1–16.

The evolution of a system of weakly interacting particles is described as a Markov process in which the transition rates, between system states specified by the occupation numbers for unperturbed single-particle states, are determined from first-order perturbation theory. A suitable generating function is introduced and used to study the equation of motion for the full joint-probability distribution of the occupation numbers. The general treatment is carried out for Maxwell-Boltzmann, Fermi-Dirac, and Einstein-Bose statistics. Specific application is made to a system of harmonic oscillators relaxing by interaction with a reservoir. *R. H. Kraichnan* (New York)

7342:

Mazo, Robert M. Partition function of an atom in a spherical box. Amer. J. Phys. **28** (1960), 332–335.

A formula for the density of spherical-wave states, used in the calculation of low-temperature second virial coefficients, is found to be invalid for large angular momenta. The formula therefore cannot be used for calculating partition functions. *O. Penrose* (London)

7343:

Luttinger, J. M. Fermi surface and some simple equilibrium properties of a system of interacting fermions. Phys. Rev. (2) **119** (1960), 1153–1163.

Some time ago, Migdal [Ž. Èksper. Teoret. Fiz. **32** (1957), 399–400; MR **19**, 813] considered the average occupation number of different single-particle momentum states for a system of fermions in the ground state and gave a proof that this occupation number is a discontinuous function of the momentum even in the case of interacting fermions. His proof made use of certain analyticity properties of the one-particle ground state propagator without attempting to show that the propagator possesses these analyticity properties. In the paper being reviewed, the author points out that the perturbation expansion for the propagator possesses the properties necessary to obtain Migdal's result. This leads to the definition of a sharp Fermi surface for interacting fermions in those cases where the perturbation expansion (even if it does not converge) can be taken

as an indication of the properties of the exact solution. It is well known that for effectively attractive forces, such as those giving rise to superconductivity, the perturbation expansion breaks down. In this case, one also knows that the Fermi surface is not sharp.

The author proceeds to apply the mathematical results obtained to a discussion of the low-temperature heat capacity of the system and of spin paramagnetism. Finally the effect of a periodic external potential, due to the crystal lattice, is considered. *B. Zumino* (New York)

7344:

Tareeva, E. E. Free-energy computation for a gas of low density. Dokl. Akad. Nauk SSSR **132** (1960), 568–571 (Russian); translated as Soviet Physics. Dokl. **5**, 551–555.

Consider a system of Fermions described by the following Hamiltonian:

$$H = \sum T(f)a_f^+a_f + \tfrac{1}{2}\sum U(f_1f_2|f_2'f_1')a_{f_1}^+a_{f_2}^+a_{f_2'}a_{f_1'}.$$

Here, a and a^+ have their usual meaning of annihilation and creation operators, T is the difference between the kinetic energy and the chemical potential, while U is some kind of interaction, in general depending on spins and momenta. If we denote an ensemble average of some quantity A by the symbol

$$\langle A\rangle = Q^{-1}\,\mathrm{Sp}\,[Ae^{-H/\theta}];\quad Q = \mathrm{Sp}\,[e^{-H/\theta}],$$

where θ is the absolute temperature $\theta = kT$, we can define the "two particle Green function" from

$$G_{f_1f_2g_1g_2}(t-t') = \varepsilon(t-t')\langle[a_{f_1}(t)a_{f_2}(t),\, a_{g_2}^+(t')a_{g_1}^+(t')]\rangle.$$

The "two particle correlation function" F_2 is defined from the Fourier transform $G(w)$ of the $G(t)$ by the relation

$$F_2 = \int_{-\infty}^{+\infty}\frac{G(w+i\varepsilon)-G(w-i\varepsilon)}{e^{w/\theta}-1}\,dw.$$

This correlation function is related to the free energy F of the system and consequently also to other thermodynamic quantities through relations of the form

$$\frac{\partial(\Delta F)}{\partial g} = \mathrm{Sp}\,[UF_2(g)].$$

Here, ΔF means the correction of the free energy of the unperturbed system when the Hamiltonian is changed from T to $T+gU$.

In the paper to be reviewed here the author first writes down an exact equation of motion for the Green function G. Such an equation is easily obtained from the ordinary quantum mechanical formalism where time derivatives are computed with the aid of commutators with the Hamiltonian H. The author considers the low density limit where only a negligible number of states has an occupation number close to unity. This means that higher order terms in the occupation numbers can be neglected and the resulting equation can be solved for G. The solution is given in terms of a complete set of eigenfunctions for the two particle Hamiltonian. The formalism sketched above then allows the author to write down an expression for the free energy involving a summation over the discrete states of the two particle Hamiltonian and an integration over the continuous spectrum. Afterwards, the low temperature and the high temperature limits are considered. In particular, the author finds in the low temperature limit that the system can be described as a mixture of Fermions and Bosons, where the latter are made up of bound two particle states of the original Fermions.

G. Källén (Lund)

7345:

Reiner, A. S. On the use of formal operator techniques in quantum statistical mechanics. Physica **26** (1960), 700–716.

By means of formal operator techniques, it is shown that a close parallel exists between iterative solutions of the partition function, in either its "time"-dependent form $e^{-\beta H_N}$ or in its "time"-independent resolvent form $(z-H_N)^{-1}$. In particular, a multiple scattering treatment of the resolvent operator is equivalent to an iteration in terms of the binary collision kernel of T. D. Lee and C. N. Yang [Phys. Rev. (2) **113** (1959), 1165–1177; MR **21** #1725], each leading to analogous linked cluster expansions for the cluster coefficients. As an application the author recalculates, in the scattering matrix formalism, the first three cluster coefficients for a hard sphere boson gas. *J. R. Klauder* (Murray Hill, N.J.)

7346:

Krieger, Irvin M.; Gans, Paul J. First-order stochastic processes. J. Chem. Phys. **32** (1960), 247–250.

The authors find the general solution to birth and death equations for which the transition probabilities from state i to state j are given by $\alpha_{ij}n_i$ where n_i is the occupation number of level i. Although the derivation begins with an incorrect equation, a succession of cancelling errors enable the authors to arrive at the known result. The solution to the difference-differential equations requires only a straightforward application of the generating function. For the authors' particular system it is shown that "a first-order system which has attained equilibrium will, upon introduction of a disturbance which changes the transition probabilities, relax to a new equilibrium through a set of multinomial distributions". The authors' contention that their method is simpler than that of Siegert [Phys. Rev. **76** (1949), 1708–1714] is questionable.

G. Weiss (Washington, D.C.)

7347:

Rumer, Yu. B. Negative and limiting temperatures. Ž. Èksper. Teoret. Fiz. **38** (1960), 1899–1902 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1365–1367.

The author writes the partition function $z(\lambda)$ for a thermodynamic system for the temperature variable in the entire complex plane as a Laplace integral (1) $z(\lambda) = \int_0^\infty \rho(E)e^{-\lambda E}dE$, where $\rho(E)$ is the spectral density for the energy of the system. Then on inverting the transformation (1) we get $\rho(E) = (2\pi i)^{-1}\int_{\sigma-i\infty}^{\sigma+i\infty} z(\lambda)e^{\lambda E}d\lambda$, where the integral is taken along a straight line $\mathrm{Re}\,\lambda = \sigma$ which lies entirely in the region of analyticity of the function $z(\lambda)$. For large values of the energy E the integral (1) can be calculated approximately by the method of steepest descent. With logarithmic accuracy we have $\ln\rho(E) = \ln z(\beta) + \beta E$, where the saddle point β is a root of the equation $z'(\beta)+z(\beta)E = 0$. It is shown that the saddle point lies on the real axis and is unique. In conclusion, the author studies two examples of "unusual" thermodynamic systems for which negative and limiting temperatures exist.

The two examples are the Ising model and the gas of noninteracting particles in an external field with an asymptotically logarithmic potential.

C. Y. Wang (Minneapolis, Minn.)

7348:

Friedman, Harold L. Density expansions of correlation functions for equilibrium systems. J. Chem. Phys. **34** (1961), 73–78.

Author's summary: "A method of deriving density expansions of correlation functions from density expansions of the excess free energy is based on certain thermodynamic relations between derivatives of the grand partition function and derivatives of the excess free energy. The method also depends on using the most general expansion of the interaction energy into components; it is not assumed that this energy is limited to pairwise components. Expansions are obtained for the correlation functions in multicomponent gases and for the correlation functions for solute species in solutions. The method is also applicable to ionic solutions. A mathematical difficulty limits the systems for which these equations are proved to those in which there is a certain flexibility in treating the model."

7349:

Van Hove, Léon. Lectures on statistical mechanics of non equilibrium phenomena. La théorie des gaz neutres et ionisés (Grenoble, 1959), pp. 149–183. Hermann, Paris; Wiley, New York; 1960.

This lecture deals mainly with the author's own work [Physica **21** (1955), 517–540; **23** (1957), 441–480; MR **17**, 115; **19**, 695] on derivation of the master equation from the first principle of quantum mechanics. But it begins with a short review of Boltzmann equation, Pauli equation and the derivation of the former from the latter. This classical method is then compared to a more modern method of the statistical-mechanical theory of irreversible processes by the example of electrical conductivity which is now expressed in terms of an equilibrium correlation of current. An approximation to this general formula leads to the result which is obtained from the older theories. The last parts of the lecture are devoted to a sketch of the derivation of Pauli equation. It is shown that a generalized Pauli equation for an infinite system can be obtained for the coarse grained transition probability by the technique of resolvent operators and diagrams. For small coupling or low density of scatterers, the generalized Pauli equation is easily reduced to the ordinary Pauli equation. The condition of this is also discussed briefly. *R. Kubo* (Tokyo)

ELASTICITY, PLASTICITY

See also A7076, 7441, 7885, 7886.

7350:

Long, Robert R. ★Mechanics of solids and fluids. Prentice-Hall Series in Engineering of the Physical Sciences. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1961. x+156 pp. $9.00.

This book gives an elementary account of the subject and assumes only a limited mathematical knowledge. The theory is developed using Cartesian tensors and a careful introduction to these is given in the first chapter. In the section on elasticity the problems of flexure, plane stress and plane strain are briefly discussed and the compatibility conditions for strain and stress are derived. In the section on fluids the Lagrangian and Eulerian equations of motion are derived and an introduction is given to the idea of a boundary layer. A chapter is also devoted to an examination of the experimentally observed physical properties of real materials. Each chapter is followed by exercises for the student and by references to more advanced treatments. The book provides a good elementary introduction to the subject particularly suited to engineers and physicists. *J. E. Adkins* (Providence, R.I.)

7351:

Freiberger, W. F. (Editor-in-chief). ★The international dictionary of applied mathematics. D. Van Nostrand Co., Inc., Princeton, N.J.-Toronto-London-New York, 1960. viii+1173 pp. $25.00.

This dictionary is intended to explain briefly the terms, concepts and methods used in the application of mathematics to physical and engineering problems. The subjects covered are: acoustical engineering, acoustics, aerodynamics and hydrodynamics, astronomy, atomic structure, automatic control, chemistry, differential geometry, elasticity, electromagnetic theory, general mathematics, mechanics, molecular structure, network topology, nuclear engineering, nuclear science, numerical analysis, optics, probability and statistics, quantum mechanics, relativity, solid-state physics, spectroscopy, statistical mechanics, strength of materials, structural analysis and design, tensor analysis, thermodynamics, vector analysis, viscoelasticity and plasticity. Faced with so wide a range of subjects, a reviewer can only read critically the entries in his own field and see if he can understand the entries in other fields, a sampling process whose result may depend on the interests of the reviewer. Subject to this qualification, the dictionary entries are generally clear, precise and as comprehensive as can be expected from the division of 8000 entries among 32 subjects, and the layout and cross-referencing is so good that there is no difficulty in finding all the information about a particular topic. This is made easy for speakers of languages other than English by providing indices in German, Russian, French and Spanish with the English equivalents.

I would expect the dictionary to be useful as an aid to the understanding of work not in one's own field that was relevant to a particular problem, but it is probable that more detailed information would be required than any comprehensive dictionary could supply. For this reason, I would like more references to standard texts than have been included. On the whole, it is a good example of its class of technical reference books and many people will find it useful. *A. A. Townsend* (Cambridge, England)

7352:

Prager, William. An elementary discussion of definitions of stress rate. Quart. Appl. Math. **18** (1960/61), 403–407.

The author discusses stress rate definitions of Jaumann, Cotter and Rivlin, Oldroyd and Truesdell, referred to a fixed, rectangular Cartesian coordinate system, which differ mutually by terms which vanish if the motion is

(locally) a rigid body motion. Only Jaumann's definition implies that the invariants of the stress tensor are stationary for vanishing stress rates. For this reason the author prefers Jaumann's definition for application in the constitutive equations of plasticity in which the stress invariants play a major role. Discussion in the paper is lucid and informative. *W. T. Koiter* (Delft)

7353:

Truesdell, C. Stages in the development of the concept of stress. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 556–564. SIAM, Philadelphia, Pa., 1961.

An informative historical review.

7354:

Paul, B. A simplified geometric proof of the reciprocal stress theorem. Quart. Appl. Math. **18** (1960/61), 395–396.

7355:

Argyris, J. H.; Kelsey, S. The validity of the initial strain concept. J. Roy. Aero. Soc. **65** (1961), 129–136.

7356:

Rüdiger, D. Eine Verallgemeinerung des Prinzips vom Minimum der potentiellen Energie elastischer Körper. Ing.-Arch. **27** (1960), 421–428.

The paper is concerned with a variational formulation of the boundary value problem of linear elasticity consisting of the differential equations

$$\sigma_{x,x}+\cdots+\overline{P}=0, \quad \sigma_x=2\mu\varepsilon_x+\lambda(\varepsilon_x+\varepsilon_y+\varepsilon_z), \quad \text{etc.}$$

and of the boundary conditions

$$p=\overline{p}, \text{ etc. on } S_s, \quad u=\overline{u}, \text{ etc. on } S_d,$$

where

$$\varepsilon_x=u_{,x}, \quad p=\sigma_x\cos(n,x)+\cdots, \quad \text{etc.}$$

The author states that "among all possible displacements which on the entire surface can assume arbitrary values, the actually occurring ones make an extremum of the expression"

$$\Pi=\tfrac{1}{2}\iint_{S_s}[(p-2\overline{p})u+\cdots]\,dS-\tfrac{1}{2}\iint_{S_d}[(u-2\overline{u})p+\cdots]\,dS+\tfrac{1}{2}\iiint[(P-2\overline{P})u+\cdots]\,dV.$$

According to the author this statement differs from the standard form of the principle of minimum potential energy by the fact that the comparison displacement functions need not satisfy the prescribed displacement boundary conditions. It may be inferred, although this is not explicitly stated, that the quantities P, Q, R in Π are to be taken as combinations of second derivatives in u, v, w, of the form

$$-P=\sigma_{x,x}+\cdots=[2\mu\varepsilon_x+\lambda(\varepsilon_x+\varepsilon_y+\varepsilon_z)]_{,x}+\cdots=[2\mu u_{,x}+\lambda(u_{,x}+\cdots)]_{,x}+\cdots, \quad \text{etc.},$$

and similarly on S

$$p=[2\mu u_{,x}+\lambda(u_{,x}+\cdots)]\cos(n,x)+\cdots, \text{ etc.}$$

{A more conventional form of such generalization of the principle of minimum potential energy may be obtained by specialization of formulas given by the reviewer [J. Math and Phys. **32** (1953), 129–135; MR **15**, 369]. Starting with the general variational equation $\delta I=0$ where, here,

$$I=\iiint[u_{,x}\sigma_x+\cdots-(u\overline{P}+\cdots)-W(\sigma_x,\cdots)]\,dV-\iint_{S_d}[(u-\overline{u})p+\cdots]\,dS-\iint_{S_s}[\overline{p}u+\cdots]\,dS,$$

one eliminates stresses by means of stress strain relations. In this way one obtains

$$\Pi=\iiint[W^*(u_{,x},\cdots)-(\overline{P}u+\cdots)]\,dV-\iint_{S_d}[(u-\overline{u})p+\cdots]\,dS-\iint_{S_s}[\overline{p}u+\cdots]\,dS$$

This reduces to the conventional form of the theorem, if it is assumed that all comparison functions satisfy the boundary conditions on S_d.

The author's less familiar looking form of the theorem follows upon first writing in I

$$W(\sigma_x,\cdots)=\tfrac{1}{2}[u_{,x}\sigma_x+\cdots]$$

and upon then using the integration-by-parts formula

$$\iiint[u_{,x}\sigma_x+\cdots]\,dV=\iint[up+\cdots]\,dS-\iiint[(\sigma_{x,x}+\cdots)u+\cdots]\,dV.\}$$

E. Reissner (Cambridge, Mass.)

7357:

Grzedzielski, Alex L. M. The initial strain concept. J. Roy. Aero. Soc. **65** (1961), 127–129.

7358:

Nowacki, W. On certain boundary problems of the theory of elasticity. J. Sci. Engrg. Res. **1** (1957), 29–36.

The author advocates use of integral equations to solve boundary value problems in elasticity; the difference between the author's treatment and conventional treatment seems to be primarily in the author's interpretation of results in terms of structural methods.

C. E. Pearson (Cambridge, Mass.)

7359:

Rivaud, Jacques. Sur la flexion circulaire en théorie des déformations finies. C. R. Acad. Sci. Paris **251** (1960), 32–34.

In this paper the author considers a cylinder whose axis, lying along the x-axis of coordinates, undergoes a circular bending as well as longitudinal dilatation. The author defines the circular deflection to be that transformation

which transforms a point (x, y, z) of the cylinder to a point (X, Y, Z) such that

$$X = (\rho+y+v)\sin\frac{x(1+\varepsilon)}{\rho}, \quad Y = (\rho+y+v)\cos\frac{x(1+\varepsilon)}{\rho},$$
$$Z = z+w,$$

where ρ is the radius of the circle into which the axis of the cylinder is transformed, $\varepsilon = \varepsilon(\rho)$ is the dilatation coefficient and v and w are functions independent of x. The functions v, w and ε are obtained by successive approximation, such that each approximation satisfies the conditions of the possibility of the problem. These conditions are that both the resultant force and the couple in a plane section of the cylinder vanish. The author demonstrates the validity of his solution by an induction process. In fact, writing

$$v = \sum_1^\infty \frac{v^{(p)}}{\rho^p}, \quad w = \sum_1^\infty \frac{w^{(p)}}{\rho^p}, \quad \varepsilon = \sum_1^\infty \frac{\varepsilon^{(p)}}{\rho^p},$$

and assuming that the first, second, $\cdots$, pth approximations satisfy the conditions of the possibility of the problem, the author shows that the $(p+1)$th approximation also satisfies the same conditions.

The author also demonstrates the convergence of the successive approximations if ρ is taken to be suitably large.

M. Nassif (Assiut)

7360:

Teodorescu, Petre P. Sur le problème du coin plan élastique. C. R. Acad. Sci. Paris **250** (1960), 3446–3448.

In this note the author considers the elastic infinite quadrant $x \geqq 0$, $y \geqq 0$, acted along one of its edges ($y=0$, say) by a normal load of the form

$$p(x) = \int_0^\infty b(\alpha)\cos\alpha x\, d\alpha.$$

The author assumes the following expressions for the normal tensions σ_x, σ_y and the tangential tension τ_{xy}:

$$\sigma_x = -\int_0^\infty A(\alpha)(1-\alpha y)e^{-\alpha y}\cos\alpha x\, d\alpha - \int_0^\infty B(\beta)(1+\beta x)e^{-\beta x}\cos\beta y\, d\beta,$$

$$\sigma_y = -\int_0^\infty A(\alpha)(1+\alpha y)e^{-\alpha y}\cos\alpha x\, d\alpha - \int_0^\infty B(\beta)(1-\beta x)e^{-\beta x}\cos\beta y\, d\beta,$$

$$\tau_{yx} = \tau_{xy} = -y\int_0^\infty \alpha A(\alpha)e^{-\alpha y}\sin\alpha x\, d\alpha - x\int_0^\infty \beta B(\beta)e^{-\beta x}\sin\beta y\, d\beta.$$

Applying the boundary conditions $\sigma_x = 0$, $\tau_{xy} = 0$ along $x=0$ and $\sigma_y = p(x)$, $\tau_{yx} = 0$ along $y=0$, the author shows that the functions $A(\alpha)$ and $B(\beta)$ satisfy the integral equations

$$A(\alpha) + \frac{4}{\pi}\int_0^\infty \frac{\alpha^2\beta B(\beta)}{(\alpha^2+\beta^2)^2}\, d\beta = -b(\alpha),$$

$$B(\beta) + \frac{4}{\pi}\int_0^\infty \frac{\alpha\beta^2 A(\alpha)}{(\alpha^2+\beta^2)^2}\, d\alpha = 0.$$

The problem is now completely solved by calculating the displacement components from σ_x, σ_y and τ_{xy} and by applying the condition that the origin 0 of coordinates remains fixed.

M. Nassif (Assiut)

7361:

Deresiewicz, H. A note on second-order Hertz contact. J. Appl. Mech. **28** (1961), 141–142.

7362:

Polozii, G. N. General solution of the problem of contact with a rigid edge for an arbitrary polygon and an arbitrary polygonal aperture. Kiïv. Derž. Univ. Nauk Zap. **16** (1957), no. 2 = Kiev. Gos. Univ. Mat. Sb. **9** (1957), 35–51. (Russian)

The paper is devoted to the solution of two problems: (1) the solution of the equations of plane strain appropriate to a region of the plane bounded by the straight lines A_1A_2, A_2A_3, $\cdots$, A_nA_1, it being assumed that the normal component of displacement and the normal component of stress are prescribed at each point of each segment A_iA_{i+1} and, in particular, that the side A_nA_1 of the polygon is in contact with a rigid line; (2) the similar problem for an infinite two-dimensional region with a polygonal opening, one edge of the polygon being kept rigid.

The solutions of both problems are interesting exercises in the use of Muskhelišvili's method of Cauchy integrals. Closed expressions are obtained for the complex potentials occurring in the Krylov-Muskhelišvili equations but no calculations are made using these expressions.

I. N. Sneddon (Glasgow)

7363:

Mitchell, T. P.; Weese, J. A. Stress distributions analyzed in bispherical co-ordinates. J. Appl. Mech. **27** (1960), 726–732.

The authors apply the general solution of the field equations of classical elastostatics in terms of the Papkovich-Neuber stress functions, referred to spherical bipolar coordinates, to the solution of three specific axisymmetric boundary-value problems. The problems considered are: (a) the stress distribution in a half-space with a spherical cavity, due to a concentrated normal load at the plane boundary, the load axis passing through the center of the cavity; (b) the stress distribution in a half-space with a spherical cavity, due to a uniform pressure at the boundary of the cavity; (c) the stress distribution in a body bounded by two non-concentric spheres, induced by a uniform pressure at the internal boundary. Each of the foregoing problems is reduced to the solution of an infinite system of linear algebraic equations. The solution of this system of equations by the usual approximative numerical method is, in each instance, illustrated for one choice of the governing geometric parameter.

E. Sternberg (Providence, R.I.)

7364:

Kämmel, G. Zur Theorie des räumlich gekrümmten Stabes. Ing.-Arch. **27** (1959), 255–267.

Es werden die allgemeinen Gleichungen zur Berechnung des Spannungs- und Formänderungszustandes des räumlich gekrümmten Stabes abgeleitet. Die sonst übliche Annahme, daß die Krümmungshalbmesser der Stabachse groß sind

im Vergleich zu den Querschnittsabmessungen wird nicht verwendet. Nachdem die geometrischen Zusammenhänge dargelegt sind, wird der Verformungszustand der Stabachse und der Verschiebungszustand der Querschnittspunkte beschrieben, wobei von der Bernoullischen Hypothese vom Ebenbleiben der Querschnitte Gebrauch gemacht wird. Die 6 Gleichgewichtsbedingungen werden in üblicher Weise durch die Schnittgrößen (Resultierende der Spannungen) ausgedrückt. Die Schnittgrößen werden dann als Funktionen der Formänderungsgrößen der Stabachse dargestellt. Damit ergeben sich 10 Differentialgleichungen des Spannungs- und Verformungszustandes. Eine weitere Gleichung für das Torsionsmoment wird aus der Änderung des Drillwinkels um die Stabachse gewonnen. Die allgemeine Theorie wird auf den Träger, dessen Stabachse eine Schraubenlinie beschreibt, angewendet. Dieses Problem führt auf eine Differentialgleichung 4. Ordnung mit konstanten Koeffizienten. Der Spannungs- und Verschiebungszustand wird als Funktion der Belastung explizit angegeben. Numerische Ergebnisse finden sich nicht.

W. Zerna (Hannover)

7365:

Shanley, F. R. ★Weight-strength analysis of aircraft structures. 2nd ed. Dover Publications, Inc., New York, 1960. xiii + 404 pp. $2.45.

The second edition of this book is an unabridged republication of the first edition published in 1952 (McGraw-Hill, New York), to which have been added separate bibliographies on Optimum Design of Structures and Creep Buckling. The book has served as text book for the course in Optimum Design of Structures at the University of California. Although it is the most competent all-embracing treatise on the subject, it is regrettable that the works contained in the bibliography attached and which appeared after the first edition have not been used to bring the text up to date.

J. Solvey (Melbourne)

7366:

Frisch-Fay, R. Non-linear bending of a cantilever under several concentrated loads. Austral. J. Appl. Sci. **11** (1960), 233–243.

The title problem is solved assuming that the loads are parallel, but this can be generalized for loads acting in arbitrary directions.

H. D. Conway (Ithaca, N.Y.)

7367:

Nariboli, G. A. Torsion function for built-up bodies. Proc. 4th Congress Theoret. Appl. Mech. 1958, pp. 65–72. Indian Soc. Theoret. Appl. Mech., Kharagpur.

This paper deals with the solution of the torsion problem for a cylinder whose cross section consists of a symmetrical region made up of two rectangles and a triangle. When the triangle takes certain particular shapes, the torsion function for each sub-region can be determined under given boundary conditions and continuity requirements between the different sub-regions. The method leads to the solution of an infinite set of simultaneous equations for the unknown constants introduced.

R. M. Morris (Cardiff)

7368:

Yonekawa, Motonobu. The analysis of frame vibrating systems. Rev. Elec. Comm. Lab. **8** (1960), 397–409.

Numerical examples are worked out for the forced and free vibrations of a portal frame composed of three uniform beams.

G. Kron (Schenectady, N.Y.)

7369:

Szidarovszky, J. Die Beanspruchung von Stäben mit veränderlichem Querschnitt auf Biegung bei gleichzeitigem Druck. Acta Tech. Acad. Sci. Hungar. **31** (1960), 125–138. (English, French, and Russian summaries)

7370:

Sato, Takeshi B. Equations of motion for curved and twisted beam with noncoincident mass and elastic axes. Proc. Fac. Engrg. Keio Univ. **11** (1958), 66–70.

The equations of motion for a curved and twisted beam with non-coincident mass and elastic axes are deduced using the Hamiltonian principle. It is assumed that the radius of gyration and the distance between the mass and elastic axes are small compared with the length of the beam.

R. C. DiPrima (Troy, N.Y.)

7371:

Guldan, Richard. ★Rahmentragwerke und Durchlaufträger. 6te erweiterte Aufl. Aus dem Nachlass des Verfassers, herausgegeben und bearbeitet von Horst Reimann. Springer-Verlag, Vienna, 1959. xxiii + 501 pp. $20.00.

Dies weitverbreitete Spezialwerk über Rahmstatik liegt nunmehr in erweiterter Form als sechste Auflage vor. Der erste Teil des Buches ist den statischen Rechnungsgrundlagen gewidmet. Im zweiten Teil finden sich Zahlenbeispiele. Der dritte Teil enthält eine Anzahl von Hilfstafeln zur Berechnung von Rahmentragwerken und Durchlaufträgern. Im Vergleich zu früheren Auflagen sind vor allem folgende Erweiterungen zu bemerken. Das Wesen der unverschieblichen und verschieblichen Tragwerke wird eingehend betrachtet. Die theoretischen Grundlagen sind ausführlicher dargelegt, insbesondere im Hinblick auf Rahmentragwerke mit und ohne Stabendverstärkungen, die auch gelenkige Stabanschlüsse aufweisen. Um die Vorteile des Drehwinkelverfahrens auch für die Berechnung von lotrecht verschieblichen Rahmentragwerken mit gelenkigen Stabanschlüssen voll zur Geltung zu bringen, wurden hierfür zweckmäßige Mustergleichungen aufgestellt und deren Anwendung an Beispielen gezeigt.

W. Zerna (Hannover)

7372:

Chmelka, Fritz; Melan, Ernst. ★Einführung in die Festigkeitslehre: für Studierende des Bauwesens. 4te, umgearbeitete und ergänzte Aufl. Springer-Verlag, Vienna, 1960. viii + 369 pp. $7.60.

Das Buch wendet sich hauptsächlich an Studierende des Bauingenieurwesens, aber auch für den Maschinenbauer enthält es manch Wissenswertes. Es bringt die für die Durchführung einfacher Bemessungsaufgaben grundlegenden Gedanken der technischen Festigkeitslehre in leicht verständlicher Weise. In 8 Kapiteln werden die allgemeinen Grundlagen der Festigkeitslehre, die Flächenmomente, Biegungs- und Schubbeanspruchung gerader Träger, die Biegelinie, Torsionsprobleme, das Knickverhalten von

Stäben, sowie einige Grundlagen der statisch unbestimmten Tragwerke behandelt. Der Stoff wird mit zahlreichen Beispielen erläutert, wobei sowohl die deutschen als auch die österreichischen Normen Berücksichtigung finden.

W. Zerna (Hannover)

7373:

Nowinski, J. Theory of thin-walled bars. Appl. Mech. Rev. **12** (1959), 219–227.

Survey article.

7374:

Szelągowski, Franciszek. A semi-infinite plate acted on by an internal load. Rozprawy Inż. **7** (1959), 541–549. (Polish. Russian and English summaries)

7375:

Nash, William A.; Modeer, James R. Certain approximate analyses of the nonlinear behavior of plates and shallow shells. Proc. Sympos. Thin Elastic Shells (Delft, 1959), pp. 331–354. North-Holland, Amsterdam, 1960.

Berger's method of neglecting the second invariant of the middle surface strains is extended to obtain uncoupled equations for the finite vibrations of plates and the equilibrium and buckling of shallow spherical shells. The results obtained compare favourably with those already known.

The period of a vibrating hinged plate is found in terms of the complete elliptic integral of the first kind, while the solution for the shallow spherical shells is expressed in terms of Bessel's functions. *B. R. Seth* (Kharagpur)

7376:

Galimov, K. Z. On large deflections of a rectangular cylindrical panel. Inžen. Sb. **25** (1959), 20–36. (Russian)

The first part is devoted to the problem of large displacements of a rectangular cylindrical shell segment all edges of which are subjected to the action of forces perpendicular to them and free to slide in tangent planes. The non-linear equations for the stress function and the displacement are taken in the form given by V. Z. Vlasov [*Obščaya teoriya oboloček i eë priloženiya v tehnike*, Gosudarstv. Izdat. Tehn.-Teoret. Lit., Moscow, 1949; MR **11**, 627]. The displacement is assumed in the form of a double trigonometric series, such that all geometric boundary conditions are satisfied. The stress function is taken in the form of a sum of rather complicated double series and certain functions, the coefficients being so chosen that (1) the compatibility condition, (2) the statical boundary conditions and (3) the Bubnov-Galerkin equation for the second equation of the problem are satisfied. As a particular case a square and a rectangular shell segment with the ratio of sides 1:2 are examined. Only one or two terms of the expansion for the stress function have been taken.

The second part is concerned with a segment simply supported along the edges, by a method analogous to that in the first part. Finally, a segment clamped along all edges is investigated. *W. Urbanowski* (Warsaw)

7377:

Conway, H. D. On some systems of equations encountered in thin plate and elasticity theory. J. Appl. Mech. **28** (1961), 143–144.

7378:

Verma, G. R. Note on bending of rectangular plates by concentrated couples on its edge. J. Appl. Mech. **28** (1961), 142.

7379:

Bufler, H. Der Spannungszustand in einem aus zwei elastischen Halbscheiben bestehenden Verbundkörper. Ing.-Arch. **29** (1960), 233–249.

In a previous paper in Z. Angew. Math. Mech. **39** (1959), 218–236 [MR **21** #4607] the author calculated the normal and shearing tractions between two semi-infinite plates of equal thicknesses connected with each other along a finite portion of their boundaries and subjected to various types of loading. In this paper the state of stress at a point inside one of the two plates is completely determined by means of the complex variable method. The types of loading on the plates are the same as before, including normal force, shear force without twisting, pure shear, pure twisting, and thermal and initial stresses. Numerical results are presented for the case of two plates made of the same material and for the case of one plate being rigid.

Yi-Yuan Yu (Brooklyn, N.Y.)

7380:

Kurutz, I. Die Berechnung radialer Laufräder mit konischer Deckscheibe. Period Polytech. Engrg. **4** (1960), 267–276.

7381:

Mansfield, E. H. On the tension loads in rivets connecting stringers to shear-buckled skin. J. Roy. Aero. Soc. **65** (1961), 59–60.

7382:

Valenta, J. État élastique des récipients cylindriques à couches multiples embattues à chaud. Acta. Tech. Acad. Sci. Hungar. **31** (1960), 227–240. (German, English, and Russian summaries)

Author's summary: "The author solves several cases of the stress distribution in multi-layer, cylindrical vessels assembled at higher temperature, which consist of cylinders with equal wall thicknesses. The results are obtained by solving the difference equation of the first order, and are expressed by digamma and trigamma functions. The author cites also the case of an ideally pre-stressed vessel and defines the limit between a pre-stressed cylindrical vessel and a thick-walled cylinder."

7383:

Flügge, Wilhelm. ★Stresses in shells. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1960. xi+499 pp. DM 58.80.

About two hundred pages of this book are devoted to linear membrane theory (which is that aspect of shell theory where bending moments and transverse shear forces, as well as non-linear effects, are assumed negligible. For many problems this is a practically adequate assumption.) The treatment here is quite thorough and comprehensive, except concerning the nature of appropriate boundary conditions in problems where this represents a

significant question (as for instance in the problem of unsymmetrical deformations of spherical shells).

About one hundred pages are concerned with bending theory of circular cylindrical shells. Unsymmetrical deformations are analyzed to a considerable extent by means of the author's own system of basic equations.

Another one hundred pages cover the subject of linear-theory bending of shells of revolution. This includes the classical material on symmetrical bending and edge effects associated with the names of H. Reissner, Meissner and Geckeler, but does not include more recent results and simplified methods of derivation. Unsymmetrical bending of spherical shells of constant thickness is treated following A. Havers [Ing.-Arch. **6** (1935), 282–312]. In addition to this, there is a treatment of conical shells with wall thickness proportional to distance from the apex where the differential equations of the problem become a system of equidimensional equations. Not included here are results on shallow spherical shells, in spite of their practical significance and amenability to elegant treatment.

Seventy pages of the book are devoted to an introductory treatment of buckling problems. In addition to classical material on buckling of circular cylindrical shells, the author presents some non-conventional material concerned with the linear buckling theory of cylindrical shells of finite length. Seven pages are an outline of non-linear buckling theory as put forward by Cox, Donnell and von Karman, and six pages describe aspects of the linear buckling theory of spherical shells.

The book concludes with a two-page appendix on circular ring theory and with thirteen pages of bibliographical notes including references to many relatively recent publications. *E. Reissner* (Cambridge, Mass.)

7384:

Muštari, H. M.; Teregulov, I. G. The theory of shells of moderate thickness. Dokl. Akad. Nauk SSSR **128** (1959), 1144–1147 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1129–1132.

7385:

Varga, L. Bestimmung der Spannungen in den durch konzentrierte Einzelkräfte belasteten und durch Stringer ausgesteiften Schalenkonstruktionen. Period. Polytech. Engrg. **4** (1960), 251–265.

7386:

Stuiver, Willem. An approximate solution for the symmetrical end problem of conical shells. J. Aerospace Sci. **28** (1961), 71–72.

7387:

Reuss, E.; Thamm, F. Der Membranspannungszustand in einer Kugelschale in der Umgebung eines konzentrierten Momentes. Period. Polytech. Engrg. **4** (1960), 217–226.

7388:

Saelman, B. A note on the minimum-weight design of spherical and cylindrical pressure surfaces. J. Aerospace Sci. **28** (1961), 72–73.

7389:

Hieke, M. Spezialfälle der belasteten Kreismembran. Z. Angew. Math. Mech. **40** (1960), 268–274. (English and Russian summaries)

Für die Untersuchung einiger Schwingungsformen einer am Rand angespannten Kreismembran werden zwei Fälle betrachtet. Einmal wird eine plötzlich einsetzende statische Belastung und zum anderen eine rhythmische Belastung angesetzt. Die inhomogene partielle Differentialgleichung des Problems wird mit Hilfe der Weberschen Beziehungen für Besselsche Funktionen solchen Lösungen zugeführt, deren Störungsfunktionen über einen zum Mittelpunkt der Membran konzentrischen Kreis verteilt sind. Es wird gezeigt, daß dieser Lösungsweg auch gangbar ist, wenn die Störungsfunktionen längs konzentrischer Kreislinien angreifen. *W. Zerna* (Hannover)

7390:

Schaefer, Hermann. Die Analogie zwischen den Verschiebungen und den Spannungsfunktionen in der Biegetheorie der Kreiszylinderschale. Ing.-Arch. **29** (1960), 125–133.

Die sechs Gleichgewichtsbedingungen der Biegetheorie der Kreiszylinderschale werden durch sechs Spannungsfunktionen identisch erfüllt. Die Randwerte der Spannungsfunktionen werden angegeben. Für die Deformation der Schale werden sechs Verträglichkeitsbedingungen aufgestellt. Sie haben formal die gleiche Gestalt, wie die Gleichgewichtsbedingungen und lassen sich durch sechs Verschiebungsgrößen identisch befriedigen. Es wird gezeigt, daß zwischen Verschiebungen und Spannungsfunktionen sowie zwischen Deformationsgrößen und Schnittgrößen Analogie besteht. Bei Verwendung des bekannten von Reißner—Meißner eingeführten Elastizitätsgesetzes wird diese Analogie nicht gestört. Schließlich werden sämtliche Deformationen durch drei Verschiebungsgrößen und sämtliche Schnittgrößen durch drei Spannungsfunktionen ausgedrückt. Das Variationsproblem für die Spannungsfunktion wird angegeben und gezeigt, daß die hier angegebene Biegetheorie der bekannten Flüggeschen Theorie an Genauigkeit nicht nachsteht. Die Formulierung des Schalenproblems mit Hilfe der drei Spannungsfunktionen wird sich dann empfehlen, wenn die Randbelastung vorgegeben ist. Dann lassen sich die Randbedingungen in den Spannungsfunktionen übersichtlicher formulieren, als in den Verschiebungen.
W. Zerna (Hannover)

7391:

Neuber, H. Der abwickelbare Schalenträger. Z. Angew. Math. Mech. **40** (1960), 22–38.

Es wird der Membranspannungszustand einer Schale behandelt, deren Mittelfläche abwinkelbar und auf ihrer Oberfläche unbelastet ist. Die Aufgabe wird auf die Bestimmung von zwei Spannungsfunktionen zurückgeführt. Die Gleichungen zur Bestimmung dieser Spannungsfunktionen werden mit Hilfe des Kräftepotentials über das zugehörige Variationsproblem gefunden und auf dem Wege der Berechnung der Formänderungsgrößen kontrolliert. Die Lösungen der beherrschenden Differentialgleichung werden diskutiert, und die Kreiskegelschale als Beispiel behandelt. Numerische Ergebnisse werden nicht angegeben. *W. Zerna* (Hannover)

7392:

Jäger, B. Die Eigenfrequenzen verwundener Schaufeln. Ing.-Arch. **29** (1960), 280–290.

Es wird die Eigenfrequenz schlanker Schaufel, die einseitig eingespannt (starr oder elastisch) oder gelenkig gelagert sind, ermittelt. Dabei wird die Schaufel durch eine gerade mit Elastizität und Masse belegte Linie ersetzt. Die Elastizität wird mit der Richtung der Biegung als veränderlich angenommen. Die bei der umlaufenden Schaufel auftretenden Fliehkräfte werden in die Rechnung eingesetzt. Die einzelnen Schaufelschnitte werden als starr angenommen, eine Torsion der Schaufel wird nicht berücksichtigt. Zur Lösung wird die Methode der Übertragungsmatrizen angewendet. Ein Beispiel wird unter Verwendung eines digitalen Rechenautomaten durchgerechnet. Die gerechnete Eigenfrequenz stimmt gut mit der Messung überein. *L. Speidel* (Mülheim)

7393:

Dunwoody, N. T. The free vibrations of membranes with elliptical boundaries. Quart. J. Mech. Appl. Math. **13** (1960), 359–368.

A numerical method for finding normal frequencies of an elliptical membrane. The technique is to avoid Mathieu functions by use of a Fourier series, and then to solve the resulting infinite set of equations numerically. *J. W. Craggs* (Newcastle-upon-Tyne)

7394:

Mitra, A. K. Note on forced torsional oscillation of a circular cylinder by the application of a transient force at one end. J. Aerospace Sci. **28** (1961), 246–247.

7395:

Bird, J. F.; Hart, R. W.; McClure, F. T. Vibrations of thick-walled hollow cylinders. Exact numerical solutions. J. Acoust. Soc. Amer. **32** (1960), 1404–1412.

The authors consider plane strain vibrations of infinitely long thick-walled cylinders. While special cases of this problem have been treated previously, the present paper adopts a numerical approach and deals with a wider range of boundary conditions than is possible analytically. Specifically, the cases in which the outer wall is free, supported, or clamped and the inner wall is free, supported, or matched to a gas filling the inner cavity are handled. Extensive calculations are reported, the results appearing as graphs of the frequency versus ratio of inner and outer shell radii. The authors conclude that "... (i) the shear-compressional wave coupling in the solid is very weak (except for low frequency); (ii) for the gas-filled cylinder, the modes are very close either to the gas quasi-modes (gas in a rigid-walled cylinder), or to the solid quasi-modes (shell with free inner surface), depending on the geometry". *W. E. Boyce* (Troy, N.Y.)

7396:

Bird, J. F. Vibrations of thick-walled hollow cylinders. Approximate theory. J. Acoust. Soc. Amer. **32** (1960), 1413–1419.

The author adopts the conclusion (i) of the previous paper (see review above)—that coupling between shear and compression waves in a thick-walled infinitely long cylinder is weak—as his starting point in the development of an approximate theory governing the thickness modes of such cylinders. The procedure is of the perturbation type based on waves which are either purely shear or purely dilatational. The author establishes satisfactory accuracy for all cases except low modes of very thick cylinders by comparison with previous exact calculations. Since only elementary functions appear (Bessel functions being replaced by their asymptotic forms), and since only a few terms are required in most cases, the calculations are feasible on desk machines. *W. E. Boyce* (Troy, N.Y.)

7397:

Federhofer, Karl. Über Reibungseinflüsse bei den ebenen Biegungsschwingungen eines elastischen Kreisbogenträgers. Ann. Mat. Pura Appl. (4) **50** (1960), 263–271.

Eine frühere Arbeit des Verfassers [Österreich. Ing.-Arch. **10** (1956), 344–349] beschäftigte sich mit der Feststellung der Einflüsse einer äusseren Flüssigkeitsreibung und innerer Reibungen (Baustoffdämpfung) auf die Biegungsschwingungen eines elastischen Kreisringes. Die entwickelten Differentialgleichungen wurden für den geschlossenen Kreisring gelöst und der Einfluss der Reibungen auf die Kreisfrequenzen bestimmt. Aber die gewonnenen allgemeinen Ergebnisse behalten zwar auch im Falle eines offenen Kreisbogens, dessen Enden beliebig gelagert sind, ihre Gültigkeit, aber ihre Anwendung stösst auf mathematischen Schwierigkeiten. Die nachstehende Näherungsrechnung—die Erweiterung der Rayleighschen Methode- vermeidet diese Rechnungen und führt zu Ergebnissen, die numerisch unmittelbar auswertbar sind und praktisch ausreichenden Genauigkeitsgrad besitzen. Benützend die Lagrangeschen Gleichungen mit der Voraussetzung dass die Reibungskräfte den Geschwindigkeiten direkt proportional sind die Gleichungen fur genäherte Berechnung der Eigenschwingzahlen und Dämpfungsexponenten sind gegeben, ersetzend die Eigenfunktionen durch benachbarte Funktionen fur eines offenen Kreisbogenträgers mit beliebig gelagerten Kämpfern. *D. P. Rašković* (Belgrade)

7398:

Dugundji, John. On the calculation of natural modes of free-free structures. J. Aerospace Sci. **28** (1961), 164–166.

7399:

Kaliski, Sylwester. The three-dimensional dynamic problem of a cylinder of finite length. Arch. Mech. Stos. **12** (1960), 71–84. (Polish and Russian summaries)

Author generalizes his earlier solution of the finite elastic cylinder problem, in the case of axial symmetry, to the three-dimensional case under the assumption of a harmonic field of external forces. He formulates the governing equations for the general transient vibration problem by appealing to the non-homogeneous vector displacement equation of motion from linear elasticity theory, introducing cylindrical coordinates and related displacement functions and, finally, displacement bifunctions as dependent variables. The governing equations in the latter are transformed (Laplace), and solved by first constructing the Green tensor for the system. Introduction of boundary conditions (author treats first boundary-value problem; displacement $\bar{u}=0$ at surface; second and mixed problems may be treated similarly) generates a system of Fredholm

integral equations of the first kind. These reduce to an infinite system of algebraic equations in which the unknowns depend on the Laplace transformation parameter p (real here). To obtain the solution for the harmonic external force field, p^2 is replaced by $-\omega^2$, where ω is the frequency.

For the transient case all that is proved is that a unique solution of the infinite system of algebraic equations (involving real p) exists. Inversion is not attempted. In both cases the author discusses regularity of these algebraic equations. Numerical evaluation is not given here, but the author mentions a suitable technique given in one of his earlier papers. *J. Miklowitz* (Pasadena, Calif.)

7400:

Chu, Hu-Nan. On simple thickness vibrations of thin sandwich cylinders. J. Appl. Mech. **28** (1961), 145–146.

7401:

Fettis, Henry E. Some simplifications in the treatment of rotary-inertia effects for transverse vibration of beams. J. Aerospace Sci. **28** (1961), 252–253.

7402:

Göcke, Hermann. Rheolineare Schwingungen mit periodisch veränderlicher Dämpfungs- und Federkraft. Wiss. Z. Hochsch. Schwermaschinenbau Magdeburg **4** (1960), 135–137, 193–196.

7403:

Lorenz, Hans. ★Grundbau-Dynamik. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1960. viii+308 pp. DM 46.50.

Im vorliegenden Buch werden Probleme der dynamischen Beanspruchung von Baukonstruktionen und des Baugrundes behandelt. Nach Anlage und Auffassung ist das Buch für Bauingenieure bestimmt und liefert diesen wertvolle Ergebnisse und Berechnungsverfahren. Auch die Beispiele sind fast ausschließlich dem Bereich des Bauingenieurwesens entnommen. Neben einer allgemeinen Einleitung und einer ausführlichen Zusammenfassung der wichtigsten Ergebnisse am Schluß des Buches ist der Stoff in den 3 Kapiteln: Schwingungssysteme mit konzentrierten Massen, homogene Systeme und Stabwerksdynamik, Dynamik des Baugrundes zusammengefaßt. Man findet im ersten der 3 genannten Kapitel eine Schwingungslehre der Systeme mit mehreren Freiheitsgraden, die durch die Fülle der durchgerechneten Beispiele überrascht. Auch bei der Behandlung der Stabwerke können wichtige Informationen aus zahlreichen Nomogrammen übernommen werden; z.B. für einfache Rahmentragwerke bei verschiedenen Belastungsfällen die ersten 3 Eigenwerte. Hier schließt sich der Verfasser eng an die Ergebnisse des Buches *Dynamik der Stabwerke, eine Schwingungslehre für Bauingenieure* [J. Springer, Berlin, 1933] von Hohenemser und Prager an. In dem sehr umfangreichen Kapitel über die Dynamik des Baugrundes werden neben vielen Versuchsergebnissen auch einige theoretische Ansätze gebracht. Die den Praktiker interessierenden Fragen der dynamischen Baugrunduntersuchungen, der Rüttelverdichtung und der Schwingrammung dürften dem Stande der Technik entsprechend und fast erschöpfend behandelt sein. Das bei sorgfältig ausgewählter theoretischer Darstellung stets mit dem Blick auf die Erfordernisse der Praxis abgefaßte Buch wird als wichtige Arbeitsgrundlage sicher begrüßt werden. Es ist bestens geeignet der dynamischen Betrachtungsweise im Bauwesen zum Durchbruch zu verhelfen. *K. Magnus* (Stuttgart)

7404:

Dulaney, E. N.; Brace, W. F. Velocity behavior of a growing crack. J. Appl. Phys. **31** (1960), 2231–2236.

7405:

Safronov, Yu. V. The stability of plates with stepwise varying rigidity. Bul. Inst. Politehn. Iaşi (N.S.) **5 (9)** (1959), no. 3–4, 79–82. (Russian. English and Romanian summaries)

7406:

Christopher, P. A. T. The stability of the short-period motion of an airframe having non-linear normal force and pitching moment curves. Aero. Quart. **11** (1960), 255–268.

The phase plane method of analysis is used to determine stability criteria. Results are compared with those of linear theory. *A. W. Babister* (Glasgow)

7407:

Czerwenka, G. Recherches théoriques et expérimentales sur la capacité de charge de coques cylindriques minces et courtes renforcées par des profils circonférentiels minimum de faible et moyen écartement et soumises à une pression latérale. Proc. Sympos. Thin Elastic Shells (Delft, 1959), pp. 137–166. North-Holland, Amsterdam, 1960.

A résumé of theoretical and experimental investigations concerning elastic stability of a ringstiffened cylindrical shell subjected to a lateral pressure. "Mathematical developments" have been omitted by referring to the author's dissertation, not yet published. *S. Drobot* (Notre Dame, Ind.)

7408:

Chudzikiewicz, Andrzej. The influence of deformability of the cross-section of the Eulerian critical force of a thin-walled beam. Rozprawy Inż. **8** (1960), 101–119. (Polish. Russian and English summaries)

The influence of a distortion of cross-sections of a double-tee beam, in the elastic range, on the critical value of the over-all buckling Euler's force is investigated, assuming an indeformability of the flanges of the beam in the planes of the cross-sections. If the normal stress does not surpass the critical buckling stress of the wall, the problem reduces to that of a stability of flanges, considered as bars being twisted and bent by transverse forces, transmitted from the nonlinearly deforming wall-plate. Solution of the corresponding system of partial differential equations shows that for a very thick or very thin wall the influence of the distortion of cross-sections on the Euler load is insignificant, and for standard dimensions does not surpass a few per cent. *J. Nowinski* (Austin, Tex.)

7409:

Foti, Cesare. L'instabilità flesso-torsionale al di là dei limiti elastici di una trave appoggiata agli estremi con un carico concentrato in un punto generico. Rend. Accad. Sci. Fis. Mat. Napoli (4) **26** (1959), 189–201. (English summary)

7410:

Nash, William A. Recent advances in the buckling of thin shells. Appl. Mech. Rev. **13** (1960), 161–164.

Survey article.

7411:

Craggs, J. W. On the propagation of a crack in an elastic-brittle material. J. Mech. Phys. Solids **8** (1960), 66–75.

In this paper the Griffith theory of brittle cracking is extended to a dynamical problem, in which a semi-infinite crack in an infinite medium is extended by finite forces. The body is assumed to be in a state of plane strain parallel to the xy-plane and to have a cut over the half-plane $y=0$, $x<Vt$, where V is a constant velocity and t denotes the time. It is further assumed that the surface tractions σ_{yy}, σ_{xy} are equal on the two sides of the cut and of the form $\sigma_{yy}=-f(x-Vt)$, $\sigma_{xy}=g(x-Vt)$, $-\infty<x-Vt<0$, where f and g are prescribed functions, differentiable any number of times except at isolated points, and that $R(\sigma_{xx}-A)$, $R(\sigma_{xy}-B)$, $R(\sigma_{yy}-C)$ all tend to zero as $R=\{(x-Vt)^2+y^2\}^{1/2}\to\infty$, A, B and C being given constants.

The method of solving the equations of motion is that due to the reviewer [Rend. Circ. Mat. Palermo (2), **1** (1952), 57–62; MR **17**, 802]. The analysis is carried out completely in the case where f and g are simple step functions and interesting physical conclusions are deduced from this simple model. It is shown that the force required to maintain a steady rate of extension of the crack decreases as the rate increases. It is also shown that various criteria which may be assumed for crack division lead to limiting velocities of propagation of a single crack.

I. N. Sneddon (Glasgow)

7412:

Baron, M. L. Response of nonlinearly supported cylindrical boundaries to shock waves. J. Appl. Mech. **28** (1961), 135–136.

7413:

Sunčeleev, R. Ya. Oscillation of an inhomogeneous anisotropic half-space with an axis of elastic symmetry of rotation with given displacements on the boundary. Akad. Nauk Uzbek. SSR. Trudy Inst. Mat. Meh. **21** (1957), 83–89. (Russian)

The author considers the propagation of waves in the inhomogeneous anisotropic half-space $x_3>0$, the x_3-axis being an axis of symmetry in regard to the elastic constants of the body. The disturbance is assumed to be due to the boundary condition $\mathbf{u}(x_1, x_2, x_3, t)|_{x_3=0}=\mathbf{f}(x_1, x_2, t)$, where $\mathbf{u}$ denotes the displacement vector and $\mathbf{f}$ is a prescribed vector. It is further assumed that, at $t=0$, $\mathbf{u}=\partial\mathbf{u}/\partial t=\mathbf{0}$. The equations of motion are solved by means of a Fourier-Bessel transform, the results being obtained in the form of triple integrals over an infinite region of integration. The analysis is purely formal; no attempt is made to discuss the formal problem of evaluating these triple integrals for particular values of the vector $\mathbf{f}$.

I. N. Sneddon (Glasgow)

7414:

Baltrukonis, J. H.; Gottenberg, W. F.; Schreiner, R. N. Dynamics of a hollow, elastic cylinder contained by an infinitely long rigid circular-cylindrical tank. J. Acoust. Soc. Amer. **32** (1960), 1539–1546.

From the authors' summary: "Dispersion equations are derived for the propagation of transverse waves within an infinitely long thick-walled hollow elastic cylinder which is perfectly bonded along its outer cylindrical surface to an infinitely long rigid circular-cylindrical tank. In the case of infinite wavelength the dispersion equations reduce to two uncoupled frequency equations; one defining the natural frequencies of free vibration of the hollow elastic core in the antisymmetric axial shear mode and the other defining the natural frequencies of plane strain vibrations. Some numerical results are presented for the dispersion equations and the two frequency equations."

R. N. Goss (San Diego, Calif.)

7415:

Perri, E. Teoria e caratteristiche di due onde superficiali dispersive vibranti nel piano principale. Ann. Geofis. **13** (1960), 1–41. (English summary)

If one violates a radiation condition and if one assumes that the real part of a complex wave number is wavelength independent, which it is not, then it is possible to generate "new"" types of waves. Two such waves are investigated; the author correctly claims these have not been investigated heretofore in the literature of elastic wave theory. The procedure involves the writing of the general solution, in cartesian coordinates, to the two dimensional wave equation. This solution is then generalized by subjecting it to a rotation of coordinates. The author also claims to investigate surface waves, although these waves, in his formulation, are allowed to propagate in directions which are not parallel to the surface of a half-space. The difficulties associated with the two errors are insuperable; the author has not obtained new results.

L. Knopoff (Los Angeles, Calif.)

7416:

Chao, Chi-Chang. Dynamical response of an elastic half-space to tangential surface loadings. J. Appl. Mech. **27** (1960), 559–567.

Transform techniques are used to find useful closed expressions for the displacement at the surface or directly below a concentrated time-dependent tangential force applied to the surface of an elastic half space.

C. E. Pearson (Cambridge, Mass.)

7417:

Bolt, B. A.; Butcher, J. C. Rayleigh wave dispersion for a single layer on an elastic half space. Austral. J. Phys. **13** (1960), 498–504.

The dispersion of Rayleigh waves in a medium with one finite homogeneous layer over a semi-infinite homogeneous layer has been studied. Using the frequency equation derived by Sezawa and others, the phase velocity and the group velocity curves for different values of RH has been

worked out with the help of an electronic digital computer, for eleven different cases with assumed values of α, β and ρ_2/ρ_1, approximately fitting the structure of the crust of Western Australia. Results have been given in tabular form for the fundamental and the first higher mode. The results indicate that (i) the Airy wave velocity is sensitive to shear velocity in the upper layer, (ii) the velocity curves are not very sensitive to the density contrast between the two layers. The author concludes that the observed Rayleigh wave dispersion alone cannot give precise information about the elastic parameters of the different layers.

S. K. Chakrabarty (Howrah)

7418:

Lee, E. H.; Radok, J. R. M. The contact problem for viscoelastic bodies. J. Appl. Mech. **27** (1960), 438–444.

At the surface of contact of a smooth rigid sphere and a viscoelastic half-space, the normal stress and displacement distributions are obtained from the elastic solution by replacing the elastic moduli by the corresponding viscoelastic operators. The validity of this procedure is investigated. Graphs of the pressure distribution are given for the indentation of a Maxwell material at constant velocity. The initial elastic response of the material is shown by the presence of 'an elastic boundary layer'. The reviewer thinks that this concept may be useful in the solution of more complex viscoelastic boundary value problems.

D. R. Bland (Manchester)

7419:

Distéfano, José Néstor. Sulla stabilità in regime viscoelastico a comportamento lineare. I, II. Atti. Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **27** (1959), 205–211, 356–361.

A linear viscoelastic beam is subject to arbitrary but bounded transverse loading and to axial compression. Small deviations from the straight of the axis of the beam in the initial state are included. The problem is to determine under what axial loads the transverse displacement is bounded for large values of the time. Using integral equations and a theorem of Paley and Wiener for their solutions at large times, a surprisingly simple answer to the problem is obtained. The second paper extends the solution to ageing viscoelastic beams of varying cross-section. (In the relation $\varepsilon(t)=\sigma(t)/E+\int_{-\infty}^{t}\sigma(\tau)f(t,\tau)d\tau$, f is a function of the single variable $t-\tau$ for a non-ageing material only.)

D. R. Bland (Manchester)

7420:

Lee, E. H.; Symonds, P. S. (Editors). ★Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics. Held at Brown University, R.I., April 5–7, 1960. Sponsored by the Office of Naval Research and Brown University. Pergamon Press, Oxford-London-New York-Paris, 1960. xviii+611 pp. $10.00; 70s.

A set of 30 lectures. Those of mathematical interest will be reviewed separately in these pages.

7421:

Sankaranarayanan, R. A note on the impact pressure loading of a rigid plastic spherical shell. J. Aerospace Sci. **28** (1961), 77–78.

7422:

Imegwu, E. O. Plastic flexure and torsion. J. Mech. Phys. Solids **8** (1960), 141–146.

The paper deals with combined flexure and torsion of prismatic beams loaded by terminal bending and twisting moments. The beam material is rigid-perfectly plastic and may obey either the Tresca or Mises yield condition. The equations of Handelman [Quart. Appl. Math. **1** (1944), 351–353; MR **5**, 252] are solved by a relaxation method to obtain the interaction curve, i.e., the locus of stress states which are sufficient to cause plastic flow. In terms of suitable dimensionless variables this interaction curve is found to be invariant with respect to the cross-sections considered (circular, square, and triangular). It is also shown to be closely approximated by the lower bound curve of Hill and Siebel [J. Mech. Phys. Solids. **1** (1953), 207–214].

W. E. Boyce (Troy, N.Y.)

7423:

Lippmann, Horst. Ebenes Hochkantbiegen eines schmalen Balkens unter Berücksichtigung der Verfestigung. Ing.-Arch. **27** (1959), 153–168.

The author considers bending of a narrow beam of rectangular cross-section, in the plane parallel to the long axis. As concerns the material it is in fact assumed rigid-plastic, but the elastic lag after the deformation is accounted for approximately. The strain-hardening law is assumed in the form

$$Y = Y_0\left[1+\nu\left(\frac{A}{Y_0}\right)^{\lambda}\right],$$

Y_0, Y being the yield points and A the density of bending energy; ν is a constant and λ a non-negative integer. As the plasticity condition two alternatives are taken: the Mises condition and Tresca condition; the flow law is taken in the form following from the theory of plastic potential. The solution of the equations is obtained by numerical methods and the results given in the form of graphs. The problem of existence and uniqueness of the solution has been discussed.

W. Urbanowski (Warsaw)

7424:

Eason, G. The elastic-plastic bending of a curved bar by end couples in plane stress. Quart. J. Mech. Appl. Math. **13** (1960), 334–358.

Author's summary: "The stresses and displacements within an elastic-plastic curved bar of perfectly plastic, non-hardening, compressible material subjected to a pure bending moment are calculated for both the Tresca and von Mises yield conditions and their associated flow rules. Plane-stress conditions are assumed and the applied bending moment is determined in the four cases which occur for either yield condition. Some numerical results are presented and the results for the two yield conditions are compared."

R. Hill (Nottingham)

7425:

Andriankin, È. I. A converging wave in a plastic medium. Dokl. Akad. Nauk SSSR **131** (1960), 769–772 (Russian); translated as Soviet Physics. Dokl. **5**, 242–245.

The problem of the propagation of spherical plastic waves in a compressible medium is considered. The plastic

shock wave is produced by a pressure applied at the free boundary of a spherical layer; at the internal surface of the spherical layer the pressure is assumed to be equal to zero. The initial density of the material is assumed to depend on the radius. Behind the front of the shock wave the medium becomes incompressible and satisfies the Prandtl plasticity condition. The solution of the problem is obtained numerically in the case when the external pressure is variable in time. *N. Cristescu* (Bucharest)

7426:

Landau, H. G.; Zwicky, E. E., Jr. Transient and residual thermal stresses in an elastic-plastic cylinder. J. Appl. Mech. **27** (1960), 481–488.

Authors' summary: "Equations are given for the stress rates in solid cylinders subject to transient temperature distributions, based on the assumption of an elastic, perfectly plastic material obeying a von Mises temperature-dependent yield condition. A numerical procedure for integrating the equations is developed and applied to a temperature distribution approximating a phase transformation and to a quenched cylinder. The effect of various factors on the residual stresses is noted."

F. J. Lockett (Sevenoaks)

7427:

Hwang, Chintsun. Thermal stresses in an elastic, work-hardening sphere. J. Appl. Mech. **27** (1960), 629–634.

A method is presented for obtaining the transient and residual thermal stresses in a spherically symmetrical situation. The elastoplastic material is assumed to obey the von Mises yield criterion and to work-harden isotropically; the thermal and mechanical properties are assumed temperature independent. The problem is reduced to the solution of a single non-linear differential equation, which is solved numerically for the problem of the cooling of a sphere, initially at uniform temperature, when immersed in a medium at zero temperature.

F. J. Lockett (Sevenoaks)

7428:

Barrekette, E. S. Thermoelastic stresses in beams. J. Appl. Mech. **27** (1960), 465–473.

Series solutions are obtained for the stresses in a free elastic beam of arbitrary constant cross-section under an arbitrary temperature distribution. Inertia effects and thermoelastic coupling are neglected and the material constants are assumed to be independent of temperature. The general three-dimensional theory is presented and is illustrated by the special case of a right circular cylindrical beam subjected to temperatures varying only with the axial coordinate. A numerical example is given and is compared with the strength-of-materials solution.

F. J. Lockett (Sevenoaks)

7429:

Derski, Włodzimierz. The state of stress and displacement in a thick circular plate due to a nonstationary temperature field. Rozprawy Inż. **7** (1959), 191–233. (Polish. Russian and English summaries)

The quasi-static state of stress in a thick circular plate due to the action of a non-steady axially symmetric temperature field is considered. Three cases of heating of the plate are examined. In the first and second ones the temperature field is symmetric and antisymmetric, respectively, with respect to the middle plane of the plate. The third one is the sum of the two first states. In order to solve the equations of thermoelasticity, Goodier's thermoelastic potential is introduced and standard mathematical methods are used. The boundary conditions on the upper and lower surfaces are satisfied rigorously but along the entire edge $r=b$ in an approximate manner only. Some limiting cases are considered, e.g., for $b\to\infty$ and the steady-state. No numerical examples are given.

Witold Nowacki (Warsaw)

7430:

Wilde, Piotr. A thermoelastic problem for an orthotropic rectangular plate. Rozprawy Inż. **7** (1959), 557–569. (Polish. Russian and English summaries)

This is a classical steady-state thermoelastic problem. Applying the double Fourier series, the solution of the heat conduction equation for the orthotropic case is found. The author assumes that the temperature at the edges of the plate is known and heat flow across the surfaces of the plate is proportional to the temperature difference between the plate and the environment. To determine the stresses a stress function in a form of double Fourier series is used. From the boundary conditions (the stress function and its normal derivatives vanish along the entire edge) one obtains two infinite systems of linear algebraic equations. Majorant of these infinite systems is the solution of the case of very intense heat absorption on upper and lower surfaces of the plate. A numerical example for a square isotropic plate is given. *Witold Nowacki* (Warsaw)

7431:

Rozovskiĭ, M. I. Application of a method of two-dimensional integral equations to investigation of creep of a spherical shell with dependence of the physical parameters of the material on temperature and time. Izv. Akad. Nauk Armyan. SSR. Ser. Fiz.-Mat. Nauk **11** (1958), no. 2, 41–50. (Russian. Armenian summary)

The paper is concerned with investigation of the thermal stresses in a spherical shell in which a spherically-symmetrical temperature field $T(r, t)$ has been established. The stress-strain relation assumed is of the type

$$\sigma_r = \lambda_1\theta + 2\mu_1\frac{\partial u}{\partial r} - f(T) - \int_{t_0}^{t}\{\varphi_1(t,\tau;r)\theta(r,\tau) + 2\psi_1(t,\tau;r)(\partial u/\partial r)\}\,d\tau,$$

where λ_1 and λ_2 are Lamé's constants, supposed to depend on r, t and T, $f(T)=(3\lambda_1+2\mu_1)\alpha T$, where α is the coefficient of linear expansion, also supposed to vary with $T(r, t)$ and t. θ denotes the dilation of the solid, and u the radial component of displacement. The equations are solved using Volterra's theory of integral equations.

The case in which $\lambda_1=\mu_1=\lambda_0 e^{-m\tau}$, $\varphi_1=\psi_1=\varphi_0(t,\tau)e^{-m\tau}$ is considered in more detail and the method of obtaining practical solutions is illustrated by a simple numerical example. *I. N. Sneddon* (Glasgow)

7432:

Liu, Hsien-chih. The elastic stresses and deformations produced in a semi-infinite elastic solid by a point source of heat beneath its free surface. Sci. Sinica **9** (1960), 604–651.

The steady-state problem indicated in the title is solved by superposing a particular solution of the thermoelastic equations upon a purely elastic solution which is chosen so that the final solution satisfies the elastic boundary conditions. Much space is occupied in rediscovering the general solutions to which the special case is applied. A numerical example is treated exhaustively, all the non-zero components of stress and displacement being plotted against the spacial coordinates, both for the component solutions and for the final solution, and also for different values of the parameter h (the distance of the point source beneath the surface). The latter is unnecessary if r/h and z/h are taken as coordinates. The paper contains 54 figures.

F. J. Lockett (Sevenoaks)

7433:

Weiner, J. H.; Landau, H. G. Thermal stresses in elasto-plastic bodies. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 369–384. Pergamon, Oxford, 1960.

Paper is essentially a review of work on the subject published elsewhere by the authors and their co-workers. Comparison with experimental results has been added.

H. Parkus (Vienna)

7434:

Bellamy, C. J. Thermal stresses in hollow cylinders of finite length. Austral. J. Appl. Sci. **11** (1960), 217–232.

An approximate solution is obtained to the steady-state problem indicated in the title in which the temperature distribution $T=T(r)$. It is assumed that $\sigma_z=f(z)\sigma_{zc}$, $\sigma_r=\sigma_{rc}$, $\sigma_\theta=\sigma_{\theta c}+\sigma_t$, where the suffix c denotes the solution to the corresponding problem for the infinite cylinder, σ_t represents the end-effects and $f(z)$ is to be found. The equilibrium equations allow σ_t and τ_{rz} to be expressed in terms of $f(z)$, and this function is then found using the theorem of stationary strain energy for thermoelasticity. To this end $f(z)$ is expressed as a polynomial whose degree is governed by the accuracy required. For a special case it is shown that the tensile hoop stress can be as much as 28 per cent greater at the ends than at the centre, this maximum occurring when the length and diameter of the cylinder are equal.

F. J. Lockett (Sevenoaks)

7435:

Lieb, Burton A. Cylindrical bending of a heated long rectangular plate. J. Roy. Aero. Soc. **65** (1961), 26–30.

7436:

Deresiewicz, H. Thermal stress in a plate due to disturbance of uniform heat flow by a hole of general shape. J. Appl. Mech. **28** (1961), 147–149.

STRUCTURE OF MATTER

7437:

Seitz, Frederick; Turnbull, David (Editors). ★Solid state physics: Advances in research and applications, Vol. 11. Academic Press, New York-London, 1960. xvi+438 pp. $12.50.

The prominence of the study of semi-conductors in present-day solid state physics is shown by the fact that three of the five articles in this eleventh Advances volume are entirely devoted to semi-conductors. The first, "Semiconducting properties of gray tin", by G. A. Busch and R. Kern (pp. 1–40), reviews experimental results for this material, which tend to be subject to considerable scatter because of the difficulties of obtaining suitable specimens. The article also includes a useful compendium of the general physical properties of gray tin and its transformation to and from white tin. The properties of other semiconducting materials are described in the article "Imperfection ionization energies in CdS-type materials by photoelectric techniques", by R. H. Bube (pp. 223–260), in which special emphasis is put on photoconductivity in the study of ionization and excitation processes in insulators. The third article is "The effects of elastic deformation on the electrical conductivity of semiconductors", by R. W. Keyes (pp. 149–221). This begins with a general discussion of the relationships between stress, strain, conductivity and resistivity, and of the methods of measurement of the effect of stress on resistivity; it continues with a detailed discussion, in terms of the electronic structure, of the experimentally-determined effects of hydrostatic pressure and of shear stress on the resistivity of semiconductors.

A wide range of topics is included in C. A. Swenson's review "Physics at high pressure" (pp. 41–147), dealing with advances since about 1945. The first third of the article is a useful account of experimental methods for pressures up to about 25000 atmospheres at both normal temperatures and low temperatures. This is followed by results of measurements on P-V-T relations, electrical and magnetic properties of metals, semiconductors and superconductors, optical absorption edges, and various other properties, special attention being given to properties at low temperatures. Theoretical aspects are dealt with only briefly.

Finally there is a long article "Cyclotron resonance", by B. Lax and J. G. Mavroides (pp. 261–400). The study of cyclotron resonance during the superposition of an alternating electric field and a steady magnetic field has been applied only in the last decade to solids, but very notable results have been achieved, especially in the measurement of effective masses of charge carriers and the study of energy band structures in metals and semiconductors. After a brief résumé of work with ionized gases, a very detailed account is given of these applications in solids, covering both the theoretical background to experiments and the experimental results themselves. The main work has been at microwave frequencies, but some work with millimeter and infrared radiation is also described. Other topics under "new developments" include the effect of elastic strain on cyclotron resonance, magneto-acoustic resonance, and possible practical applications.

M. S. Paterson (Canberra)

7438a:

★Физика твердого тела. I. [Solid state physics. I]. Collection of articles. Akad. Nauk SSSR, Otd. Fiz.-Mat. Nauk. Izdat. Akad. Nauk SSSR, Moscow-Leningrad, 1959. 298 pp. 20 r.

7438b:

★Физика твердого тела. II. [Solid State physics. II]. Collection of articles. Akad. Nauk SSSR, Otd. Fiz.-Mat. Nauk. Izdat. Akad. Nauk SSSR, Moscow-Leningrad, 1959. 328 pp. 22 r.

A collection of 87 articles on solid state physics, appearing under the imprimatur of the Academy of Sciences (USSR) and the editorship of the late Academician A. F. Joffé. The work reported is primarily experimental in nature, covering a wide variety of topics, with concentration on the properties of semiconductors. It represents a fair sample of the active work in experimental solid state physics in the Soviet Union as of 1958. The table of contents is given in both Russian and English.

E. L. Hill (Minneapolis, Minn.)

7439:

Jones, H. ★The theory of Brillouin zones and electronic states in crystals. Series in Physics. North-Holland Publishing Co., Amsterdam; Interscience Publishers, Inc., New York; 1960. ix + 268 pp. $9.50.

The wave-mechanical theory of the structure of solids, particularly metals, has become one of the major fields of theoretical physics during the last three decades. This success has been bound up to a large degree with the dominating influence of three mathematical properties of the Schrödinger equation; namely, its linearity, its invariance under the groups of translations and rotations in ordinary space, and its invariance under the group of permutations of identical particles (Pauli principle). It may not be unfair to claim that the most certain of the results derived from non-relativistic quantum mechanics arise directly from these properties, while for relativistic theories the Lorentz group exerts a similar influence.

The volume under review deals with the theory of the Schrödinger equation, on the one-electron model, in ideal crystal lattices. The underlying mathematical theme is the structure of the crystallographic groups and their linear representations. With this in mind, the pure mathematician may find the pace of somewhat pedestrian slowness, but the student of theoretical physics will undoubtedly welcome the detail and completeness and the attention paid to the intuitive background of the physical theory.

The discussion starts at the beginning; that is, with Floquet's (now known as Bloch's) analysis of the Mathieu equation. The extension to the 3-dimensional space groups and their influence on the topology of the eigenvalue distribution, and the eigenfunctions, follows standard lines but in greater detail than can be given in the usual texts on solid state theory. The final two chapters deal with the explicit, and still incompletely solved, problem of construction of approximate eigenfunctions (atomic and molecular orbital methods), and with spin-orbit coupling effects. This book will be welcomed as an important addition to the literature of applied mathematics in general, and of solid state theory in particular.

E. L. Hill (Minneapolis, Minn.)

7440:

Maradudin, A. A.; Montroll, E. W.; Weiss, G. H.; Herman, Robert; Milnes, H. W. Green's functions for monatomic simple cubic lattices. Acad. Roy. Belg. Cl. Sci. Mém. Coll. in-4° (2) **14** (1960), no. 7, 176 pp.

Green's functions for monatomic simple cubic lattices are defined by the integral

$$I(a, b, c; \alpha; \beta) = \frac{1}{\pi^3} \iiint_0^\pi \frac{\cos ax \cos by \cos cz}{(2+\alpha)\beta - \cos x - \cos y - \alpha \cos z}\, dx\, dy\, dz.$$

A tabulation of this integral to six significant figures is presented for the following parameter values: $\mu = \beta^{-1} = 0.00(0.01)1.00$; $\alpha = 1, 2, 4, 8, 16$; $0 \leq a^2 + b^2 + c^2 \leq 15$.

Closed expressions for special values of the parameters, the recurrence formula satisfied by the integral, and asymptotic expressions valid in various ranges of the parameters are given. Some examples of the use of the tables in connection with actual physical problems are discussed. *E. L. Hill* (Minneapolis, Minn.)

7441:

Pipkin, A. C.; Rivlin, R. S. Galvanomagnetic and thermomagnetic effects in isotropic materials. J. Mathematical Phys. **1** (1960), 542–546.

A formal mathematical discussion of phenomenological constitutive relations in solids is given for the electric current density, heat flux, and magnetic intensity field when an electric field, magnetic induction field, and a temperature gradient are present. The methods used are the standard ones of invariant theory, given the physical (tensor) nature of the quantities considered and the symmetry group of the material. No direct connection is made with experimental results beyond the statement of the existence of certain effects of galvanomagnetic and thermomagnetic nature. *E. L. Hill* (Minneapolis, Minn.)

7442:

Kuhlmann-Wilsdorf, Doris. Frictional stress acting on a moving dislocation in an otherwise perfect crystal. Phys. Rev. (2) **120** (1960), 773–781.

It is asserted that earlier calculations of the frictional stress suffered by a moving dislocation are unreliable because they depend sensitively upon a function (the core energy) which is known only approximately. Here, the calculation of the core energy is circumvented by considering instead the stresses and strains on the slip planes. Values for the maximum value of the frictional stress obtained here exceed the earlier results by factors greater than 100. It is pointed out that atomic vibrations introduce an uncertainty into the position of a dislocation (even at the lowest temperatures), and it is shown that this causes a reduction in the frictional stress. This effect is found to be significant for glide dislocations in fcc and hcp crystals at all temperatures, and for NaCl-like crystals above room temperature, but not for diamond-like crystals.

H. B. Rosenstock (Washington, D.C.)

7443:

Gosar, P. On the relation between Zener breakdown and residual resistance in crystals. Nuovo Cimento (10) **18** (1960), 241–250. (Italian summary)

Insulators are, by definition, solids in which the application of an electric field F of moderate strength produces no electric current. This is attributed to the fact that the "valence band" of energy states, which is completely filled with electrons, is separated from the empty "conduction band" by an energy gap ΔE which the electrons cannot cross. Very high electric fields, however, will produce such a transition and thus a current ("Zener breakdown"). In this paper the effects of even higher electric fields are studied; the current is shown to be proportional to $F \exp(-k(\Delta E)^{3/2} F^{-1})$ (where k depends on various lattice and geometrical parameters), and the conductivity thus

remains finite. A physically very different situation exists in a conductor: the highest band is only partly filled, and even very small fields F produce a current; the conductivity would be infinite, were it not for the fact that all real crystals contain impurities which have the effect of producing very small "energy gaps" ΔE inside the conduction band. The author shows that a mathematically analogous treatment of the two situations is possible in spite of the very different magnitudes of the quantities involved, and thus interprets residual resistance due to impurities in a conductor as resulting from Zener breakdown on a smaller scale. *H. B. Rosenstock* (Washington, D.C.)

7444:

Klyačkin, V. I. A contribution to the theory of phase transitions in molecular crystals. Fiz. Tverd. Tela **2** (1960), 929–939 (Russian); translated as Soviet Physics. Solid State **2**, 847–856.

The systems of nonlinear integrodifferential equations describing the equilibrium orientation state of a crystal whose molecules have rotational degrees of freedom are derived by applying N. N. Bogolyubov's method. The application of the superposition approximation for the binary partition function permits one to find clear expressions for the unary partition function and to derive the existence conditions and the temperature of phase transitions. The influence of the short-range order orientation correlation on the temperature behavior of order parameters and on the temperature dependence of the thermal capacity is explained. *Werner Nowacki* (Bern)

7445:

Mahanty, J.; Maradudin, A. A.; Weiss, G. H. Vibrational thermodynamic properties of lattices with defects. I. The linear lattice. Progr. Theoret. Phys. **20** (1958), 369–394.

Several methods of analysing vibrational properties of crystal lattices with defects are developed. Integral expressions for additive functions of normal mode frequencies are derived following the work of Montroll and his collaborators. It is shown that the Helmholtz free energy can be evaluated at high and low temperatures without performing the integrations. The methods presented are valid for lattices of all odd dimensions, although specific results are presented here for one dimensional monatomic and diatomic lattices. Using a method similar to that developed by Lifshitz, it is shown that the properties of a lattice with defects can be expanded in a series of powers of the concentration of defects. The coefficient of the nth power depends on the properties of a lattice with n defects. Examples of such expansions are given. An exact expression for the frequency distribution function of a monatomic linear chain with an isotope defect is given.

Werner Nowacki (Bern)

7446:

Mahanty, J.; Maradudin, A. A.; Weiss, G. H. Vibrational thermodynamic properties of lattices with defects. II. Two- and three-dimensional simple cubic lattices. Progr. Theoret. Phys. **24** (1960), 648–660.

The paper is a continuation and application to two- and three-dimensional crystals of the methods of part I [#7445] for the evaluation of the changes in thermodynamic properties of a lattice due to the presence of defects of various kinds. In particular the effects of an isotope defect and a pair of isotope defects on the zero point energy and free energy of two- and three-dimensional cubic lattices are considered. In addition the free energy and self entropy of a vacancy in a three-dimensional cubic lattice is evaluated. *Werner Nowacki* (Bern)

7447:

Maškevič, V. S. Higher approximations in the Ewald method. Fiz. Tverd. Tela **2** (1960), 908–912 (Russian); translated as Soviet Physics. Solid State **2**, 828–832.

The macroscopic electromagnetic field that is used in the Ewald method is found for multipoles of arbitrary order in the form of an expansion in powers of the ratio of the lattice constant to the wavelength.

Werner Nowacki (Bern)

7448:

Erdmann, Joachim. Zur Symmetrie der Wellenfunktionen in Kristellen mit hexagonal dichtester Kugelpackung. Z. Naturforschg. **15a** (1960), 524–531.

Zur Berechnung der Energiebänder in Kristallen nach der Blochschen Methode (Methode der festen Bindung) ist die Lösung von Säkulardeterminanten erforderlich. Für Wellenvektoren, die in der Brillouin-Zone symmetrisch gelegen sind, ergeben sich Aufspaltungen der Determinante in Faktoren entsprechend der Symmetrie der Wellenfunktionen. Es wird gezeigt, wie man mit Hilfe bekannter gruppentheoretischer Verfahren die Aufspaltung der Determinante in Faktoren angeben kann. Die Methode wird auf das Gitter mit hexagonal dichtester Kugelpackung angewandt. *Werner Nowacki* (Bern)

7449:

Scholz, Alfred. Statistische Formulierung von chemischen Reaktionen zwischen Fehlstellenteilchen in kubischen Kristallgittern. Ann. Physik (7) **5** (1960), 353–372.

Die kinetischen Gleichungen von Störstellenreaktionen in kubischen Kristallen werden mit Hilfe von Wahrscheinlichkeitsansätzen abgeleitet. Hierbei fungiert als entscheidende Grösse die Aufenthaltswahrscheinlichkeit eines Störatoms auf einem Gitterplatz; für diese wird eine statistische Deutung gegeben. Bei den chemischen Reaktionen zwischen Störstellen wird die Korrelation nächst benachbarter Fremdionen berücksichtigt. Konkret durchgerechnet werden: 1. die Bildung von Lücken in Kristallen vom NaCl-Typ, 2. die chemische Reaktion $Ag_o^{\cdot}+S'_G \rightleftarrows [Ag_o^{\cdot}S'_G]$ in Silberhalogeniden. Die statistische Ableitung ergibt für die kinetischen Gleichungen eine gewisse Modifikation gegenüber dem sonst üblichen phänomenologischen Ansatz für die kinetischen Gleichungen. Dies betrifft die kinetischen Konstanten, welche noch eine Zeitabhängigkeit enthalten. *Werner Nowacki* (Bern)

7450:

Pekar, S. I. Identification of excitons with light waves in a crystal and the macroscopic theory of excitons with and without account of retardation. Ž. Èksper. Teoret. Fiz. **38** (1960), 1786–1797 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1286–1293.

Excitons are identified with light waves in a crystal. The

theory of excitons is reduced to the macroscopic theory of light waves. The energy bands, limiting energies for $\mathbf{k} \to 0$, effective masses etc. of excitons are expressed in terms of the dielectric constant tensor $\varepsilon(\omega, \mathbf{k})$. In the limit $c \to \infty$, we obtain the results of the conventional exciton theory, based on the Schrödinger equation and neglecting retardation of interactions between particles. The results show that retardation can be neglected when the electromagnetic wave that accompanies an exciton is longitudinal. If however, the wave possesses a rotational field, retardation considerably affects the results in regions where the refractive index is not very large. Exciton energies and effective masses then differ from those derived by means of the Schrödinger equation. Crystals of different symmetries are considered. *Werner Nowacki* (Bern)

7451:

Kane, Evan O. Theory of tunneling. J. Appl. Phys. **32** (1961), 83–91.

"Tunneling" or the theory underlying electronic transitions from valence to conduction band in semiconductors subjected to high electric fields is reviewed. The equations used, in general, are not derived but references to original work are given. Direct tunneling is reviewed first in which the lattice vibrations do not assist the process. In this case, the momentum of the electron is conserved. Then indirect tunneling is considered in which lattice vibrations are instrumental by emitting or absorbing a "phonon". Finally possible explanations are given for experimentally observed "excess currents" which are not accounted for by the developed theory. *H. Statz* (Waltham, Mass.)

7452:

Baltensperger, W.; de Graaf, A. M. Long range interactions between magnetic moments in semiconductors. Helv. Phys. Acta **33** (1960), 881–888.

Authors' summary: "The interaction between magnetic moments embedded in a non-degenerate electron gas is calculated. In the fully degenerate gas the Ruderman, Kittel interaction applies whereas in the Boltzmann gas the interaction is ferromagnetic only and of comparatively long range. Between these extreme cases the behaviour changes gradually. In a semiconductor an additional interaction comes from the virtual excitations of the valence band. These two interactions and the magnetic dipole interaction may be of comparable magnitude in dilute magnetic semiconducting alloys."

7453:

Vyatskin, A. Ya. Concerning the theory of inelastic electron scattering in solids. Fiz. Tverd. Tela **2** (1960), 122–132 (Russian); translated as Soviet Physics. Solid State **2**, 112–122.

Der Verfasser benützt die Bezeichnungen schwache und starke Bindung (Kopplung) in Metallgittern in dem Sinne, dass es sich im ersteren Falle um weite und im letzteren um enge erlaubte Energiebänder handelt. In zwei vorangehenden Arbeiten [Z. Tehn. Fiz. **28** (1958), 2217–2227, 2455–2468] wurde die unelastische Streuung von Elektronen in Metallen unter der Annahme der schwachen Bindung zwischen den Metallelektronen behandelt. Ziel der vorliegenden Arbeit ist das selbe Problem im Falle der starken Bindung zu berechnen.

In den gewohnten Bezeichnungen hat man für die Wellenfunktion des auf das Metall einfallenden Elektrons

$$F_0 = N_0 e^{i(KX - E_k(2\pi t/h))}, \quad N_0 = \Omega^{-1/2}, \tag{1}$$

N_0 ist der Normierungsfaktor. Die Gitterelektronen werden mit Hilfe von Blochschen Eigenfunktionen

$$\varphi_{\alpha\mathbf{k}} = n_0 \sum_{\mathbf{m}} e^{i\mathbf{k}\mathbf{a}_\mathbf{m}} \Phi_\alpha(\mathbf{r} - \mathbf{a}_\mathbf{m})^{-(E_{\alpha,\mathbf{k}}(2\pi t/h))} \tag{2}$$

beschrieben, wo Φ die Eigenfunktion eines Atoms bedeutet. Bei enge Energiebänder muss jedoch schon der Umstand berücksichtigt werden, dass Eigenfunktionen vom Typ (2) aufeinander nicht mehr ganz orthogonal sind. Diese Schwierigkeit wird mit Hilfe von einer von N. N. Bogolyubov angegebenen Methode beseitigt und eben darin liegt der Unterschied gegenüber der Annahme der schwachen Bindung. Weiter wird mit Hilfe der Bornschen Näherung gerechnet, der Störungsoperator ist selbstverständlich die Coulombsche Wechselwirkung. Aus den erhaltenen Resultaten folgt, dass man zwei verschiedene Typen von Übergängen unterscheiden muss, für die hat man $\mathbf{k}' = \mathbf{k} + \mathbf{q}$ bzw. $\mathbf{k}' = \mathbf{k} + \mathbf{q} + \mathbf{n}$ wo $\mathbf{q} = \mathbf{K} - \mathbf{K}'$ ist und $\mathbf{n}/2\pi$ den reziproken Gittervektor bedeutet. Von diesen interzonalen freien und interzonalen n-Übergängen verursachen die ersteren einen verschmierten Hintergrund bei der elastischen Streuung, die letzteren dagegen das Erscheinen von spezifischen Maxima.

Die erhaltenen Resultate lassen sich noch weitgehend vereinfachen und da man auch unter der Annahme der schwachen Bindung qualitativ ähnliche Resultate erhält, so ist es wahrscheinlich, dass die auch in intermediären Fällen gültig sein werden. *T. Neugebauer* (Budapest)

7454:

Fechner, Bogdan. On the statistics of spin waves by the Bethe method. Acta. Phys. Polon. **19** (1960), 289–293.

Author's summary: "The exact solution of Slater-Bloch's secular equations by the Bethe method without any additional conditions is considered. The bearing of the results upon the question of statistics of spin waves is discussed. It is shown that there exists an ambiguity which cannot be overcome without additional assumption."

7455:

Dornberger-Schiff, K.; Grell-Niemann, H. On the theory of order-disorder (OD) structures. Acta Cryst. **14** (1961), 167–177.

Authors' summary: "OD-structures consisting of equivalent layers are first characterized as having pairs of adjacent layers which are all equivalent. Then a slightly more general condition—the *'vicinity condition'*—is formulated which is satisfied not only by all ordered structures but also by all OD-structures. Partial operations (POs) are seen to be of fundamental importance for characterizing the symmetry properties of OD-structures and the set of POs of a certain structure is called an OD-groupoid. OD-structures of the same substance, built of the same kind of layers with the same kinds of pairs of adjacent layers, are said to belong to the same family, the corresponding OD-groupoids to the same OD-groupoid family. Twins of one particular type are described as

special members of families of OD-structures. A report on the deduction of a complete list of OD-groupoid families is given, and the resulting numbers of such families with different symmetry characteristics are listed in tables. There are 333 in all."

7456:

Cullwick, E. G. Electromagnetic momentum and electron inertia in a current circuit. Proc. Inst. Elec. Engrs. C **103** (1956), 159–170.

Author's summary: "In the second volume of his *Treatise on electricity and magnetism* [3rd ed., Dover, New York, 1954; MR **16**, 99] Clerk Maxwell developed the theory of electric current-circuits from general dynamical principles, and discussed the experimental effects which should occur if an electric current is a true motion of some substance possessing inertia. Since none of these effects had at that time been observed, Maxwell developed his general electromagnetic theory on the assumption that they do not exist, or at least that they produce no sensible effect.

"It is now known, however, that an electric current in a conductor consists of moving electrons, and the inertia effects which were discussed by Maxwell have been observed experimentally. They are extremely small, and have not been brought within the scope of electromagnetic theory. A conduction current is usually assumed to be due to the drifting along the conductor, with a very small mean velocity, of all the available conduction electrons, so that the kinetic energy of the electrons due to this motion is negligible in comparison with the magnetic energy of the current. Electron-inertia effects in current circuits have therefore been accepted as something outside classical electromagnetic theory—a position which is illogical if, as is usual, we identify the kinetic and magnetic energies of a free electron.

"It is shown in the paper that it is possible to identify the kinetic energy of the conduction electrons in a current circuit with the magnetic energy of the current, so that electron-inertia effects can be included in the general electromagnetic scheme. In consequence, a current circuit can be said to possess an electromagnetic mass whose motion, when current flows, entails electromagnetic momentum. This momentum accounts for the known effects of electron inertia and also for the force on the end wire of a long rectangular circuit.

"The relativistic form of the theory indicates the possibility that electromagnetic laws may depart from the classical form, becoming non-linear in circuits where a high inductance per unit length of conductor is combined with a current greater than is usually found in practice.

"The inadequacy of classical theory also extends to the known electromagnetic properties of superconductors, and the present hypothesis suggests the possibility of a unified theory in which there would be no necessity to distinguish between a superconductor and a perfect conductor."

7457:

Cullwick, E. G. Magnetic energy and electron inertia in a super-conducting sphere. Proc. Inst. Elec. Engrs. C **103** (1956), 441–446.

Author's summary: "The hypothesis that the magnetic energy of a current circuit is the kinetic energy of the effective conduction electrons, developed in a previous Monograph [see preceding review], is applied to the case of a conducting sphere without resistivity in a uniform magnetic field. A surface current is induced which prevents the growth of a magnetic field within the sphere, and expressions are found for the number and velocity of effective conduction electrons which carry the current. It is found that these electrons are in stable radial equilibrium, moving in circular orbits under the action of magnetic forces.

"The well-known Meissner effect in pure superconductors is shown to be an expected rather than an unexpected phenomenon, since its absence would require, under certain conditions, a supercurrent lacking equilibrium.

"The theory is shown to lead, by means of a simple assumption, to the basic equations of the London theory of superconductivity, but with a different interpretation of the velocity parameter. Finally, the inertial supercurrent and magnetic field which should be produced by the steady rotation of a superconducting sphere, as deduced by the new theory, are shown to be exactly the same as those forecast by the London theory."

7458:

Bernardes, N.; Primakoff, H. Theory of solid He^3. Phys. Rev. (2) **119** (1960), 968–980.

7459:

Lundquist, S. Mercury experiments. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 34–37. Zanichelli, Bologna, 1960.

The author shows that for a fluid with finite conductivity, the magnetic field diffuses through the fluid, but if the fluid velocity is much larger than the effective diffusion velocity, the field is compelled to follow the fluid motion and the conductivity is effectively infinite. The author calculates that for laboratory size containers and magnetic flux densities, the field diffuses through liquid mercury, but the conductivity is effectively infinite for the interior of the earth or the sun. A few experiments are mentioned which show that a magnetic field stiffens the mercury and delays the transition from laminar to turbulent flow to a higher critical velocity. *D. Nelson* (Seattle, Wash.)

7460:

Hyvärinen, L. P. The coherent scattering of x-rays in the direction of the primary beam. Ann. Acad. Sci. Fenn. Ser. A. VI No. 45 (1960), 10 pp.

The author treats the problem of determining the intensity of x-rays scattered by a homogeneous and amorphous slab of matter in the direction of the primary beam. He simplifies the problem by considering the surface (an ellipsoid of revolution) consisting of those points in the slab which scatter the incident radiation with constant phase. By projection along these surfaces he is able to compute the effect of multiple scattering by the electrons, assumed uniformly distributed. In this way he obtains the experimentally observed result that $I = I_p \exp(-\mu z)$, where $I_p(I)$ is the intensity of the incident (emergent) beam and z is the thickness of the slab.

H. A. Hauptman (Washington, D.C.)

7461:

Hayashi, Mitsuhiko. Psuedokinematical approximation in electron diffraction by crystal. J. Phys. Soc. Japan **15** (1960), 2054–2063.

In der strengen dynamischen Theorie der in Kristallgittern auftretenden Interferenzerscheinungen von Röntgen- und Elektronenstrahlen wird der ganze Kristall als eine Einheit betrachtet und damit werden Mehrfachstreuungen, Umweganregungen usw. berücksichtigt. In der kinematischen (geometrischen) Theorie werden dagegen bloss Einfachstreuungen in Betracht genommen. Bei der Elektronenstreuung von Gasmolekülen genügt schon oft zur Deutung der experimentellen Ergebnisse die einfache kinematische Theorie, wenn jedoch die Gasmoleküle auch schwere Atome enthalten, so beobachtet man ausgesprochene Abweichungen von dieser Theorie. R. Glauber und V. Schomaker [Phys. Rev. **89** (1953), 667–671] erweiterten deshalb die kinematische Theorie, die eigentlich der Bornschen Näherung—in der Mehrfachstreuungen ganz unberücksichtigt bleiben—entspricht, auf die Weise, dass Sie statt den Atomformamplituden die aus der quantenmechanischen Stosstheorie folgenden strengeren Ausdrücke einführten. Diese sogenannte pseudokinematische Theorie, welche schon auch Mehrfachstreuungen, jedoch nur innerhalb eines Atoms berücksichtigt, lieferte dann im erwähnten Fall mit der Erfahrung übereinstimmende Resultate. Ziel der vorliegenden Arbeit ist die Frage theoretisch zu untersuchen, dass in welchem Masse diese pseudokinematische Theorie auch noch zur Deutung der in Kristallgittern auftretenden Elektroneninterferenzerscheinungen brauchbar ist.

Den Ausgangspunkt der Berechnungen bildet die gewohnte Methode, nach der man die ψ-Funktion der einfallenden ebenen Welle nach der Streuung in eine solche von einer ungestörten ebenfalls ebenen Welle und von auslaufende Wellen darstellende ψ-Funktionen zerlegt. Es werden Glieder bis zur zweiten Ordnung berücksichtigt und die erhaltenen Resultate werden mit dem für den ganzen Kristall aus der Bornschen Theorie mit Hilfe von sukzessiven Näherungen hergeleiteten Resultate, das als exakt betrachtet wird, vergleichen. Als wichtigstes Resultat folgt, dass die pseudokinematische Theorie entschieden besser als die kinematische ist und dass ihre Resultate im Falle von dünnen Kristallen—in denen von der Richtung des einfallenden Strahles aus betrachtet, die Atome sich kaum überdecken—mit der strengen Theorie übereinstimmen. *T. Neugebauer* (Budapest)

7462:

Szigeti, B. The infra-red spectra of crystals. Proc. Roy. Soc. London. Ser. A **258** (1960), 377–401.

The higher-order effects in the intrinsic infra-red absorption of crystals are investigated in a systematic way. In agreement with a previous paper which dealt with the static dielectric constant, it is found that in the case of ionic crystals the third- and fourth-order potential, the second- and the third-order dipole moment, and the cross-terms between the second-order moment and the third-order potential, all contribute terms of the same order to the infra-red spectrum. In the lowest approximation, the third-order moment and the fourth-order potential only affect the absorption in the immediate neighbourhood of the maximum and hence have little effect on the shape of the spectrum. The broadening of the main band is due mainly to the third-order potential, while the side bands may be caused by the second-order moment as well as by the third-order potential and by cross-terms between the two. But due to an internal field effect, in strongly ionic crystals a large second-order moment automatically leads to a large third-order potential; thus a large second-order moment may increase the width of the main band as well as the intensity of the side bands. Although the intrinsic infra-red absorption of valency, such as diamond or germanium, is due to the second-order moment only, nevertheless, there is a strong similarity between the expressions for the infra-red absorption of valency crystals and for the side-band absorption of ionic crystals. This similarity suggests that the spectra of all ionic crystals should exhibit a number of secondary maxima. The available experimental evidence does not seem sufficient to decide whether this suggestion is correct. *Werner Nowacki* (Bern)

7463:

Sekanina, Josef. Development of morphological terminology in crystallography. Práce Brn. Českoslov. Akad. Věd. **31** (1959), 521–580. (Czech. Russian summary)

7464:

Belov, N. V.; Borisov, S. V.; Golovachev, V. P. On the arbitrarily chosen signs in direct methods for solving crystal structures. Proc. Nat. Inst. Sci. India, Part A **25** (1959), 220–229.

In der Form von 3 Tabellen wird angegeben, wieviele Vorzeichen bei gegebenen Bravais-Gittern und bei zentrosymmetrischen Kristallen willkürlich vorgegeben werden können, um die statistische Methode zur Vorzeichenbestimmung bei der Fouriersynthese anwenden zu können. Auch der 2 dimensionale Fall der 17 ebenen Gruppen wird behandelt. *Werner Nowacki* (Bern)

FLUID MECHANICS, ACOUSTICS

See also A6936, A6947, A7076, 7350, 7351, 7603, 7607, 7616, 7861, 7864, 7865, 7866, 7879.

7465:

★Proceedings of the 1960 Heat Transfer and Fluid Mechanics Institute. Held at Stanford University, June 15, 16, 17, 1960. Edited by D. M. Mason, W. C. Reynolds, W. G. Vincenti. Printed and distributed by Stanford University Press, Stanford, Calif. for the Heat Transfer and Fluid Mechanics Institute, 1960. x + 259 pp. $8.75.

The papers of mathematical interest in this volume will be given separate reviews in these pages.

7466:

Ghildyal, C. D. Unsteady motion of an infinite liquid due to the uniform rotation of a sphere, $r = a$. Quart. Appl. Math. **18** (1960/61), 396–399.

7467:

Suzuki, Makoto. Theoretical and experimental studies on the vortex-tube. Sci. Papers Inst. Phys. Chem. Res. **54** (1960), 43–87.

Author's summary: "The vortex-tube is a device which utilizes the centrifugal force produced by the vortex motion in a cylindrical tube. The flow of gases is a three-dimensional flow moving toward the axial and radial direction with vortex motion, and there is no main flow else. It is very difficult to carry out a theoretical analysis. Therefore, setting up some bold assumptions concerning the velocity profile, approximate analyses have been attempted from the aerodynamic, thermodynamic and acoustic standpoints, obtaining results which are found to agree with the experiments quantitatively."

7468:

Blahó, M. Die Berechnung der Strömung für ein doppeltes Schaufelgitter. Period. Polytech. Engrg. **4** (1960), 227–249.

The incompressible, nonviscous flow past two cascades of blades in series is investigated by means of two methods. The first one is based on the replacement of each cascade by a row of single vortices. The mutual interference of the two rows is assumed to be such that the velocities induced by the second row at the blades of the first one, and vice-versa, are uniform. The first method is restricted to comparatively large axial distances between the rows; otherwise, it is only slowly—if at all—convergent. The second method is based on Schlichting's singularity method.

M. J. Schilhansl (Cranston, R.I.)

7469:

Richter, W. Berechnung der Druckverteilung von ebenen Schaufelgittern mit stark gewölbten dicken Profilen bei inkompressibler Strömung. Ing.-Arch. **29** (1960), 351–372.

Die Berechnung der Potentialströmung (Geschwindigkeitsverteilung an der Kontur, Zu- und Abströmrichtung) für ein vorgegebenes Schaufelgitter aus dicken stark gewölbten Profilen wird durchgeführt. Hierzu werden die Schaufelprofile durch eine Singularitätenverteilung (Quell- und Wirbelverteilung) ersetzt, die im Innern der Profile auf einer geeignet gewählten gekrümmten Linie angeordnet sind. Für die Singularitätenverteilung wird ein Reihenansatz gemacht, dessen Glieder durch ein lineares Gleichungssystem bestimmt werden. Das Gleichungssystem erfüllt die Bedingung verschwindender Normalkomponenten der Strömung an einer entsprechenden Anzahl von Konturpunkten. Hierzu müssen die von den einzelnen Reihengliedern induzierten Geschwindigkeiten mit Hilfe universeller Diagramme und Tabellen bestimmt werden. Rechenbeispiele zeigen gute Übereinstimmung mit Messungen sowie mit Rechnungen, die mit konformen Abbildungen durchgeführt worden sind.

L. Spiedel (Mülheim)

7470:

Riegels, Friedrich W. Fortschritte in der Berechnung der Strömung durch Schaufelgitter. Z. Flugwiss. **9** (1961), 2–15. (English and French summaries)

Author's summary: "A report is given on the progress having been obtained during the last five years in Göttingen concerning the field of radial and straight cascade design. The methods of calculation for straight cascades have been developed in such a way that pressure distributions on cascades of arbitrary shaping and initial direction of air flow can nowadays be determined within a very short time by means of automatic computers."

7471:

Serbin H. Flow induced by a cavity in a supersonic stream. J. Aerospace Sci. **28** (1961), 247–248.

7472:

Legendre, Robert. Méthode simple de calcul d'une aile élancée. Rech. Aéro. No. 78 (1960), 7–11.

Author's summary: "Le calcul est effectué dans le cadre de la théorie des corps élancés de R. T. Jones et G. N. Ward. La condition de Joukowsky est appliquée sur tout le contour de l'aile. La donnée est le potentiel sur l'aile dont le potentiel dans tout l'espace est déduit, sans intégration, par un artifice. La forme de l'aile est calculée à partir du potentiel par dérivation."

7473:

Stanišić, Milomir M. On the solution of an integral equation appearing in the delta-wing theory. Z. Angew. Math. Mech. **40** (1960), 397–414. (German and Russian summaries)

The author derives an approximate solution of the integral equation which arises in the theory of the unsteady motion of a delta wing. When the velocity potential is expanded in powers of the frequency a series of integral equations arises, all the equations are singular and have unsymmetric kernels, furthermore the domain of integration is complicated. The method of solution is explained in detail only for the case of transonic flow, but it can be applied to the supersonic case by increasing the number of terms considered. The author describes his method as "perturbation of the domain of integration".

G. N. Lance (Winfrith)

7474:

Sims, Joseph L.; Saunders, Lee M. Local normal-force ratio around an expansion corner. J. Aerospace Sci. **28** (1961), 170.

7475:

Frankl', F. I. Streamlines for profiles in subsonic flow with local supersonic zones bounded by curved surfaces of condensation. Prikl. Mat. Meh. **21** (1957), 141–143. (Russian)

7476:

Frankl', F. I. On the problem of symmetrical flow past a given symmetrical profile with subsonic velocity at infinity and local supersonic velocities. Prikl. Mat. Meh. **23** (1959), 776–780 (Russian); translated as J. Appl. Math. Mech. **23**, 1107–1114.

In earlier papers [Prikl. Mat. Meh. **20** (1956), 196–202; **21** (1957), 141–143; MR **18**, 255; preceding review] the author has formulated for the Tricomi equation $\eta\psi_{\eta\eta}+\psi_{\xi\xi}=0$ boundary value problems for flows about profiles with local supersonic zones terminated by (I) straight and (II) curved shocks. Now he formulates the problem (II) as a perturbation of (I). Since the perturbed problem is intractable, the author solves an analogous boundary value problem for $U_{yy}+U_{xx}\operatorname{sgn} y=0$ on a domain in the first and fourth quadrants. The part of the domain in the first quadrant is symmetrical with respect to $y=x$; that in the

fourth satisfies $x-y \leqq 1$. After the part of the domain above the x-axis has been conformally mapped onto a half-plane, the determination of U for $y>0$ reduces to a Riemann-Hilbert problem which can be solved explicitly.

J. H. Giese (Havre de Grace, Md.)

7477:

★**Second Symposium on Naval Hydrodynamics: Hydrodynamic noise, cavity flow.** Sponsored by the Office of Naval Research and the National Academy of Sciences—National Research Council, August 25–29, 1958, Washington, D.C. U.S. Government Printing Office, Washington, D.C., 1960. vi+583 pp. $4.00.

A set of 26 lectures; those of mathematical interest will be reviewed separately.

7478:

Proudman, Joseph. A damped Kelvin-wave of general form. J. Marine Res. **17** (1958), 424–428.

This short paper is concerned with a long free progressive wave in a channel of uniform width and depth when the component of current transverse to the channel is neglected. The density of the water is supposed uniform, and the rotation of the earth and friction (proportional to velocity) are taken into account. The equations are linear.

A solution is given for the stream velocity and for the surface elevation (which vary in both horizontal directions). It is supposed that the wave travels in the positive direction, that the elevation is prescribed at one point as a function of time, and that the effect of friction is small. A numerical example illustrates the last assumption.

F. Ursell (Cambridge, England)

7479:

Tadjbakhsh, Iradj; Keller, Joseph B. Standing surface waves of finite amplitude. J. Fluid Mech. **8** (1960), 442–451.

The waves are two-dimensional gravity waves on the surface of an inviscid incompressible fluid of finite constant depth, periodic in time and in the horizontal direction. The ratio of amplitude to wavelength is assumed small. The surface profile, potential function, pressure and frequency are found correct to the third order in this small parameter; the uniqueness of the solution breaks down in certain circumstances (e.g., for infinite depth) but this difficulty is not discussed in detail here. Graphs of the surface profile and of pressure as a function of depth are included. The authors correct an error in the work of Penney and Price [Philos. Trans. Roy. Soc. London. Ser. A **244** (1952), 236–284] where a time-dependent pressure term independent of depth was erroneously omitted, and show that their own method requires less labour which is however still considerable.

F. Ursell (Cambridge, England)

7480:

Quilghini, Demore. Sul problema generale dell'equilibrio spontaneo di una massa fluida gravitante e ruotante. Ann. Mat. Pura Appl. (4) **48** (1959), 281–303.

Verfasser untersucht gravitierende Flüssigkeitsmassen, die wie starre Körper mit der konstanten Winkelgeschwindigkeit ω um eine raumfeste Achse rotieren und sich im relativen Gleichgewicht befinden. Er zeigt, daß sich solche Gleichgewichts figuren von einer einparametrigen Schar von Flächen lückenlos und einfach erfüllen lassen, derart, daß auf jeder der Druck p, die Dichte ρ und das Gesamtpotential konstant sind. Verschwindet überdies der Druck auf der Randfläche Σ, so finden sich eine Reihe Bedingungen, denen ρ, p und ω genügen müssen. Eine solche lautet z.B. $\lim p(P)/\rho(P)=0$, falls der Punkt P auf Σ rückt. Der Fall $\omega=0$ wurde in einer früheren Note des Verfassers [Riv. Mat. Univ. Parma **8** (1957), 27–41; MR **20** #6260] betrachtet.

K. Maruhn (Giessen)

7481:

Constantine, T. On the movement of ships in restricted waterways. J. Fluid Mech. **9** (1960), 247–256. (1 plate)

As a ship moves in a canal the distance between the keel and the canal bottom decreases with increasing speed. The author investigates this phenomena using simple considerations based on Bernoulli's equation and the equation of continuity supplemented by standard shallow water theory. It is found that for sufficiently high velocities fluid is piled up ahead of the ship in the form of a bore.

R. C. MacCamy (Pittsburgh, Pa.)

7482:

Szebehely, V. G. Hydrodynamic impact. Appl. Mech. Rev. **12** (1959), 297–300.

Survey article.

7483:

Pierson, J. D. Water waves. Appl. Mech. Rev. **14** (1961), 1–3.

Expository article.

7484:

Miles, John W. Free surface oscillations in a rotating liquid. Phys. Fluids **2** (1959), 297–305.

Waves in a rotating, vertical circular cylinder are investigated using the linearized equations of motion instead of shallow water theory. This immediately leads to the result that a vertical gradient of velocity produces relative vorticity; thus the motion relative to the rotating reference frame cannot remain irrotational.

The case of a deep cylinder (depth $\gg$ radius) is studied in some detail and the frequencies of free oscillations calculated. In addition to the modes which are the counterparts of the non-rotating modes, a new set is discovered for which the fluid motion does not fall off exponentially with depth. The results are applied to the forced transverse oscillations of the tank.

M. H. Rogers (Bristol)

7485:

Kapur, J. N. Flow of a compressible viscous fluid round a corner. Math. Student **27** (1959), 51–54.

M. Ray [Proc. Nat. Inst. Sci. India. Part A **21** (1955), 155–160; MR **17**, 551] discussed the Prandtl-Meyer problem of flow of a gas with the inclusion of the effects of viscosity and heat transfer. The aim of the paper under review is to point out that Ray's solution is not correct as it does not satisfy all the equations of the problem. The author attributes this inconsistency to the wrong assumption about the viscosity (μ)-enthalpy (i) relation ($\mu \propto i^n$) in Ray's work. The author proves that Prandtl-Meyer

solution holds even in the present case if $\mu \propto [i(1-i)]^{-1/2}$. This expression is in error in as much as the term $1-i$ should be replaced by i_0-i, where i_0 is the value of enthalpy on the sonic line. But then μ will be infinite on the sonic line. The value of the paper would have been enhanced if the author had mentioned the physical assumptions under which such a law for viscosity will hold. It is not clear to the reviewer how the "no-slip" condition on the wall is satisfied, and it appears that the solution holds only far away from the boundary.

P. L. Bhatnagar (Bangalore)

7486:

Han, L. S. Hydrodynamic entrance lengths for incompressible laminar flow in rectangular ducts. J. Appl. Mech. **27** (1960), 403–409.

The author considers the problem of determining the entrance length (defined as the length during which the center line velocity u_0 attains ninety-nine per cent of its final value) in the viscous incompressible flow of a fluid in a rectangular duct. By assuming that: (1) the transverse components of the velocity v, w are negligible in comparison to the component u in the z-direction; (2) the term $\partial^2 u/\partial z^2$ is negligible in comparison to $\partial^2 u/\partial x^2$, $\partial^2 u/\partial y^2$ (here x, y are the variables in any cross section); (3) $\partial p/\partial z$ is dependent only on z, where p is the pressure, the flow equations of the problem reduce to a single linear second order equation (a modified Poisson equation), whose coefficients vary with the cross section. The method of solution consists of: (1) expanding $f(x, y)=1$, $-a \leqq x \leqq a$, $-b \leqq y \leqq b$, in double Fourier series and then expressing the pressure drop $(\partial p/\partial z)$ as a double Fourier series (an identity); (2) solving for u as a product of the Fourier series and the pressure drop; (3) eliminating the pressure drop by use of an integrated form of the equation of continuity. This leads to an expression for u which furnishes a smooth transition from the fully developed velocity profile to uniform flow. The pressure drop is calculated by a momentum principle. Graphs of velocity profiles at three cross sections are given and compared with the results of other authors.

N. Coburn (Ann Arbor, Mich.)

7487:

Wadhwa, Y. D. Unsteady rotation of a plane lamina in accelerated motion. Proc. 4th Congress Theoret. Appl. Mech. 1958, pp. 187–194. Indian Soc. Theoret. Appl. Mech., Kharagpur.

The author considers the non-steady flow of a viscous incompressible fluid past an infinite thin plate which initially rotates about an axis perpendicular to the plate. The flow velocity and pressure are assumed to be independent of the polar angle but the velocity varies directly with the radial distance from the axis of rotation. By use of this assumption, the flow equations and the equation of continuity are expressed in a dimensionless form involving only derivatives with respect to time, T, and the variable, Z, measuring the distance perpendicular to one face of the plate. Solutions are to be determined in terms of powers of T and functions of the variable, $\eta = \frac{1}{2} Z T^{1/2}$. Boundary conditions, which correspond to the velocity varying initially with a power of T, are considered. Various special solutions (first approximations) are analyzed.

N. Coburn (Ann Arbor, Mich.)

7488:

Yen, K. T. On the compressibility effects of the lubricant for two-dimensional slider bearings. J. Appl. Mech. **27** (1960), 609–612.

By variational method the author finds that for two-dimensional slider bearings with compressible lubricant, no optimal film shape exists to yield a maximum total load. However, when the mass flow is specified as a subsidiary condition, a bearing with stepped-film shape and isothermal lubricant is found to give the maximum load. But its exact shape still depends on the value of the maximum pressure.

L. N. Tao (Chicago, Ill.)

7489:

Bloom, Martin H.; Rubin, Stanley. High-speed viscous corner flow. J. Aerospace Sci. **28** (1961), 145–157.

The authors examine the high-speed viscous steady flow within an interior corner formed by the intersection of two semi-infinite surfaces. The flow proceeds along the line of intersection. The analysis is applied to the case of constant pressure, constant corner angle, and isothermal surfaces. It is assumed that the Crocco velocity-enthalpy relation is valid. The fundamental equations governing the motion are those of the boundary layer: momentum in the direction of the main motion; continuity; energy; and state. The Prandtl number is assumed to be equal to unity. The equations are transformed in a standard manner to form the momentum integral equation. In the next step there is applied the density transformation of the type of Dorodnitzyn-Howarth. As a particular case, the authors consider zero-pressure gradient. The assumed velocity profiles are symmetric and of the fourth-degree. The integral equation is solved in a standard way. Two cases are considered: incompressible and compressible fluid. Favorable agreement between some results obtained by this method and by other methods is demonstrated for the isothermal, constant-density case. Results show an increasingly sharp merger of the outermost isovels of streamwise velocity as the Mach number increases. This sharp merging of the outer isovels is increased by increasing corner angle and by insulation or heating of the surfaces. The surface shear stress and heat flux, related by Reynolds analogy in the disturbed region, increase linearly from zero at the apex to their undisturbed two-dimensional values. The drag of the surface area disturbed by the merging is one half the drag due to two-dimensional shear over the same area. The total heat flux over this area is one fourth the two-dimensional heat flux over an equal area.

M. Z. v. Krzywoblocki (E. Lansing, Mich.)

7490:

Venkates, H. G. Motion of a viscous liquid past an ellipsoid. Phys. Fluids **4** (1961), 33–39.

The usefulness of Seth's synthetic method is demonstrated by obtaining a solution for the motion of a viscous liquid past an ellipsoid in terms of an arbitrary parameter n. The solution includes the irrotational, the slow viscous and the boundary layer solutions as particular cases. This is done by introducing an external force so that the equations of motion are exactly solvable, and then making the external force tend to zero in the limit. Viscous flow past a sphere and an elliptic cylinder are derived as particular cases.

The method adopts ellipsoidal coordinates and indicates

analytically the existence of the boundary layer. It is further shown that the boundary layer thickness varies with two parameters depending upon the vorticity allowable near the body and the outer edge of the layer. It is found that the order of this thickness should be greater than the conventional value $R^{-1/2}$, R being the Reynolds number. *B. R. Seth* (Kharagpur)

7491:

Rosen, Gerald. Integration theory for one-dimensional viscous flow. Phys. Fluids **2** (1959), 517–520.

The equation of continuity for the one-dimensional flow of compressible liquid of density ρ with velocity u is satisfied if

$$\partial\psi/\partial x = \rho, \quad \partial\psi/\partial t = -\rho u.$$

Hence u can be eliminated and an equation found for the pressure with ψ, t as independent variables; the equation is solved exactly in some cases in which the pressure and the viscosity coefficient are appropriate functions of ρ.

W. R. Dean (London)

7492:

Nanda, R. S. Boundary layer growth with suction. Proc. 4th Congress Theoret. Appl. Mech. 1958, pp. 241–250. Indian Soc. Theoret. Appl. Mech., Kharagpur.

The effect of suction is examined in yet another problem of boundary layer growth. As usual the end-point of the investigation is the determination of the first approximation to the onset of separation in spite of the fact that the most important feature of these flows is the boundary layer growth in the reversed flow regime.

K. Stewartson (Durham)

7493:

Cess, Robert D. Laminar-film condensation on a flat plate in the absence of a body force. Z. Angew. Math. Phys. **11** (1960), 426–433. (German summary)

Let saturated vapor flow across a cooled flat plate and condense in a (continuous) liquid film. Under the assumption that the (laminar) motion of the film is caused solely by frictional effects of the vapor, the author applies a standard "similarity" transformation to the resulting boundary-layer problem. Series solutions of the ensuing ordinary differential equations are terminated after a few terms and the resulting approximations discussed graphically. *W. C. Rheinboldt* (Syracuse, N.Y.)

7494:

Akatnov, N. I. The application of Von Mises' variables to the problem of laminar boundary layer propagation along a wall. Prikl. Mat. Meh. **24** (1960), 154–156 (Russian); translated as J. Appl. Math. Mech. **24**, 208–212.

At the leading edge of a semi-infinite plate is an infinitely thin slot source. The fluid coming out of the source is identical to that which surrounds the plate; it flows along one of the sides of the plate. The fluid in the surrounding space travels at constant velocity u_0 (parallel to the wall). For $u_0 = 0$ the problem reduces to that of the "wall jet" [see M. Glauert, J. Fluid Mech. **1** (1956), 625–643; MR **19**, 87]. The author uses the incompressible boundary layer equation in the "von Mises" form. In addition to the boundary condition at the wall and at infinite distance from the wall the solution has to satisfy an integral condition which contains a constant depending on the source intensity, u_0, and the kinematic viscosity of the fluid. The author gives a similarity solution for the case $u_0 = 0$ and he develops a solution by series expansion for the general case; however this is only valid for large distance ξ from the leading edge. Diagrams show the velocity $u(y)$, where y is the coordinate normal to the wall.

I. Flügge-Lotz (Stanford, Calif.)

7495:

Šidlovskiĭ, V. P. Laminar boundary layer on an infinite disc rotating in a gas. Prikl. Mat. Meh. **24** (1960), 161–164 (Russian); translated as J. Appl. Math. Mech. **24**, 221–226.

An infinite plane disc is rotating in its plane in a space filled with a perfect gas which has constant Prandtl number and viscosity varying as a power of the absolute temperature. For this problem approximate equations are obtained for large values of a non-dimensional parameter which is essentially the ratio of the Reynolds number to the square of the Mach number where both are based on the local distance from the axis of rotation. By suitable transformations these equations are reduced to a form analogous to that obtained in the exact solution of the corresponding incompressible flow and hence a solution can be obtained. *J. J. Mahony* (St. Lucia)

7496:

Rao, G. N. V. Note on the sufficient conditions for the stability of axially symmetric flows to axially symmetric disturbances. J. Aerospace Sci. **28** (1961), 248–249.

7497:

Hocking, L. M.; Michael, D. H. The stability of a column of rotating liquid. Mathematika **6** (1959), 25–32.

The stability of a column of rotating, inviscid liquid is discussed, including the effects of surface tension, for two cases of the basic flow, one being the rigid rotation and the other a special configuration of non-uniform rotation. The disturbances considered are confined to the two-dimensional fluctuations in the cross-sectional planes of the liquid column. A similar problem dealing with the instability of rotating cylindrical jets, which includes the three-dimensional disturbances, has been recently treated by J. Ponstein [Appl. Sci. Res. A **8** (1959), 425–456; MR **21** #6858]. *T. Yao-tsu Wu* (Pasadena, Calif.)

7498:

Clenshaw, C. W.; Elliott, D. A numerical treatment of the Orr-Sommerfeld equation in the case of a laminar jet. Quart. J. Mech. Appl. Math. **13** (1960), 300–313.

A numerical method is described for obtaining the stability characteristics of the plane laminar jet with non-dimensional velocity profile $w = \operatorname{sech}^2 y$. By making the transformation $t = \tanh y$, the range of the independent variable is changed to $(-1, +1)$. The perturbation stream function φ is then represented by an expansion in terms of the Chebyshev polynomials $T_n(t) = \cos(n \cos^{-1} t)$ with complex coefficients. On substituting this expansion into the Orr-Sommerfeld equation, a five-term recurrence relation is obtained involving α, R, and c. Approximate results

are then obtained by retaining N terms in the expansion; the results presented were obtained by machine calculation for $N=24$, 48, and 96. For large values of α, good accuracy is obtained for quite small values of N, but for small values of α, a minimum value of R is obtained only for the largest value of N. From the results obtained it is estimated that $R_{cr}\approx 3.7$, $\alpha_{cr}\approx 0.25$, and that the re (c) lies between about 0.05 and 0.10. The behavior of R for small values of α is known to depend very sensitively on the behavior of φ for large values of y and it may be expected, therefore, that a method of this type may begin to fail under such circumstances. *W. H. Reid* (Providence, R.I.)

7499:

Spielberg, Kurt; Timan, Hans. On three- and two-dimensional disturbances of pipe flow. J. Appl. Mech. **27** (1960), 381–389.

In the study of the stability of two-dimensional parallel flows, Squire's well-known theorem shows that it is sufficient to consider only two-dimensional disturbances. The present paper explores the possibility of proving a similar result for the case of Poiseuille flow. By considering three-dimensional disturbances of the form

$$\mathbf{q}(r)\exp\{i(\alpha z+\beta\varphi-\alpha ct)\},$$

it is shown that such a result does not hold, even in the inviscid case. The authors then consider the case of "cross disturbances" for which $\alpha=0$ and find that they are stable. The conclusion that Poiseuille flow is stable to infinitesimal disturbances is thus further strengthened.

W. H. Reid (Providence, R.I.)

7500:

Trehan, S. K. Oscillations of an incompressible fluid sphere of varying density. Astrophys. J. **132** (1960), 264–268.

From the author's summary: "The oscillations of a self gravitating incompressible fluid sphere of varying density have been investigated. It is found that, for the same total mass, a sphere of uniform density has a longer period of oscillation than a sphere of varying density."

R. C. DiPrima (Troy, N.Y.)

7501:

Zaĭcev, A. A. The stability of a viscous layer on a solid body in a flow of gas. Dokl. Akad. Nauk SSSR **130** (1960), 1228–1231 (Russian); translated as Soviet Physics. Dokl. **5**, 49–53.

This brief paper presents an analysis of the stability of a layer of a heavy viscous fluid with a linear velocity distribution over which flows a viscous gas. It is assumed that μ_1/μ and ρ_1/ρ are small where μ and ρ denote viscosity and density respectively and a subscript 1 refers to the gas. Approximate solutions of the Orr-Sommerfeld equation are obtained in the form of a power series in αR where R is the Reynolds number and $2\pi/\alpha$ is the wavelength of the disturbance. Only the first two terms in the series are used in the computation of the neutral stability curves. The critical value of αR appears to be of the order of 50–100 and no justification for using a small parameter approach is given. *R. C. DiPrima* (Troy, N.Y.)

7502:

Rota, Gian-Carlo. Une généralisation de l'espérance mathématique conditionnelle qui se présente dans la théorie statistique de la turbulence. C. R. Acad. Sci. Paris **251** (1960), 624–626.

L'auteur considère des opérateurs linéaires R, bornés, normaux, de l'espace L_2 en lui-même, où $L_2(s, \Sigma, \mu)$ est un espace de variables aléatoires de carré sommable dans lequel est définie une mesure μ. Si en outre $R(fg)=RfRg+R[(f-Rf)(g-Rg)]$, $f, g\in L_2$, R est un endomorphisme de Reynolds.

On démontre qu'il existe deux sortes d'opérateurs R: (1) les espérances mathématiques conditionnelles, avec $R^2=R$; (2) des opérateurs moins simples, attachés à tout semi-groupe de transformations de S.

On peut décomposer d'une façon unique S en trois sous ensembles disjoints S_0, S_1, S_2, invariants par R, tels que: (a) la restriction de R à $L_2(S_0)$ soit l'opérateur nul; (b) la restriction de R à $L_2(S_1)$ soit une espérance mathématique; (c) la restriction de R à $L_2(S_2)$ soit de la forme R_1R_0, où R_0 est une espérance mathématique et où R_1 est un opérateur du second type. *J. Bass* (Paris)

7503:

Bourret, R. C. Turbulent diffusion in two and three dimensions by the random-walk model with memory. Canad. J. Phys. **39** (1961), 133–140.

Partial differential equations are obtained which describe the probability of particle displacement for two- and three-dimensional Taylor-Goldstein lattice models of turbulent diffusion in a limit where the lattice spacing goes to zero.

R. H. Kraichnan (New York)

7504:

Peskin, Richard L. Some effects of particle-particle and particle-fluid interaction in two phase flow systems. Proc. 1960 Heat Transfer Fluid Mech. Inst., pp. 192–207. Stanford Univ. Press, Stanford, Calif., 1960.

The diffusion of small solid particles suspended in a turbulent flow is discussed under the approximation that the total force encountered by a particle consists of (1) a drag proportional to the particle's velocity relative to the local fluid, and (2) a random pressure due to the presence of the other suspended particles. A number of idealizations and approximations are invoked. It is found that particle interactions through the pressure forces reduce the diffusivity most strongly for large and slowly moving particles. *R. H. Kraichnan* (New York)

7505:

Stalker, R. J. Sweepback effects in turbulent boundary-layer shock-wave interaction. J. Aero/Space Sci. **27** (1960), 348–356.

The results are reported of a combined theoretical and experimental investigation of the interaction of shock waves and turbulent boundary layers for certain simple three-dimensional configurations in which the flow pattern is invariant in one direction parallel to the surface. It is shown that provided this direction is not too closely aligned to the mainstream, the important features of the flow can be predicted accurately by using suitable modifications of two-dimensional theory and experimental results. *J. J. Mahony* (Sydney)

7506:

Ellison, T. H.; Turner, J. S. Mixing of dense fluid in a turbulent pipe flow. I. Overall description of the flow. J. Fluid Mech. 8 (1960), 514–528.

The flow of a layer of dense salt solution introduced on the floor of a sloping rectangular pipe along which water is flowing has been studied for turbulent flow of the ventilating stream. At low velocities, the salt solution flows some distance downhill before it is entrained and swept upwards by the ventilating flow but this effect is small at most of the speeds used in these experiments. The measurements of mean flow and of concentration distributions are analysed to give the rate of growth of the salt layer as a function of the pipe Richardson number formed from the density of the fully-mixed discharge. The actual thickness depends on initial conditions of some complexity and an approximate theory is given for horizontal flow without flow reversal. A relation is found between floor concentration and depth of the salt layer.

A. A. Townsend (Cambridge, England)

7507:

Ellison, T. H.; Turner, J. S. Mixing of dense fluid in a turbulent pipe flow. II. Dependence of transfer coefficients on local stability. J. Fluid Mech. 8 (1960), 529–544.

Further measurements of concentration and velocity in turbulent flow of a stably-stratified fluid along a sloping rectangular pipe are described, and they are compared with the predictions of a simple theory that assumes the modification of the velocity distribution to be due to change of weight alone, the momentum transfer coefficient remaining unchanged by stability. Observed velocities agree closely with those predicted from the observed distributions of density. Closer comparison shows that the momentum transfer coefficient is about 20% less in stable than in neutral flow. The ratio of the transfer coefficients for salt and momentum is calculated as a function of the local Richardson number and the results are in agreement with Ellison's speculative theory with a critical flux Richardson number of about 0.15.

A. A. Townsend (Cambridge, England)

7508:

Guiraud, Jean-Pierre. Écoulement plan de Couette d'un gaz rayonnant. C. R. Acad. Sci. Paris **250** (1960), 2997–2999.

La présente note se propose l'étude d'un problème d'aérothermodynamique lorsque le couplage entre mouvement et rayonnement n'est pas négligeable. Après avoir formulé les équations fondamentales régissant le problème de Couette lorsque l'on tient compte du rayonnement, l'auteur définit un paramètre sans dimension fondamental caractérisant l'importance relative de l'énergie rayonnée et de l'énergie dissipée par frottement, puis résoud le problème mathématique ainsi posé après avoir opéré une ingénieuse transformation qui ramène le problème initial à une équation intégro-différentielle dont l'étude peut être menée à son terme. *P. Germain* (Paris)

7509:

Sauer, Robert. ★Einführung in die theoretische Gasdynamik. 3te verbesserte Aufl. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1960. xi+214 pp. DM 29.70.

The second edition [1951] was reviewed in MR **14**, 107. This third edition has been brought up to date in several respects. In view of the increased use of electronic digital computers in gas dynamics, graphical methods have largely been superseded by numerical ones: this change is reflected in the contents. There is also a new section on asymptotic expansions in slender body theory and one on transonic and hypersonic flow.

7510:

Fraeijs de Veubeke, B. Principes variationnels en mécanique des fluides compressibles. Ann. Soc. Sci. Bruxelles. Sér. I **74** (1960), 157–174.

L'auteur étudie l'application du principe de Hamilton aux écoulements isentropiques d'un fluide parfait compressible. Cette application peut être développée de deux manières distinctes suivant qu'on utilise les coordonnées lagrangiennes ou les cordonnées eulériennes. On montre ici comment passer de l'une à l'autre et on souligne l'intérêt qu'il y a à se libérer de la condition de conservation de la masse en utilisant un multiplicateur. On a ainsi une méthode élégante pour retrouver les équations générales de la mécanique des fluides parfaits. *P. Germain* (Paris)

7511:

Bouyer, Roger. ★Application de la méthode optique du contraste de phase a l'étude des écoulements gazeux. Publ. Sci. Tech. Ministère de l'Air, No. 361, Paris, 1960. x+144 pp. 32.00 NF.

7512:

Couvertier, Pierre. ★Application des décharges électriques à l'exploration des écoulements gazeux aux grandes vitesses. Publ. Sci. Tech. Ministère de l'Air, No. 365, Paris, 1960. ix+106 pp. 23.50 NF.

7513:

Filimon, Ioan. L'application correcte de la méthode de Tchaplyguine aux mouvements gazeux subsoniqués autour d'un obstacle elliptique. Bull. Math. Soc. Sci. Math. Phys. R. P. Roumaine (N.S.) **1 (49)** (1957), 269–279.

Cet article consiste en l'application d'une méthode d'approximations successives développée dans un travail du même auteur (à paraître).

Après un exposé rapide de la méthode dans le cas général (le gaz réel étant remplacé par un gaz fictif satisfaisant à la loi de Tchaplyguine, qui s'écoule autour d'un obstacle (ω) donné à l'avance, on lui fait correspondre un écoulement liquide dans le plan $z=x+iy$ autour d'un obstacle (ω_1). Il s'agit de connaitre la fonction $z(\zeta)$ qui détermine la transformation conforme de l'extérieur de (ω_1) sur l'extérieur du cercle $|\zeta|=1$ du plan $\zeta=\xi+i\eta$), l'auteur s'intéresse au cas de l'obstacle elliptique.

La fonction $z(\zeta)$ est recherchée sous forme d'un développement suivant les puissances croissantes de $(\rho_\infty-\rho_0)/2\rho_\infty$ (ρ_∞, ρ_0 sont respectivement la densité du gaz réel à l'infini et au repos) dont les deux premiers termes sont calculés exactement. L'auteur en déduit alors la distribution des vitesses sur l'obstacle, la vitesse maximum, et le nombre de Mach critique en fonction du nombre de Mach à l'infini et de l'excentricité.

Les résultats sont comparés à ceux de I. Imaï (dans le cas d'un obstacle circulaire), C. Jacob et Ko Tamada.

J. Naze (Marseille)

7514:
Hosokawa, Iwao. Transonic flow past a wavy wall. J. Phys. Soc. Japan **15** (1960), 2080–2086.

The author has developed a method of solving the small perturbation transonic equation [same J. **15** (1960), 149–157; MR **22** #1263] and applies this to title problem. The velocity and pressure distributions on the wall are found for a range of free-stream Mach numbers. Shocks are included and it is claimed that the results give a reasonable model for understanding the transition of the flow pattern from subsonic to supersonic.

However, the reviewer is not convinced that the original solution method cited satisfies the necessary order-of-magnitude requirements. *H. C. Levey* (Perth)

7515:
Nocilla, Silvio. Sull'interazione tra un corpo rigido ed una corrente di molecole libere. I. Scambi di energia. Atti. Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. **94** (1959/60), 445–477.

This is a review with some detailed calculation (or recalculation) of a few details of the fundamentals of the free molecular flow. In chapter I, the author presents the fundamental geometry of the collision phenomenon with the following items: Knudsen number; energy incoming and reflected; local and global coefficients of interaction; accommodation coefficient; etc. Following the standard approach in that regime the author quotes the expressions for the distribution function; plots of this function are given for a cylinder, flat plate, etc. In chapter II, the author discusses the energy balance during the collision process. In chapter III, the author compares the experimental results with the analysis in the cases of adiabatic and non-adiabatic conditions for both monatomic and diatomic gases. The results are extensively tabulated. The whole work is based primarily upon the American literature with the addition of some French results.

M. Z. v. Krzywoblocki (E. Lansing, Mich.)

7516:
Sauer, R. Théorie des écoulements des fluides compressibles à vitesse supersonique. Les mathématiques de l'ingénieur, pp. 150–160. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

Quelques exemples empruntés à la dynamique des gaz (propagation des ondes et calcul des ailes, méthode de l'hodographe, méthode des caractéristiques) illustrent la nécessité d'une collaboration entre mathématiques pures, mathématiques appliquées et analyse numérique.

J. P. Guiraud (Meudon)

7517:
Syčev, V. V. Hypersonic flow about a thin body at large angles of attack. Dokl. Akad. Nauk SSSR **131** (1960), 776–779 (Russian); translated as Soviet Physics. Dokl. **5**, 249–252.

Preliminary version of paper reviewed below.

M. D. Van Dyke (Stanford, Calif.)

7518:
Syčev, V. V. Three-dimensional hypersonic gas flow past slender bodies at high angles of attack. Prikl. Mat. Meh. **24** (1960), 205–212 (Russian); translated as J. Appl. Math. Mech. **24**, 296–306.

Hypersonic small-disturbance theory is extended to slender bodies at arbitrary angle of attack. The similarity rule (now involving two parameters instead of one) and unsteady analogy are shown to remain valid. Further approximations are studied for high angles of attack, and for bodies of invariant cross-sectional shape. Generalization to real gases in thermodynamic equilibrium is discussed briefly. *M. D. Van Dyke* (Stanford, Calif.)

7519:
Swigart, R. J. Third-order blast wave theory and its application to hypersonic flow past blunt-nosed cylinders. J. Fluid Mech. **9** (1960), 613–620.

The self-similar solution of G. I. Taylor and Sedov for a point explosion in a perfect gas is exact only for infinite Mach number M. Sakurai [J. Phys. Soc. Japan **8** (1953), 662–669; **9** (1954), 256–266; MR **15**, 909] calculated a correction of order M^{-2}; the next term, of order M^{-4}, is calculated here for the cylindrical case. Application is made to steady hypersonic flow past a blunt-nosed cylinder, using Hayes' unsteady analogy. Comparison is made with experiment and with numerical solutions.

M. D. Van Dyke (Stanford, Calif.)

7520:
Fenain, M. Étude théorique de prises d'air à rampe dièdre, d'envergure finie, en régime supersonique. Rech. Aéro. No. 76 (1960), 5–16.

L'auteur utilise la théorie des écoulements coniques linéarisés pour calculer le débit et la traînée additive de deux types de prises d'air dont la rampe est constituée d'un dièdre de faible ouverture.

Dans l'un de ces cas la rampe est limitée en envergure par deux plaques en delta sur lesquelles s'appuie la carène. La solution du problème ainsi posé est complètement explicité. Des comparaisons avec les résultats expérimentaux montrent que cette théorie rend convenablement compte du phénomène étudié. *P. Germain* (Paris)

7521:
Brocher, Eric F. Comments on the behavior of Sedov's blast-wave solution as $\gamma \to 1$. J. Aerospace Sci. **27** (1960), 955–956.

7522:
Rosciszewski, Jan. Hypersonic flow around bodies of revolution. J. Aerospace Sci. **28** (1961), 168–170.

7523:
Ehlers, F. Edward. On the lift, drag, and moment on a ring concentric to a cylindrical body in supersonic flow. J. Aerospace Sci. **28** (1961), 170–171.

7524:
Hartsell, C. W.; Hays, W. Derivation of Meyer's inlet criteria for real-gas flows. J. Aerospace Sci. **28** (1961), 159–160.

7525:
Gundersen, Roy M. Shock response to downstream disturbances. J. Aerospace Sci. **28** (1961), 174–175.

7526:

Ehlers, F. Edward; Shoemaker, E. M. A three-dimensional linearized analysis of the forces exerted on a rigid wing by a shock wave. J. Aerospace Sci. **27** (1960), 257–260.

Previous work of the authors in same J. **26** (1959), 75–80, 107 [MR **21** #4717] on the interaction between a plane acoustic wave and a moving flat plate is extended to the case when the wave is oblique to the edge of the plate. *J. J. Mahony* (Sydney)

7527:

Sherman, Frederick S. Shock-wave structure in binary mixtures of chemically inert perfect gases. J. Fluid Mech. **8** (1960), 465–480.

A method is presented for obtaining the structure of a shock wave in the steady one-dimensional, equilibrium flow of a binary mixture of non-reacting perfect gases. The calculations are made for a continuum with allowance made for viscosity, thermal conduction, thermal, ordinary and baro-diffusion with suitable variations of the transport coefficients with temperature, molecular mass and composition. Series solutions are given for weak shocks and results are given of sample numerical solutions for stronger shocks. A discussion of the results suggests that the model used is deficient if the components have greatly differing molecular masses because of the importance then of relaxation effects. *J. J. Mahony* (Sydney)

7528:

Glass, I. I.; Hall, J. Gordon. ★Handbook of supersonic aerodynamics. Section 18. Shock tubes. Navord Report 1488 (Vol. 6). U.S. Government Printing Office, Washington, D.C., 1959. xxxviii+604 pp. (not consecutively paged) $3.75.

This is a valuable handbook for all with an interest in shock-tubes and shock-tube techniques. Chapters 1–3 are contributed by Glass and the remainder, chapters 4–7, by Hall.

After a brief introduction in chapter 1, an extensive account of waves and their interactions is given in chapter 2. Also here, and in four supplements, an account of real gas effects is presented. Chapter 3 deals with observed flows in a constant area shock-tube and compares the idealized theory with experiment. Included is the effect of diaphragm rupture and a reasonably detailed account of boundary-layer effects.

Chapter 4 deals with various techniques for obtaining strong shocks including heating of the driver gas and cross-sectional area changes. Applications of the shock tube are covered briefly in chapter 5; for example, its use as a wind-tunnel, as a means of studying complicated wave interactions, high-temperature gas physics and chemical kinetics. Chapter 6 deals with shock-tube materials, design and construction, and chapter 7 covers shock-tube flow measurement and instrumentation.

Altogether the handbook is well illustrated, has numerous charts and tables and an extensive bibliography.

H. C. Levey (Perth)

7529:

Marshak, R. E. Effect of radiation on shock wave behavior. Phys. Fluids **1** (1958), 24–29.

Assuming that the radiation pressure and energy are given by Stefan's law, the equations governing the flow behind a shock are written down. The Rankine-Hugoniot relations are also found using the isothermal property of shocks with radiation. Some similarity solutions are found for the flow behind shocks with the additional assumptions that the radiation terms may be neglected except in the equation for the energy flux and that the flow is plane. Their properties are discussed at some length.

K. Stewartson (Durham)

7530:

Elliott, L. A. Similarity methods in radiation hydrodynamics. Proc. Roy. Soc. London. Ser. A **258** (1960), 287–301.

This paper continues the work of Marshak [#7529], concentrating mainly on spherically symmetric flows, although some reference is made to other possibilities towards the end of the paper. The conditions under which the equations can be reduced to ordinary form are discussed (negligible material energy or radiation energy or special form of the ambient density) and also the effect on the speed of propagation of the bounding shock of the properties of the total energy. The flow properties at a shock are considered but the rôle of the isothermal shock is not made clear.

Taylor's spherically symmetric flow [Proc. Roy. Soc. London. Ser. A **201** (1950), 159–186] is generalised to include the effect of radiation energy flux, other radiation effects being neglected, and taking a special form for the mean free path of radiation. The divergences from Taylor's result are considerable, particularly near the centre of the flow. *K. Stewartson* (Durham)

7531:

Gustafson, W. A. On the Boltzmann equation and the structure of shock waves. Phys. Fluids **3** (1960), 732–734.

Dans cet article l'auteur compare les méthodes d'étude de la structure des ondes de choc dues à Mott-Smith d'une part, et Rosen d'autre part; les deux méthodes conduisent à la même équation différentielle pour la densité. La méthode de Mott-Smith est ici utilisée pour étudier le profil de température dans le cas d'un gaz monoatomique ou diatomique. *J. Naze* (Marseille)

7532:

Doak, P. E. Acoustic radiation from a turbulent fluid containing foreign bodies. Proc. Roy. Soc. London. Ser. A **254** (1960), 129–145.

A formal solution, using Green's functions, is given for the problem of acoustic radiation from a turbulent fluid containing foreign bodies. Particular attention is given to the dipole surface radiation, and it is shown that specification of a single scalar variable (pressure, or density, or the normal gradient of either) over the surfaces is sufficient to determine the radiation from the surface.

As a representative example, the author considers the surface radiation from a turbulent boundary layer, and obtains for the power radiation an expression of the type $K\rho_0 a_0^3 M_0^6$, where $K \sim 10^{-5}$, M_0 is the free stream Mach number, ρ_0 and a_0 are the density and sound speed respectively in the undisturbed fluid. Among the ingredients for this result are limited experimental observations on the

mean square wall pressure fluctuations and the correlation area for the surface pressures. These data are too sketchy to be definitive, and there are good theoretical reasons, discussed by Powell [J. Acoust. Soc. Amer. **32** (1960) 982–999; MR **22** #4303], for believing that the correlation area is, if not exactly zero, very much smaller than the present author assumes. *O. M. Phillips* (Baltimore, Md.)

7533:

Paščenko, N. T. Flow of highly rarefied gases around oscillating surfaces. Prikl. Mat. Meh. **23** (1959), 760–765 (Russian); translated as J. Appl. Math. Mech. **23**, 1081–1089.

Cet article contribue à l'étude des écoulements moléculaires libres autour d'un obstacle dont le mouvement uniforme est perturbé par une faible oscillation. La distribution des vitesses des molécules avant rencontre avec l'obstacle est supposée Maxwellienne à la température de l'écoulement à l'infini; on tient compte des effets de réflexion diffuse et spéculaire. En supposant tout d'abord qu'une molécule ne peut subir qu'un choc avec l'obstacle, on évalue l'impulsion et l'énergie reçues par unité de surface de l'obstacle, et la pression. Ce calcul est ensuite repris dans l'hypothèse où, l'obstacle étant concave, une molécule peut subir plus d'un choc. L'auteur en déduit certaines conditions de validité de l'hypothèse de l'écoulement moléculaire libre autour des obstacles concaves.

Les résultats de la première partie sont appliqués au cas d'une plaque plane en mouvement uniforme sous un angle d'attaque nul, oscillant faiblement dans la direction normale à son plan, et à un cône de petit angle dans les mêmes conditions. *J. Naze* (Marseille)

7534:

Tyutekin, V. V. Diffraction of a plane sound wave by an infinite cylindrical cavity in an elastic medium with an arbitrary angle of incidence. Akust. Ž. **6** (1960), 101–106 (Russian); translated as Soviet Physics. Acoust. **6**, 97–102.

Solutions are obtained for the scalar and vector potentials as superposition of cylindrical waves of integral order. The zero-order waves are considered in some detail and a few numerical results are indicated.

C. J. Bouwkamp (Eindhoven)

7535:

Aymerich, Giuseppe. Diffrazione e riflessione di un'onda di rarefazione completa in un canale. Rend. Sem. Fac. Sci. Univ. Cagliari **28** (1958), 150–160.

The rarefaction wave produced by a piston moving in a two-dimensional duct, one wall of which is rectilinear and the other nearly rectilinear, is studied. Following Chester [Quart. J. Mech. Appl. Math. **7** (1954), 247–256; MR **16**, 85], the effect of the departure from rectilinearity is made to reside in a perturbation added to the velocity potential. A suitable change of variable reduces Chester's equation to the canonical form for hyperbolic equations, and the solution is obtained as an integral over a distribution of sources. When the contour is an angle, the solution is shown to agree with that obtained by Powell [J. Fluid Mech. **3** (1957), 243–254; MR **19**, 1005]. The method of images is used to study the effect of reflection of the perturbed wave from the straight wall. *R. N. Goss* (San Diego, Calif.)

7536:

Bellin, J. L. S.; Beyer, R. T. Scattering of sound by sound. J. Acoust. Soc. Amer. **32** (1960), 339–341.

Experiments performed in water indicate no scattering due to nonlinear interaction of two finite-amplitude sources. This negative result is in agreement with a theoretical prediction by Westervelt [same J. **29** (1957), 199–203; MR **19**, 915] but is contrary to a prediction by Ingard and Pridmore-Brown [ibid. **28** (1956), 367–369].

J. W. Miles (Los Angeles, Calif.)

7537:

Sălceanu, Constantin; Zăgănescu, Mircea. Sur la définition du facteur d'amortissement d'une impulsion sonore non périodique. C. R. Acad. Sci. Paris **251** (1960), 1615–1617.

7538:

Mawardi, Osman K. Magnetohydrodynamics—a survey of the literature. Appl. Mech. Rev. **12** (1959), 443–446.

Survey article.

7539:

Globe, Samuel. Laminar steady-state magnetohydrodynamic flow in an annular channel. Phys. Fluids **2** (1959), 404–407.

An incompressible fluid with constant electrical conductivity and kinematic viscosity flows axially in the annular channel between two concentric, rigid, perfectly conducting cylinders. The axial electric field is maintained at zero, and the flow is driven by a constant axial pressure gradient.

The radial component of the magnetic field at the channel walls is that appropriate to a radial vacuum field. The author finds an explicit steady solution in which the fluid velocity and the magnetic field additional to the vacuum field are axial and vary only in the radial direction. The various limiting forms of the result agree with the results of previous workers.

G. E. Backus (La Jolla, Calif.)

7540:

Lewellen, W. S. Magnetohydrodynamically driven vortices. Proc. 1960 Heat Transfer Fluid Mech. Inst., pp. 1–15. Stanford Univ. Press, Stanford, Calif., 1960.

Steady, incompressible, viscous, plane flow with symmetry about a center is considered first. The equation of this motion can be solved approximately for small values of the magnetic Reynolds number based on the radial velocity component and the radius. Two cases are shown: (1) the magnetic vector lies in the flow plane, (2) a uniform axial magnetic field is applied; the flow is then nearly plane, i.e., varies slowly in the axial direction if the above-mentioned magnetic Reynolds number is small. The solutions for both cases are applied to flow between two rotating porous cylinders. It is shown that in case (1) the velocity profiles can be controlled over a wide range by changing the Hartmann number. In case (2) the flow is the classical viscous vortex plus an electromagnetic driving term.

The analysis is then extended to compressible flow by observing that the tangential-momentum equation is unchanged for this geometry. Temperature, Mach number, and pressure distributions are plotted for both cases

mentioned, for several values of the controlling parameters. Finally, numerical values for two technical cases, one a driven vortex and one a generator, are given.

W. R. Sears (Ithaca, N.Y.)

7541:

Morioka, Shigeki. Jets of a perfectly conducting inviscid gas in the presence of a magnetic field parallel to the stream. J. Phys. Soc. Japan **15** (1960), 1516–1522.

The jet may be two-dimensional or axisymmetric, issuing as a uniform parallel stream into fluid at rest or in uniform motion parallel to this stream. The regime of flow within the jet is supersonic-hyperbolic; i.e., the jet is supersonic and super-Alfvénic. Two cases are considered; viz., (i) the magnetic field may be imposed on both the jet and the surrounding fluid, or (ii) on the jet only.

Within the scope of small-perturbation theory, these various combinations are treated; the results can usually be related, by analogy, to various results of ordinary gasdynamics. In case (i), however, there may occur an interesting situation wherein the characteristics of the flow outside the jet run upstream, i.e., this outside flow may be subsonic-hyperbolic. To study this type of flow more closely, the author considers reflection and refraction of a weak plane magneto-gasdynamic shock at the interface between two streams. *W. R. Sears* (Ithaca, N.Y.)

7542:

Chandrasekhar, S. The virial theorem in hydromagnetics. J. Math. Anal. Appl. **1** (1960), 240–252.

An inviscid fluid of zero electrical resistivity is considered in which a magnetic field prevails. The fluid is a perfect gas and the effect of its self-gravitation is taken into account. The virial theorem for this distribution of matter is then proved. It is shown incidentally that the total angular momentum of the system is constant, and that the existence of this integral of the equations of motion has not been affected by the presence of the magnetic field. For configurations in equilibrium and in a steady state, the virial theorem indicates that, in general, spherical symmetry is incompatible with the presence of fluid motions and magnetic fields. A variational form of the virial theorem governing small departures from an initial static state is also obtained. *G. C. McVittie* (Urbana, Ill.)

7543:

Roberts, P. H.; Tatsumi, T. The decay of magneto-hydrodynamic turbulence. J. Math. Mech. **9** (1960), 697–713.

Equations are formulated for the evolution of the wave-number spectra of kinetic and magnetic energy in isotropic incompressible hydromagnetic turbulence under the dynamical approximation that fourth-order cumulants vanish for the joint distribution of simultaneous velocity and magnetic field amplitudes. A solution for a particular normal initial distribution is obtained to order R^2, where R is an appropriate turbulent Reynolds number. The authors admit non-vanishing initial correlation between velocity and magnetic fields, and find that the strength of this correlation importantly affects the asymptotic ratio of kinetic and magnetic energies at long times. The reviewer would like to point out that, if the algebra is correct, the authors' results are quite independent of their dynamical approximation, to order R^2. To this order, they are exact consequences of the assumed normal initial state.

R. H. Kraichnan (New York)

7544:

Polovin, R. V. The motion of a piston in a conducting medium. Ž. Èksper. Teoret. Fiz. **38** (1960), 1544–1555. (Russian. English summary); translated as Soviet Physics. JETP **11**, 1113–1120.

The author considers the piston problem when the external magnetic field (H_{0x}, H_{0y}, 0), the velocity of piston (u_x, u_y, 0) assumed constant, and normal to the piston all lie in the same plane under the assumptions that the gas is inviscid, electrical conductivity is infinite and the magnitude of the external magnetic field is so small that the Alfvén wave velocity is less than the velocity of sound C_0 in the undisturbed medium. This interesting investigation generalizes the work of Bazer [Astr. J. **128** (1958), 686–712; MR **20** #5635] who considered the case when the external magnetic field and the piston velocity are respectively parallel and perpendicular to the normal to the piston and also the work of Lyubarskiĭ and Polovin [Dokl. Akad. Nauk SSSR **128** (1959), 684–687; MR **22** #3379] who discussed the case in which the motion of the piston is longitudinal, i.e., along the normal to the piston.

The types of the waves that are produced due to the motion of the piston depend on (u_x, u_y). The author determines the various domains in the (u_x, u_y) plane in which slow and fast shock waves, slow and fast self-similar rarefaction waves, Alfvén discontinuity and cavitation can exist. For sufficiently large amplitude of the slow rarefaction wave, the density of the medium in the back of the wave vanishes, i.e., cavitation results. In ordinary gasdynamics cavitation is produced when the piston is withdrawn with velocity exceeding $2C_0/(\gamma-1)$, where γ is the ratio of specific heats; in the present case, cavitation sets in at lower longitudinal piston velocities provided its transverse velocity is sufficiently large. In the case of purely transverse motion of the piston, the author verifies the result of Bazer [loc. cit] that cavitation sets in when the piston velocity is $3.67\,C_0$ if $\gamma=5/3$. Cavitation is also produced when the piston moves in if its velocity is greater than the velocity of sound and the inclination of the velocity to the normal to the piston is greater than 70° when $\gamma=5/3$. Cavitation cannot be produced in fast rarefaction waves.

When $u_y > C_0$, generation of magnetic field takes place. At supersonic velocities of insertion (u_x) and sliding (u_y) of piston, the magnetic field generated is directly proportional to u_x. The author also proves that an increase in u_x generally leads to an increase in the amplitude of the shock wave and to a decrease in the amplitude of self-similar rarefaction wave and points out the situation in which this statement is not true. *P. L. Bhatnagar* (Bangalore)

7545:

Chakraborthy, B. B.; Ramamoorthy, P. On the pulsations of an infinite cylinder with a force-free magnetic field. Z. Astrophys. **49**, 186–191 (1960).

The authors consider the small radial pulsations of an infinitely long gravitating cylindrical mass of electrically

conducting compressible fluid in the presence of a force-free magnetic field whose components in cylindrical coordinates are

$$[0, A_1 J_1(\alpha\tilde{\omega}), A_1 J_0(\alpha\tilde{\omega})].$$

They tabulate the displacement function and the change in the magnetic field inside the cylinder for the fundamental mode when the square of the ratio of Alfvén wave velocity to sound velocity in the undisturbed medium is 1/10, 1 and 10. *P. L. Bhatnagar* (Bangalore)

7546:

Bam-Zelikovič, G. M. Motion of an axially symmetric gas jet with small conductivity in an axially symmetric magnetic field. Dokl. Akad. Nauk SSSR **131** (1960), 47–50 (Russian); translated as Soviet Physics. Dokl. **5**, 231–234.

The author considers the axisymmetric motion of a perfect gas with finite but constant electrical conductivity in the presence of an axisymmetric external magnetic field, neglecting the viscosity and heat conduction. The solution is obtained as a power series in a parameter $\delta = (\sigma c^{-2}) \times aH_\infty^2/(\rho_\infty V_\infty)$, where σ denotes electric conductivity and c the velocity of light, and a, H_∞, ρ_∞, V_∞ are the characteristic length, magnetic field, density and velocity associated with the problem. The first term in this solution corresponds to the case when there is no interaction between the magnetic field and the velocity field and hence is the solution of the problem in the absence of magnetic field. The author evaluates only the first order perturbation in the solution when Mach number of the undisturbed flow is unity. When the Mach number is less than unity, even the first order solution has to be obtained in the form of a series of powers of the radial distance on the assumption that the jet is thin. Finally, as an illustration, the author discusses the case when the external magnetic field arises from a circular current of radius a.

P. L. Bhatnagar (Bangalore)

7547:

Greenspan, H. P. On longitudinal motion in a magnetic field. J. Fluid Mech. **9** (1960), 455–464.

A semi-infinite, rigid, non-conducting, flat plate is moved with constant velocity parallel to its edge in an incompressible, viscous, conducting fluid. A uniform applied magnetic field is obliquely incident upon the plate; i.e., the magnetic field is uniform at large distances from the plate and lies at an arbitrary angle in the plane perpendicular to the plate's velocity vector. Viscosity and electrical conductivity are assumed uniform and constant. This is a generalization of one of the cases treated by Hasimoto [same J. **8** (1960), 61–81; MR **22** #6320] who considered the field to be perpendicular to the plate. It is argued that the resulting flow is everywhere in the direction parallel to the plate's edge (z direction) and that all quantities depend on x and y only. It is also shown that the induced field must vanish on the plate.

The boundary-value problem is linear, and the solution, obtained in an Appendix by Wiener-Hopf techniques, is given in terms of exponential and error functions. It is discussed in some detail. The plate acquires a double-layer charge. Since the disturbances due to the plate are propagated along field lines, the greatest effects are found in the half-space in which the undisturbed lines would intersect the plate. The fluid boundary layer is distorted and stretched along the field lines. It is suggested that the behavior of the solution near the edge should be of interest in many problems. *W. R. Sears* (Ithaca, N.Y.)

7548:

Cole, J. D.; Huth, J. H. Some interior problems of hydromagnetics. Phys. Fluids **2** (1959), 624–626.

The authors consider a perfectly conducting, incompressible, magnetic-field-free fluid under uniform pressure p_∞ at ∞, and containing an evacuated cylindrical cavity whose collapse is prevented by a magnetic field produced in the cavity by various combinations of line currents parallel to the axis of the cavity. The authors consider the following three cases.

(i) Single line current (2 dimensional monopole singularity); fluid at rest at ∞. Formula (1) is in error and should read

$$R = (2\pi)^{-1} T_0 (\mu_0/2p_\infty)^{1/2}.$$

(ii) Double line current (2 dimensional dipole singularity); fluid at rest at ∞. The fluid comes in to the singularity in a cusp tangent to the direction of the dipole.

(iii) Single line current; fluid has uniform velocity at ∞. The solution is given as power series in fluid kinetic energy density/magnetic energy density, the zeroth and first terms being calculated. *G. E. Backus* (La Jolla, Calif.)

7549:

Yur'ev, I. M. On a solution to the equations of magnetogasdynamics. Prikl. Mat. Meh. **24** (1960), 168–170 (Russian); translated as J. Appl. Math. Mech. **24**, 233–237.

On considère l'écoulement plan stationnaire d'un fluide parfait parfaitement conducteur, tel que le champ magnétique $\mathbf{H}$ soit en tout point parallèle à la vitesse $\mathbf{W}$ et qu'en outre $\mathbf{H} = k\rho\mathbf{W}$ où k est une constante.

En considérant un écoulement fictif dont la vitesse et la masse spécifique sont données par

$$\rho^*\mathbf{W}^* = \rho\mathbf{W}, \quad \rho^* = (1 - k^2\rho/4\pi)^{-1}\rho$$

et en introduisant la fonction courant et le potentiel fictif correspondant, l'auteur ramène la résolution du problème à celle d'équations de Tchaplyguine.

J. Naze (Marseille)

7550:

Auluck, F. C.; Nayyar, N. K. The stability of a gravitating cylinder in the presence of magnetic fields. Z. Astrophys. **50** (1960), 7–13.

Les auteurs étudient la stabilité, en présence d'un champ magnétique, d'un cylindre de révolution en rotation. La matière qui constitue le cylindre est incompressible et conductrice d'électricité (résistivité nulle). Le champ magnétique est déterminé par un seul scalaire, fonction uniquement de la distance r à l'axe du cylindre; cette fonction fait intervenir la première fonction de Bessel $J_1(\alpha r)$, α étant une constante telle que la seconde fonction de Bessel $J_2(\alpha r)$ soit nulle sur le cylindre. A partir de cette solution, la stabilité est discutée en fonction de l'intensité du champ magnétique. *H. Cabannes* (Paris)

7551:

DiPrima, R. C. Some variational principles for problems in hydrodynamic and hydromagnetic stability. Quart. Appl. Math. **18** (1960/61), 375–385.

The author considers eigenvalue problems for which the differential equation (a) has constant coefficients but is of sufficiently high order to render cumbersome the general determination of its linearly independent solutions, (b) is non-self-adjoint, and (c) has a differential operator that may be factored into a pair of subsidiary equations. In most, but not all, of his examples the subsidiary equations are self-adjoint, and standard techniques then are used to establish variational solutions. Physical problems considered include the Taylor problem for stability of the flow between rotating cylinders, the inhibition of thermal convection by a magnetic field, and the Taylor problem with axial magnetic field. *J. W. Miles* (Los Angeles, Calif.)

7552:

Drazin, Philip G. Stability of a broken-line jet in a parallel magnetic field. J. Math. and Phys. **39** (1960/61), 49–53.

The inviscid fluid between planes $y=L$ and $y=-L$ moves in the x direction with constant velocity V, has density ρ_1, magnetic diffusivity η_1, and magnetic permeability μ. The inviscid fluid outside these planes is stationary and has density ρ_2, magnetic diffusivity η_2, and magnetic permeability μ. A uniform magnetic field of strength H in the x direction exists everywhere in both fluids. When the magnetic Reynolds numbers LV/η become very large, the flow is shown to be stable if and only if

$$\frac{\mu H^2}{8\pi} > \frac{1}{2}\left(\frac{1}{\rho_1^2}+\frac{1}{\rho_2^2}\right)^{-1/2} V^2.$$

For finite magnetic Reynolds numbers the flow is always unstable to disturbances of sufficiently long wavelength.

G. E. Backus (La Jolla, Calif.)

7553:

Edmonds, Frank N., Jr. Hydromagnetic stability of a conducting fluid in a circular magnetic field. Phys. Fluids **1** (1958), 30–41.

This work extends the analysis of Chandrasekhar [Amer. Math. Monthly **61** (1954), no. 7, part II, 32–45; MR **16**, 632] and that of Michael [Mathematika **1** (1954), 45–50, 131–142; MR **16**, 538, 759]. The author discusses the stability of electrically conducting fluid between two rotating coaxial cylinders. The difference in their radii is assumed to be small compared to the mean of their radii. Chandrasekhar discussed the same problem for the case when the lines of magnetic force are parallel to the common axis of the cylinders. The present author assumes that the lines of force are concentric with the cylinders. Such a magnetic field is produced by an electric current which is distributed symmetrically about and flows parallel to the common axis of the cylinders. The difference of the present analysis from that of Michael is that while Michael neglected the dissipative mechanism of viscosity and magnetic diffusivity, that mechanism is taken into account in this paper.

The method of solving the problem is the same as that given in Chandrasekhar's paper. First the boundary conditions, the so-called Fermi boundary conditions, are formulated. That makes the problem manageable. The author then solves the problem by using techniques developed by Chandrasekhar. He further compares his results with the earlier ones. Especially the comparison between the present analysis and that of Michael is very clearly brought out.

The main result of the paper is that the magnetic field inhibits the onset of instability. It turns out, however, that this effect is quite small because the hydromagnetic interaction involves a mere displacement of the magnetic field. *R. P. Kanwal* (University Park, Pa.)

7554:

Sakurai, Takeo. Correction to the paper "Two-dimensional flow of an ideal gas with small electric conductivity past a thin profile". J. Phys. Soc. Japan **15** (1960), 1135–1136.

The correction to the author's paper in same J. **15** (1960), 326–333 [MR **22** #1281], noticed by S. Ando, is that in the subsonic case the forward wake (Greenstein's precursor) appears in the direction $\theta=\pi+2\delta(1-M_0^2)^{-1/2}$ instead of $\theta=\pi+2\delta\{(1-M_0^2)^{-1/2}\pm M_0\}$, while in the supersonic case the right sides of formulae (74), (75) and (76) should be multiplied by $2^{1/2}$.

G. E. Backus (La Jolla, Calif.)

7555:

Ando, Shigenori. Some remarks on the magnetogasdynamic linearized theory. J. Phys. Soc. Japan **15** (1960), 1523–1533.

In addition to the usual magnetohydrodynamic assumption, it is assumed that the velocity and magnetic fields are both steady and uniform in the undisturbed region, but may have arbitrary orientation with respect to one another. The disturbances, which may be unsteady, are caused by a two- or three-dimensional body which makes only small perturbations in both fields. After linearization of the equation, a scalar function Φ is introduced such that the induction equation is identically satisfied. The velocity- and magnetic-field perturbation vectors and the pressure perturbation are given by rather complicated third-order operators on Φ. The differential equation for Φ is fourth-order; it involves as parameters the magnetic Reynolds number R_m, the interaction number R_mS, where S is the ratio of magnetic to static pressures, the Mach number, and the direction cosines of the angle between undisturbed-field vectors. It is shown that when $R_m=0$ the function Φ is related to the velocity potential. In another limit, when $S=0$, Φ can also be interpreted: here it is related in a different manner to the velocity potential.

To illustrate the application of this method, the author treats here some cases of steady flow with weak interaction: $R_m\approx 0$ and $R_mS\ll 1$. For plane steady flow past a two-dimensional symmetric airfoil at zero incidence, the results agree with Sakurai's [same J. **15** (1960), 326–333, 1135–1136; MR **22** #1281, 7554] for subsonic and supersonic flow. Steady axisymmetric flow past pointed bodies is also treated here. *W. R. Sears* (Ithaca, N.Y.)

7556:

Lighthill, M. J. Note on waves through gases at pressures small compared with the magnetic pressure, with applications to upper-atmosphere aerodynamics. J. Fluid Mech. **9** (1960), 465–472.

Magnetohydrodynamic waves are considered in which the Alfvén velocity a_1 is much larger than the sound velocity a_0. Apart from two modes with wave velocity a_1, the only possible waves are longitudinal ones, propagated unidirectionally along lines of magnetic force with velocity a_0. They can be interpreted as sound waves, confined to effectively rigid magnetic tubes of force. An application is made to the flow pattern around a body moving though the F_2 layer of the ionosphere. It is argued that in this layer, though neutral particles have a very large mean free path, charged particles interact electrostatically and form a continuous fluid whose movement is independent of that of the neutral particles. A body moving with the speed of an artificial earth satellite would then excite the uni-directional sound waves, but no waves at the much faster Alfvén velocity. Its movement would then be accompanied by a V-shaped pattern of electron density which might be responsible for anomalous radar echoes reported by Kraus and others.

G. C. McVittie (Urbana, Ill.)

7557:

Barthel, James R.; Lykoudis, Paul S. The slow motion of a magnetized sphere in a conducting medium. J. Fluid Mech. **8** (1960), 307–314.

The authors consider a permanently, uniformly magnetized sphere moving slowly parallel to its direction of magnetization in a viscous, electrically conducting, incompressible fluid at rest at infinity. Inertial forces are altogether neglected. If the exact solution be regarded as a power series in ordinary Reynolds number R, magnetic Reynolds number R_m, and M, the square of the Hartmann number, then the authors have found the coefficients of the terms RR_m, RM, RR_mM, and RM^2 in the expressions for the velocity and pressure fields and drag coefficient.

G. E. Backus (La Jolla, Calif.)

7558:

Poots, G.; Sowerby, L. Axially symmetric stagnation point flow with heat transfer in magnetohydrodynamics. Quart. J. Mech. Appl. Math. **13** (1960), 385–407.

The authors consider the steady axially symmetric stagnation point flow. The fluid is assumed to be incompressible, viscous and electrically and thermally conducting. The magnetic field is taken to be normal to the wall which is thermally insulated. The analysis reveals the existence of three regions of flow. In the first place there is the magneto-viscous layer of the same order of thickness as the boundary layer of non-conducting fluids. Lorentz force influences the behavior of the motion in this region. Moreover the pressure distribution throughout this layer and the velocity distribution near the edge of the layer are greatly affected by the adjacent layer: the magneto-inviscid layer. The magneto-inviscid layer is the region of the flow in which the Lorentz force is comparable in magnitude with the inertia terms, but the viscosity is unimportant. This layer plays the role of a buffer layer between the magneto-viscous layer and the region of the potential flow in the free stream.

Continuing their analysis, the authors discuss the magnetohydrodynamic effects on heat transfer. They find that, in the vicinity of the stagnation point, the presence of the magnetic field produces a considerable reduction in the local shear stress and the eigentemperature at the wall.

The paper ends with an elegant discussion of the main results. This section contains many figures and tables.

R. P. Kanwal (University Park, Pa.)

7559:

Ludford, G. S. S. The propagation of waves along and through a conducting layer of gas. J. Fluid Mech. **9** (1960), 119–132.

The author considers two infinite half-spaces of non-conducting gas separated by a layer of perfectly conducting gas (an imperfect conductor is briefly considered), and containing a uniform magnetic field perpendicular to the conducting layer. He finds that such a conducting layer will act as an almost perfect reflector of incident plane electromagnetic waves, except when the layer is an integral multiple of Alfvén half-wavelengths thick, in which case almost perfect transmission will occur. If the electric field of the incident wave is perpendicular to the directions of propagation and the steady magnetic field, a small fraction of the reflected energy will appear in a sound wave. The author also shows that such a conducting layer can transmit surface waves analogous to Rayleigh and Love waves, but that the interface between a semi-infinite conducting half-space and a semi-infinite insulating half-space cannot.

G. E. Backus (La Jolla, Calif.)

7560:

Smirnov, A. G. The theory of certain magnetohydrodynamic phenomena occurring in the free laminar thermal convection of the electrically conducting fluid in a round vertical pipe located in a weak magnetic field. Ž. Tehn. Fiz. **29** (1959), 1245–1251 (Russian); translated as Soviet Physics. Tech. Phys. **4** (1960), 1141–1147.

A round, electrically insulating pipe has its axis parallel to the z axis, the direction of gravity. In the pipe is a viscous fluid with finite electrical and thermal conductivities. The temperature of the wall of the pipe varies linearly in the x direction. A magnetic field in the x or y direction is imposed externally. The free convection in the absence of the field is assumed so weak that it does not distort the externally imposed field. The external field is assumed so weak that its effect on the velocity field is negligible. The resulting Lorentz force is calculated and used to define an effective viscosity.

G. E. Backus (La Jolla, Calif.)

7561:

Greenspan, H. P. Flat plate drag in magnetohydrodynamic flow. Phys. Fluids **3** (1960), 581–587.

The author considers the two dimensional hydromagnetic flow of a viscous, electrically conducting, incompressible fluid past a flat plate at zero angle of attack, the fluid velocity and applied magnetic field being parallel far upstream. The electric field parallel to the leading edge of the plate is made to vanish. The differential equations are linearized as in Oseen's treatment of viscous flows, and reduced to an equivalent singular integral equation.

Asymptotic series in (Reynolds number)$^{-1/2}$ are obtained for the drag coefficient C_D in the limiting cases in which (magnetic diffusivity)/(kinematic viscosity) is $\ll 1$ or $\gg 1$. For small Reynolds number R, the drag coefficient is obtained by replacing the kernel of the integral equation by the first few terms in its power series. The values of

C_D for large and small R fit smoothly together, so presumably C_D for intermediate R can be obtained by interpolation. The author emphasizes, as in his previous papers on this subject, that the fundamental physical phenomenon in super-Alfvén flow is the upstream propagation of vorticity by Alfvén waves, and the consequent production of a wake-like "precursor".

G. E. Backus (La Jolla, Calif.)

7562:

Cap, F.; Hofinger, E. Über einige Klassen von stationären, zweidimensionalen Lösungen der Magnetogasdynamik. Acta Phys. Austriaca **13** (1960), 262–264.

The authors restate, in terms of stream functions, the problem of finding steady, hydromagnetic flows of a perfectly conducting compressible fluid in which the vorticity vanishes. *G. E. Backus* (La Jolla, Calif.)

7563:

Gupta, A. S. On the flow of an electrically conducting fluid near an accelerated plate in the presence of a magnetic field. J. Phys. Soc. Japan **15** (1960), 1894–1897.

The author considers the uniformly accelerated motion of an infinite plate in an electrically conducting viscous fluid with an applied magnetic field perpendicular to the flow. Unfortunately, the induced magnetic field is neglected and the problem reduces to a conventional exercise in the use of the Laplace Transform which is of limited interest.

H. P. Greenspan (Cambridge, Mass.)

7564:

Takaisi, Yorisaburo. Wall-effect upon two-dimensional Stokes flow of an electrically conducting liquid in a uniform magnetic field. J. Phys. Soc. Japan **15** (1960), 1876–1885.

This paper deals with the steady slow motion of a semi-infinite viscous electrically conducting fluid past a circular cylinder. The applied uniform magnetic field is either parallel or perpendicular to the free stream flow. The analysis is based on Stokes approximation, Faxen's image method and an expansion in powers of the Hartmann number. The results show that the drag coefficients are almost independent of the Hartmann number (at least for small values of this parameter) for any orientation of magnetic field. This may be a consequence of completely neglecting the induced magnetic field insofar as the basic flow pattern is concerned.

H. P. Greenspan (Cambridge, Mass.)

7565:

Bleviss, Z. O. A study of the structure of the magneto-hydrodynamic switch-on shock in steady plane motion. J. Fluid Mech. **9** (1960), 49–67.

En magneto-dynamique des fluides il existe des chocs pour lesquels la vitesse et le champ magnétique sont normaux à l'onde de choc avant le choc et inclinés sur l'onde de choc après le choc. La structure de ces chocs est étudiée. L'auteur raméne cette étude à celle d'un système de quatre équations différentielles du premier ordre, dépendant de quatre paramètres: les deux coefficients de viscosité, la conductivité thermique et le résistivité électrique. Les courbes intégrales sont construites dans les cas limites d'une conductivité électrique faible d'abord, forte ensuite. *H. Cabannes* (Paris)

7566:

Greenberg, O. W.; Sen, H. K.; Trève, Y. M. Hydrodynamic model of diffusion effects on shock structure in a plasma. Phys. Fluids **3** (1960), 379–386.

Les auteurs étudient la façon dont les effets de diffusion modifient la structure des chocs dans les plasmas. Le problème consiste à étudier la solution d'un système de deux équations différentielles du premier ordre, qui passe par deux points singuliers dont l'un est un col, tandis que l'autre est soit un nœud soit un foyer, dans ce dernier cas la solution est oscillante. Le modèle envisagé conduit dans certains cas (comme c'est le cas également en aérodynamique classique lorsqu'on prend en considération seulement la conductivité thermique) à des solutions discontinues; les auteurs pensent que l'introduction de la viscosité pourra permettre dans tous les cas de trouver une solution continue. *H. Cabannes* (Paris)

7567:

Ericson, W. B.; Bazer, J. On certain properties of hydromagnetic shocks. Phys. Fluids **3** (1960), 631–640.

L'auteur établit quatre propriétés intéressantes pour les magnéto-chocs, stationnaires, non relativistes. 1° L'entropie spécifique augmente à travers le choc si celui-ci est une compression et dans ce cas seulement. 2° Elle varie alors dans le même sens que le produit de la masse par la vitesse normale. 3° Après un choc rapide, correspondant à une compression, le plus grande des célérités du son est supérieure à la vitesse de propagation du choc. 4° Après un choc lent, correspondant à une compression, le plus petite des célérités du son peut être inférieure, égale ou supérieure à le vitesse de propagation du choc. Les deux dernières propriétés supposent que le milieu est un gaz idéel polytropique. *H. Cabannes* (Paris)

7568:

Kulikovskiĭ, A. G.; Lyubimov, G. A. The simplest problems concerning gas-ionizing shock waves in an electromagnetic field. Dokl. Akad. Nauk SSSR **129** (1959), 525–528 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1195–1198.

Les auteurs indiquent que les chocs en magnéto-dynamique des fluides peuvent être considérés comme des cas limites de mouvements continus avec viscosité et conductivités thermique et électrique, seulement lorsque les grandeurs qui caractérisent l'état du fluide avant le choc sont liées par une certaine relation. Cette relation est précisée dans trois cas: (1) "viscosité magnétique" prépondérante, (2) "viscosité magnétique négligeable", (3) température avant le choc voisine de la température à partir de laquelle la conductivité électrique doit être prise en considération. Les chocs plans stationnaires (choc normal: problème du piston, et choc oblique: problème du dièdre) sont étudiés dans ces différents cas. *H. Cabannes* (Paris)

7569:

Kulikovskiĭ, A. G.; Lyubimov, G. A. Some remarks on the structure of a normal magneto-hydrodynamic shock wave. Prikl. Mat. Meh. **23** (1959), 1146–1147 (Russian); translated as J. Appl. Math. Mech. **23**, 1644–1647.

Les auteurs étudient la structure des chocs normaux, lorsque les conductivités thermique et électrique sont prises

en considération, tandis que la viscosité est négligée. Le problème consiste à trouver les intégrales qui passent par deux points singuliers d'un système differentiel du premier ordre. Le point qui correspond à l'infini amont est un nœud, celui qui correspond à l'infini aval un col ou un nœud suivant les données. Dans le dernier cas, aucune courbe intégrale continue ne relie les deux points singuliers et le solution comporte un choc à champ magnétique constant. *H. Cabannes* (Paris)

7570:

Kiselev, M. I.; Kolosnicyn, N. I. The calculation of oblique shock waves in magnetogasdynamics. Dokl. Akad. Nauk SSSR **131** (1960), 773–775 (Russian); translated as Soviet Physics. Dokl. **5**, 246–248.

A partir des équations des chocs en magnéto-dynamique des fluides, les auteurs établissent une équation cubique, qui détermine la composante tangentielle du champ magnétique après le choc. La vitesse après le choc est détermine ensuite à l'aide d'une famille de polaires de choc et le problème du choc stationnaire autour d'un dièdre est étudié. *H. Cabannes* (Paris)

7571a:

Jordan, H. L. Hydromagnetic shock waves. I. Theory. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 82–89. Zanichelli, Bologna, 1960.

7571b:

Jordan, H. L. Hydromagnetic shock waves. II. Experiments. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 90–96. Zanichelli, Bologna, 1960.

Dans la premiére partie l'auteur rappelle les équations des mouvements et des chocs en magnéto-dynamique des fluides. Le seconde partie est une revue des principales expériences réalisées dans des tubes à choc magnétiques rectilignes ou annulaires. *H. Cabannes* (Paris)

7572:

Kanwal, R. P. On magnetohydrodynamic shock waves. J. Math. Mech. **9** (1960), 681–695.

For a non-dissipative unsteady plane magneto-hydrodynamic flow, the jumps in various flow quantities across a curved shock are calculated in terms of the (assumed constant) upstream state and the local shock curvature and velocity. The magnetic field is taken normal to the plane of flow. The author employs the general magnetohydrodynamic shock conditions of Friedrichs [Nonlinear wave motion in magnetohydrodynamics, Los Alamos report LAMS-2105 (1957); see also Friedrichs & Kranzer, Nonlinear wave motion, New York University report NYO-6486 (1958)], which he rederives, using methods developed for gas dynamics by T. Y. Thomas [Math. Mag. **22** (1949), 169–189; MR **11**, 278].

H. C. Kranzer (Garden City, N.Y.)

7573:

Kanwal, R. P. Flow behind shock waves in conducting gases. Proc. Roy. Soc. London. Ser. A **257** (1960), 263–268.

L'auteur étudie, dans cette note, quelques conséquences des lois de conservation classiques pour une onde de choc dans un écoulement stationnaire de gaz parfait et parfaitement conducteur. Le résultat essentiel concerne le calcul du tourbillon et de la densité de courant créés par un choc courbe en écoulement plan. Ces quantités peuvent être obtenues comme solutions d'un système linéaire que l'auteur a écrit dans ce travail mais dont les coefficients sont fort compliqués. Dans le cas tridimensionnel il existe une relation simple entre les composantes normales au choc du tourbillon et de la densité de courant. *P. Germain* (Paris)

7574:

Raju, P. K.; Verma, Y. K. Reflection and refraction of hydromagnetic plane waves at the boundary of two compressible media. Z. Astrophys. **50** (1960), 29–34.

Les auteurs étudient la réflexion et la réfraction des ondes accoustiques et des ondes d'Alfvén en magnéto-dynamique des fluides, lorsque ces ondes rencontrent le surface de séparation de deux milieux. Les équations sont établies pour les fluides compressibles, lorsque le champ magnétique est normal à la surface de séparation des deux milieux. Les calculs explicites dans le cas des ondes d'Alfvén montrent que, pour ces ondes, la compréssibilité n'a aucune influence sur les résultats. *H. Cabannes* (Paris)

7575:

Agostinelli, Cataldo. Sull'equilibrio adiabatico magneto-dinamico di una massa fluida gassosa gravitante, in rotazione non uniforme. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **28** (1960), 278–283.

L'auteur généralise dans cet article l'étude effectuée dans de précédentes publications, en considérant maintenant que la masse gaseuse est en rotation non uniforme; on suppose seulement que la vitesse de rotation est indépendante de la latitude φ. Le fluide est supposé conducteur parfait, non visqueux. *J. Naze* (Marseille)

7576:

Peyret, Roger. Sur une correspondance entre certains écoulements de magnétodynamique des fluides et ceux de la dynamique des gaz. C. R. Acad. Sci. Paris **250** (1960), 1971–1973.

Les équations régissant les écoulements stationnaires d'un fluide parfait, compressible, parfaitement conducteur, tels que l'induction est parallèle à la vitesse, sont ramenées à celles de la dynamique des gaz pour un fluide fictif, que l'écoulement soit continu ou non. *J. Naze* (Marseille)

7577:

Carini, Giovanni. Su una relazione energetica della magneto-idrodinamica. Boll. Un. Mat. Ital. (3) **14** (1959), 477–481. (English summary)

On propose dans cet article une équation de l'énergie, déduite du premier principe de la thermodynamique, pour le système des équations magnétohydrodynamiques de Navier-Minkowski (obtenu en couplant les équations de l'hydrodynamique et le système complet des équations de l'électrodynamique). *J. Naze* (Marseille)

7578:

Coburn, N.; Ong, R. S. B. The characteristic system for three-dimensional, steady, isentropic, compressible, hydromagnetic flows. J. Math. Mech. **9** (1960), 715–732.

An analysis of the normal cone for non-dissipative steady three-dimensional magnetohydrodynamic flows leads to a demonstration of the existence at each point of three normals n, $'n$, and k such that n and $'n$ lie in the plane determined by the local velocity and magnetic field vectors, while k is orthogonal to this plane. The three families of characteristic surfaces determined by these normals are used to derive the characteristic system for these flows. This system consists of twenty-six quasi-linear partial differential equations in eighteen unknown variables plus six algebraic equations. Also found explicitly are the twelve differential equations in eleven unknowns which make up the characteristic system in the special case of axial symmetry.

H. C. Kranzer (Garden City, N.Y.)

7579:

Montgomery, David. Nonlinear Alfvén waves in a cold ionized gas. Phys. Fluids **2** (1959), 585–588.

Etude des ondes transversales dans un plasma froid sans collisions. Le système des équations différentielles se raméne à une quadrature, ce qui permet l'étude qualitative des mouvements (ondes non linéaires non superposables). Les ondes polarisées rectilignement sont impossibles. Les ondes polarisées circulairement ne s'accompagnent d'aucune compression du gaz et son depourvues de champ électrique longitudinal.

E. Schatzman (Paris)

7580:

Agostinelli, Cataldo. Sull'equilibrio radiativo magnetodinamico di una massa gassosa stellare uniformemente rotante e gravitante. Ann. Mat. Pura Appl. (4) **50** (1960), 249–261.

Les équations régissant l'équilibre radiatif d'une masse stellaire gaseuse en rotation uniforme sont écrites dans le cas où le fluide est doué d'une conductivité électrique infinie; on étudie plus particulièrement le cas d'une sphère en rotation lente et uniforme. L'auteur trouve que, si dans une masse gaseuse le champ magnétique est uniforme et parallèle à l'axe de rotation, une rotation lente peut donner naissance à une perturbation du champ magnétique qui laisse invariantes les caractéristiques de la densité et de la pression. *J. Naze* (Marseille)

7581:

Kakutani, Tsunehiko. Effect of transverse magnetic field on the flow due to an oscillating flat plate. II. J. Phys. Soc. Japan **15** (1960), 1316–1331.

Une plaque plane oscille dans un fluide visqueux, conducteur (parfait) d'électricité; un champ magnétique uniforme perpendiculaire à la plaque est impose à l'infini. L'auteur détermine le mouvement du fluide dans le cas où la plaque est également un conducteur (parfait) d'électricité, le cas de la plaque isolante a été traité pecedemment [même J. **13** (1958), 1504–1509; MR **20** #5637]. Les forces tangentielles exercées sur la plaque dans l'un et l'autre cas sont telles que le produit de leurs amplitudes et la somme de leurs phases soient constants. *H. Cabannes* (Paris)

7582:

Ahiezer, I. A.; Polovin, R. V. The motion of a conducting piston in a magnetohydrodynamic medium. Ž. Èksper. Teoret. Fiz. **38** (1960), 529–533 (Russian. English summary); translated as Soviet Physics. JETP **11**, 383–386.

Les auteurs étudient le problème du piston en magnétodynamique des fluides, la conductivité électrique du fluide et celle du piston sont infinies. Le mouvement du piston engendre soit des chocs, soit des détentes. En tête se propage un choc (ou une détente) rapide, ensuite une discontinuité d'Alfvén, enfin un choc (ou une detente) lent. D'un point de vue qualitatif, la discussion faite par les auteurs en fonction de la vitesse du piston fait apparaître 17 cas différents. *H. Cabannes* (Paris)

7583:

Uflyand, Ya. S.; Čekmarev, I. B. Investigation of transient flow of a conducting fluid in a plane channel with moving walls. Ž. Tehn. Fiz. **30** (1960), 465–471 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 437–442.

A viscous incompressible fluid of finite electrical conductivity fills a plane channel bounded by the non-conducting parallel planes $y = \pm a$. The system is embedded in a uniform external magnetic field H_0 acting normally to the walls of the channel. Initially the whole system is at rest and at time $t = 0$ the walls start to move with arbitrary velocities $V_{\pm a}(t)$ in the direction of x-axis. With the help of Laplace transforms, the authors determine exactly the velocity distribution in the fluid and the induced electromagnetic field in two cases: (i) when both the walls move in the same direction with velocity $V_1(t)$; and (ii) when they move with the same velocity in opposite directions. To satisfy the boundary conditions associated with the induced magnetic and electric fields at the walls, it is necessary to consider the induced currents in the medium surrounding the fluid. *P. L. Bhatnagar* (Bangalore)

7584:

Causse, René; Poirier, Yves. Mise en vitesse et amortissement d'un volume cylindrique de liquide conducteur en rotation dans un champ magnétique. C. R. Acad. Sci. Paris **251** (1960), 1056–1058.

Authors' summary: "Étude théorique de la mise en vitesse et de l'amortissement d'un volume cylindrique occupé par un liquide conducteur et soumis à un champ magnétique normal à l'axe de rotation. Paramètres fondamentaux. Courbes théoriques pour quelques valeurs de ces paramètres."

7585:

Čekmarev, I. B. Some problems of stationary flow of a conducting liquid in an infinitely long annular tube in the presence of radial magnetic field. Ž. Tehn. Fiz. **30** (1960), 601–605 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 565–569.

The author considers three problems relating to the stationary flow of a viscous liquid with finite electrical conductivity in an infinitely long annular tube in the presence of external radial magnetic field. In the first problem, the cylinders are both electrically conducting and permeable and the motion takes place under a

constant longitudinal pressure gradient in the presence of injection (suction) at the inner cylindrical surface. In view of the continuity equation, the radial velocity is inversely proportional to radial distance and hence there is constant suction (injection) at the outer cylinder also. In the second problem, the cylinders are non-conducting but permeable. The motion is produced by constant injection (suction) at the wall of the inner cylinder in the presence of external uniform and axial electric field. The motion is confined to the planes normal to the axis. In the third problem, the cylinders are non-conducting and impermeable. The motion takes place in the presence of constant longitudinal pressure gradient and uniform axial electric field. Here the velocity has only axial and azimuthal components. In all the cases it is possible to give the solution in the exact form.

P. L. Bhatnagar (Bangalore)

7586:

Paul, Ranjit. Solutions of concentration-dependent diffusion equation. Phys. Fluids **3** (1960), 905–907.

The theory of diffusion in a chamber of the Ney and Armistead type is considered in the case when the diffusion coefficient depends on concentration of the gases.

S. Simons (London)

7587:

Aris, R. On the dispersion of a solute in pulsating flow through a tube. Proc. Roy. Soc. London. Ser. A **259** (1960/61), 370–376.

Author's summary: "The effective dispersion coefficient of a solute in pulsating flow through a circular tube is here found. The case of a viscous flow under a pulsating pressure gradient is treated in detail and it is found that the Taylor diffusion coefficient contains terms proportional to the square of the amplitude of the pressure pulsations. However, the coefficients of these terms tend rapidly to zero, and the effect of pulsation will rarely contribute a fraction of more than $\frac{1}{115}$ (the ratio of the amplitude of pressure gradient pulsation to mean pressure gradient)^2 to the total dispersion coefficient. The methods may be applied to diffusion in any periodic flow."

7588:

Êfros, D. A. Permeability of porous media during the filtration of a gassed liquid. Dokl. Akad. Nauk SSSR **132** (1960), 311–314 (Russian); translated as Soviet Physics. Dokl. **5**, 493–496.

Experimental data are presented to show that the permeability in flow of gas-saturated liquid through porous media is different from that in flow of gas-liquid mixture, although they are generally assumed to be the same. The difference may be due to the peculiar distribution of the gas and the liquid in the pores of the structure. A new expression with two parameters is suggested for the permeability in steady linear flow of the gas-saturated liquid.

G. Paria (Kharagpur)

7589:

Êfros, D. A.; Kuranov, I. F. Analyzing the flow of an aerated fluid assuming a two-parameter permeability characteristic. Dokl. Akad. Nauk SSSR **132** (1960), 553–556 (Russian); translated as Soviet Physics. Dokl. **5**, 497–500.

In an earlier paper [see #7588 above] a two-parameter expression was suggested for the permeability in steady linear flow of the gas-saturated liquid through porous media. It is further examined here for flows having different geometry. For stationary plane-radial flow, it applies well. In plane-radial non-stationary inflow, however, it gives only reasonably approximate results.

G. Paria (Kharagpur)

7590:

Scheidegger, Adrian E.; Johnson, Edward F. The statistical behavior of instabilities in displacement processes in porous media. Canad. J. Phys. **39** (1961), 326–334.

The problem of fingering in displacement processes in porous media has been treated from a macroscopic standpoint. With the Muskat model of displacement theory as the basis, there has been deduced a differential equation satisfied by the relative area occupied by fingers. It is similar to the Buckley-Leverett equation of immiscible displacement. The method of characteristics is used to solve it. It is shown that the statistical theory does not lead to any stabilization of the fingers.

G. Paria (Kharagpur)

7591:

Wageman, W. E.; Guevara, F. A. Fluid flow through a porous channel. Phys. Fluids **3** (1960), 878–881.

To investigate the laminar flow of an incompressible fluid through a cylindrical channel with a uniformly porous wall, a dimensionless parameter, analogous to an inverse wall Reynolds number, has been defined. The Navier-Stokes equations have been solved when the parameter is very small. Two separate sets of experiments are described. These yield qualitative agreement with the theory.

G. Paria (Kharagpur)

OPTICS, ELECTROMAGNETIC THEORY, CIRCUITS

See also 7320, 7321, 7336 a-c, 7351, 7441, 7456, 7457, 7566, 7579, 7806, 7874, 7876, 7878, 7888.

7592:

Buchdahl, H. A. Optical aberration coefficients. V. On the quality of predicted displacements. J. Opt. Soc. Amer. **49** (1959), 1113–1121.

[For parts I, II, III and IV see same J. **46** (1956), 941–943; **48** (1958), 563–567, 747–756, 757–759; MR **19**, 355; **20** #2972; **22** #6337, 6338.]

This paper is devoted to a number of examples illustrating the application of the author's higher aberration coefficients in the practical design of lens systems.

G. L. Walker (Providence, R.I.)

7593:

Buchdahl, H. A. Optical aberration coefficients. VI. On computations involving coordinates lying partly in the image space. J. Opt. Soc. Amer. **50** (1960), 534–539.

In the author's monograph, *Optical aberration coefficients* [Oxford Univ. Press, London, 1954, MR **19**, 354] and in previous papers of the present series [see #7592 for references], all explicit computing schemes have hitherto referred to sets of four coordinates lying entirely in the

object space. As a preliminary to the following paper [#7594], coordinates consisting of two pairs lying entirely in the object and image spaces, respectively, are considered and using these coordinates, an explicit computing scheme is given for the primary, secondary, and tertiary aberration coefficients. *G. L. Walker* (Providence, R.I.)

7594:

Buchdahl, H. A. Optical aberration coefficients. VII. The primary, secondary, and tertiary deformation and retardation of the wave front. J. Opt. Soc. Amer. **50** (1960), 539–544.

In the author's previous work on aberration coefficients [see review #7592 for references], the geometrical aberrations of an optical system have been analyzed in terms of the displacement ε' of the intersection points with the ideal image plane of arbitrary rays, relative to ideal intersection points, ε' being expressed as a series in ascending powers of suitably chosen variables. On the other hand, the Wave Theory of Aberrations [H. H. Hopkins, *Wave theory of aberrations*, Clarendon, Oxford, 1950; MR **12**, 460] is developed through the equation of the wave front W' in the image space, i.e., of that surface whose normals constitute the congruence into which the system has transformed the pencil of rays issuing from any object point. The equation of W', when written in a suitable form, differs from that of a certain spherical surface W_0' only through the presence of a term D, here called the deformation of the wave front. D may be approximated by the terms of a power series up to a certain order, the coefficients of which (deformation coefficients) are closely related to the aberration coefficients which defined ε'. The principal object of this paper is to establish the relations between these two sets of coefficients, those of the third, fifth, and seventh orders being dealt with explicitly. In the diffraction theory of aberrations on the other hand, one is interested primarily in the normal displacement between corresponding points on W_0' and W'. A function R which describes this displacement may be called the retardation of the wave front. This function is also considered, and simple relations are established between the deformation coefficients and the retardation coefficients, i.e., the coefficients of the power series for R. *G. L. Walker* (Providence, R.I.)

7595:

Buchdahl, H. A. Optical aberration coefficients. VIII. Coefficient of spherical aberration of order eleven. J. Opt. Soc. Amer. **50** (1960), 678–683.

As a sequel to the fourth paper of the present series, in same J. **48** (1958), 757–759 [MR **22** #6338], devoted to the development of a computing scheme for the coefficient of quaternary (order nine) spherical aberration, a computing scheme is now developed for the coefficient of quinary (order eleven) spherical aberration.

G. L. Walker (Providence, R.I.)

7596:

Herzberger, M. Higher-order imagery. J. Opt. Soc. Amer. **51** (1961), 135–142.

The properties of a symmetrical optical system are completely specified by a characteristic function G of one sort or another, where G is a function of three rotational invariants e_1, e_2, e_3 formed from base vectors $\mathbf{b}_1$, $\mathbf{b}_2$. The aberration coefficients g_{nt} of order $2n-1$ are the coefficients of the homogeneous polynomial of degree n in e_1, e_2, e_3 in the Taylor expansion of G. Certain problems of geometrical optics are equivalent to the determination of the corresponding coefficients g_{nj}' in the expansion of the function $G'(e_1', e_2', e_3')$ which results from a linear homogeneous transformation of the base vectors $\mathbf{b}_1$, $\mathbf{b}_2$ to new base vectors $\mathbf{b}_1'$, $\mathbf{b}_2'$. In this paper a procedure for finding the g_{nj}' in terms of the g_{nt} is developed, so-called Smith polynomials being employed in the course of it. Some applications are briefly discussed. *H. A. Buchdahl* (Hobart)

7597:

Biot, A. Sur les aberrations périaxiales de l'œil. I. Ann. Soc. Sci. Bruxelles. Sér. I **74** (1960), 175–188.

The real eye is a very complicated type of optical system owing to an essential asymmetry, inhomogeneity of refractive index and changes in structure due to accommodation. Furthermore, measurements of the pertinent data are delicate and difficult especially since these data are changed after death. By suitably schematizing its structure, however, one can achieve a good approximate analysis of the geometrical and chromatic aberrations of the living eye. *E. W. Marchand* (Rochester, N.Y.)

7598:

Vernier, Pierre. Définition du contour du spot d'un densitomètre, effet Callier et granulation photographique. Rev. Opt. **39** (1960), 505–515. (English summary)

Author's summary: "In the first part a general formula is given that yields the transparency of a photographic emulsion surface S. On the S border, it has been allowed for edge effects due to the densitometer shortcomings. The influence of the instrument apertures on the measurement of optical densities is shown, and the importance of the degree of coherence of the light received in a point of the plate is emphasized.

"In the second part, it is shown that the statistical properties of the optical density of a uniformly exposed emulsion cannot be simply derived from those of a localized property of the emulsion (e.g., local transparency), unless a restrictive assumption were made as to the plate structure. Under precise conditions, the relative standard deviation in optical density might be considered as an independent feature of the densitometer for a given granularity."

7599:

Keller, R. Die Lichtverteilung im Bildraum optischer Systeme bei Anwesenheit von Aberrationen dritter Ordnung beliebig hohen Betrages unter Einschluss des Falles anamorphotischer Abbildung. Optik **17** (1960), 611–632. (English and French summaries)

Author's summary: "In the present work diffraction theory is applied to an anamorphic, afocal system for any desired third order aberration of unrestricted magnitude. But the procedure that has been developed can also be applied to rotationally symmetrical systems.

"The Gaussian image point is chosen as origin of a right-angled coordinate system, and is furthermore the centre of curvature of a reference sphere U, which passes through the centre of the exit pupil. The separation of the Gaussian

image point from the optical axis is denoted by either σ or σ', depending on whether the sagittal or meridian plane is under consideration, since the anamorphic system possesses different magnifications in two mutually perpendicular sections. The position of a point in the exit pupil is indicated by the polar coordinates r and φ, while a point in the image plane is represented by ρ and ψ. For a system free from aberration, U becomes a wave surface for a pencil of rays which converges on the Gaussian image.

"The departure of the actual wavefront from the reference sphere U is given by an aberration function $V(\sigma, \sigma', r, \varphi)$. For the afocal, anamorphic system discussed here, a single aberration can be expressed by the function $\beta \cdot r^n \sin^n \varphi$.

"In the second chapter, the diffraction theory treatment is carried out. Each aberration is discussed individually, and then the interaction of all the aberrations is investigated. The latter represents the case occurring in practice. Applications of the remainder theorem from the theory of functions enables one to reduce the two dimensional problem to a one dimensional one. In all cases integrals appear which depend on only one variable, namely r. The integration is carried out by means of Simpson's Rule; the numerical evaluation was performed with a program-controlled electronic computer.

"The calculated patterns of light distribution are discussed. Furthermore, several small-scale measurements are mentioned, which confirm—at least in outline—the theoretical conclusions.

"Finally, an example from the program for the magnetic drum computer IBM 650 is given in the form of a block diagram."

7600:

Macrakis, Michael S. Theoretical and experimental study of the backscattering cross section of an infinite ribbon. J. Appl. Phys. **31** (1960), 2261–2266.

Author's summary: "The geometrical optics approximation is derived for the backscattering cross section per unit length of an infinite ribbon. A comparison is made with the exact theory, the approximate theory of Sommerfeld, the variational method, and with experimental results obtained through the space-separation method for the measurement of backscattering cross sections in a parallel-plate region."

7601:

Shinoda, Gunji; Suzuki, Tatsuro. Effects of the scanning apertures on the image quality. Tech. Rep. Osaka Univ. **10** (1960), 681–691.

Authors' summary: "For rectangular, sinusoidal apertures or some others similar to them the effects on the image quality have been analysed from the point of view of the spatial frequency response of the apertures. The sinusoidal aperture or those similar to it showed more desirable results in any cases than the rectangular or circular apertures. If incoherent illumination is allowed the more desirable results will be expected because it has a sharp cut-off frequency characteristic. The overlapped aperture may be effective not only to reduce the extraneous images on the scanned image but also to reduce the appearance of scanning lines on the image."

7602:

Pipkin, A. C. Electrical conductivity of partially ionized gases. Phys. Fluids **4** (1961), 154–158.

The author considers a gas of electrons, ions, and neutrals, in which each ion has lost the same number of electrons, and in which the degree of ionization (ratio of number of ions to number of neutrals) is a parameter varying between zero and one. This mixture of gases is described by the Boltzmann equation, and is perturbed by spatially uniform D.C. electric and magnetic fields. The transfer equations for the mixture have been previously obtained by expanding the distribution function for each gas component as a (Maxwellian) × (Polynomial in velocity components), and taking velocity moments of the three Boltzmann equations. The collision integrals have been evaluated for all possible types of elastic collisions, and the coefficients in the polynomials (of 2nd degree) can be represented in terms of macroscopic quantities such as average velocity, stress tensor, etc. It is desired to solve for current density as a function of the applied fields. The first and third moments of the Boltzmann equations are used (since the first moments contain a third moment—the heat flow) and lead to a set of simultaneous algebraic equations which can be solved by elementary means. The second moment does not need to be considered since a scalar pressure and an equation of state is assumed. It is found that without a magnetic field, the conductivity is independent of the degree of ionization, except for very small degrees; whereas with a magnetic field, the conductivity depends very strongly on the degree of ionization. To the reviewer there seems to be no reason why the same methods could not be used for a fully ionized gas containing ions that have been stripped by different amounts.

R. Gerwin (Seattle, Wash.)

7603:

Alfvén, H. Plasma physics. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 1–8. Zanichelli, Bologna, 1960.

A general, largely non-mathematical, introduction to some aspects of plasma physics is presented. A very brief survey of magneto-hydrodynamics of incompressible fluids is included. There is no corresponding discussion for compressible fluids except to point out the additional complications possible. Four or five brief paragraphs are devoted to technological and astrophysical applications.

G. Morris (Seattle, Wash.)

7604:

Lundquist, S. Particle view compared with hydrodynamics. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 15–19. Zanichelli, Bologna, 1960.

The author points out that macroscopic currents can exist in a plasma without motion of guiding centers. He derives the quasi-static magnetic permeability and dielectric constant of a plasma region and points out the dependence of the plasma diffusion on conductivity and density and temperature gradients. He shows that when these are negligible, a plasma has an Einstein-de Haas effect (conservation of total angular momentum).

J. E. Drummond (Seattle, Wash.)

7605:

Lundquist, S. Run-away phenomena and relaxation effects. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 20–24. Zanichelli, Bologna, 1960.

The author begins by noting some of the important length dimensions in a plasma and their relative magnitudes. He then cites Spitzer's results for electrical resistivity for the fully ionized gases having the assumed ratios of the various lengths. The pair of equations for the electrical and heat fluxes in terms of electric field strength and temperature gradients are stated with a relation between the coefficients. The frictional force acting on a moving electron is given from the literature and the condition for "strong run-away" shown (continual acceleration of the charged particles). An algebraic error can be spotted dimensionally. *J. E. Drummond* (Seattle, Wash.)

7606:

Lundquist, S. Plasma stability. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 25–33. Zanichelli, Bologna, 1960.

The author summarizes three common approximate treatments of stability problems: perturbation of particle orbits; normal modes of the linearized equations of motion; and the variation of energy principle. To illustrate the first two methods, the author considers the stability of a plasma supported by a magnetic field against a gravitational field. The energy method is illustrated briefly by the problem of energy change under the interchange of magnetic flux tubes. Results of studies of more complicated field and plasma configurations such as cylindrical, helical, and "frozen-in" fields are considered very briefly and the authors of such studies noted. *D. Jerde* (Seattle, Wash.)

7607:

Allen, J. E. Collision-free hydromagnetic waves. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 38–50. Zanichelli, Bologna, 1960.

Propagation of hydromagnetic waves through a plasma is discussed in two cases: (a) when the propagation is along the magnetic lines of force, and (b) when the propagation is transverse to the lines of force. When a purely transverse plane wave travels along the magnetic line of force, there exists a dispersion relation between the wave velocity U and the wave number k for arbitrary values of the amplitude. For low frequencies U tends to the Alfvén velocity, but the wave is always circularly polarized. The electrons and the ions move in circles with almost equal radii.

When the wave propagates transverse to the magnetic field, it is neither transverse nor longitudinal but a mixture of both. The electrostatic field becomes important and the waves are no longer circularly polarized. For waves of small amplitude, there is a cut-off frequency beyond which no waves can propagate, and in the low frequency limit the speed of the waves tends to the Alfvén velocity. In the case of steady waves of large amplitude, the variations in the magnetic field, the electrostatic field, the relative velocity between electrons and ions, and the ion density are discussed. Defining α as the ratio of the wave velocity and the Alfvén velocity, it is shown that $1 \leq \alpha \leq 2$. The magnetic field at any point does not exceed three times the initial field. For $\alpha \sim 2$ the energy received by the ions can be very appreciable. It is further shown that if a train of hydromagnetic waves is launched into a plasma, the waves propagate and grow in amplitude, but whether this can develop into a shock phenomenon or not is unanswered. *F. C. Auluck* (Delhi)

7608:

Allen, J. E. Langmuir probes and boundary phenomena. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 51–60. Zanichelli, Bologna, 1960.

This paper is a review of the work done on Langmuir probes. The presence of a probe in the plasma tube influences the potential distribution through its boundary due to the drain of charged particles and secondary emissions. The electrostatic field does not remain entirely confined to the narrow space-charge sheath because some field penetrates the plasma as well. The different types of boundaries are considered; plane, cylindrical and spherical. The equation of plasma-sheath is obtained and its solutions discussed in two cases: (i) when the ion generation is proportional to the electron density, and (ii) when the ion generation is constant throughout the plasma. The field penetrates over distances of the order of 2 or 3 times the dimensions of the probe. *F. C. Auluck* (Delhi)

7609:

Allen, J. E. Magnetic confinement and different machines. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 61–68. Zanichelli, Bologna, 1960.

This paper discusses the physical principles underlying the classification of different types of machines used for the magnetic confinement of plasma. The inward pressure may be due to the interaction between an azimuthal current and a longitudinal field or between a longitudinal current and an azimuthal field. The current may be produced by the application of an electric field or the diffusion of charged particles across magnetic lines of forces. Various combinations of these methods are discussed in the case of infinite cylinder and these extended to systems of finite size like the stellators, Perhapstrons, the Zeta and the mirror machine. *F. C. Auluck* (Delhi)

7610:

Allen, J. E. Fast circuits. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 69–75. Zanichelli, Bologna, 1960.

In this paper the author discusses the experiments which are of interest in fast circuits required for the rapid compression of plasma, the production of current sheets, and for the effective detachment of the plasma from the walls of the discharge tubes. Two types of spark gaps, namely the "trigation" type used at Aldermaston and the four-electrode spark gap used in the Scylla apparatus at Los Alamos are described. *F. C. Auluck* (Delhi)

7611:

Hunziker, Raul R. The flow about a charged body moving in the lower ionosphere. J. Aerospace Sci. **27** (1960), 935–942.

Author's summary: "The flow about an electrically charged body traveling at high speeds through the lower ionosphere is analyzed. A simple gas model composed of electrons, ions, and neutral particles is used and the hydrodynamic description given is based on Maxwell's transfer equations for a mixture.

"The conditions under which local statistical equilibrium can be assumed are discussed, and different approaches to determine the gasdynamic force in the subsonic,

supersonic, and hypersonic cases are indicated. The reciprocal action of the electric field of the flow on the body is also analyzed, and a formula for the resultant electric force is given. The total force on the body is equal to the sum of the gasdynamic force and the electric force.

"The negative potential acquired by a plane body is also calculated. Finally, the lack of validity of Debye's linearization in this case and the solution of the exterior nonlinear problem which characterize the electric potential and the electron distribution are discussed."

R. Finn (Stanford, Calif.)

7612:

Shafranov, V. D. Propagation of an electromagnetic field in a medium with spatial dispersion. Soviet Physics. JETP **34** (7) (1958), 1019–1029 (1475–1489 Ž. Éksper. Teoret. Fiz.).

The problem considered is that of the penetration of an electromagnetic field into a semi-infinite, homogeneous, anisotropic medium with spatial dispersion. It is assumed that the functional relation between the displacement vector **D** and the electric field **E** may be expressed as

$$D_\alpha(\mathbf{r}) = \int K_{\alpha\beta}(\mathbf{r}-\mathbf{r}')E_\beta(\mathbf{r}')\,d\mathbf{r}'.$$

Fourier integral methods are employed to obtain expressions for the field components in terms of their values (assumed constant) on the plane boundary of the medium. The results in the absence of partial dispersion reduce to the well known ones for a homogeneous anisotropic medium. The general solution is obtained as a Fourier integral and a detailed examination of it requires a knowledge of the analytic properties of the Fourier transform of $K_{\alpha\beta}$. A complete analysis is presented for the propagation of a transverse wave along a magnetic field in a plasma. *W. E. Williams* (Liverpool)

7613:

Ron, A.; Kalman, G. Interaction of a test particle with a plasma. I. Distribution function of field particles. Ann. Physics **11** (1960), 240–259.

The distribution function of plasma particles is calculated in the case where a charged test particle travels through the plasma. The effect of the field particles on the test particle is described through a stochastic field which affects the test particle (i) by an average force ("polarization effect"), and (ii) by fluctuations in the varying stochastic field ("statistical effect"). Conditions for the steady state solution and for neglect of the interaction between plasma particles are investigated, and the problem is studied subject to these simplifications.

The ensuing distribution function is discussed in the light of two previous results, due to Bohm and Pines, and to Debye and Huckel. There is a disagreement (which is explicable) with Bohm and Pines, and which is of interest in connection with the energy loss of a moving test particle in a plasma. *R. M. May* (Cambridge, Mass.)

7614:

Pappert, R. A. Charge excitation of plasma motion in a magnetic field. Phys. Fluids **3** (1960), 966–972.

A study of the wake generated behind a uniformly moving charged particle in a plasma. This work is based on the linearized Vlasov equation and treats the case where there is a weak magnetic field in the direction of the particle's motion. The analysis is restricted to those cases where the velocity, v, of the particle satisfies the inequalities $c_i < v < c_e$ and $mc_s^2 \ll Mv^2$, where c refers to the thermal velocities of the electrons and ions, and m, M are their respective masses. The structure of the wake behind a point and line charge is determined.

E. A. Jackson (Princeton, N.J.)

7615:

Silin, V. P. Kinetic equation for rapidly varying processes. Ž. Éksper. Teoret. Fiz. **38** (1960), 1771–1777 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1277–1280.

An investigation of the response of a homogeneous plasma to a rapidly varying electric field. The period of this field is assumed to be small compared to the plasma period (ω_p^{-1}), but large compared to the average duration of a collision which produces large angle deflections ($(e^2/\kappa T)(m/\kappa T)^{1/2}$). The equation governing the two particle correlation function is approximated by one which only takes into account interactions between two uncorrelated particles. This basic approximation reduces the original integro-differential equation to a differential equation which can be solved in terms of the past history of the single particle distribution function, $f(v, t)$. This leads to a closed equation for $f(v, t)$ involving retarded times, which reduces to Landau's equation if the retardation is neglected. It is shown that the dissipation is modified by the replacement of the Debye length, in the Coulomb logarithm, by the distance a particle traverses during a period of the field. In the presence of a strong magnetic field, it is noted that the maximum impact parameter has a resonant character if the frequency is near the electron Larmor frequency. *E. A. Jackson* (Princeton, N.J.)

7616:

Schmidt, George. Plasma motion across magnetic fields. Phys. Fluids **3** (1960), 961–965.

In order to determine the applicability of the hydromagnetic fluid model to certain plasma flow problems, the author studies four experimental situations where a low density, collisionless plasma moves across a magnetic field. These cases are analysed by combining the guiding center approximation, to describe the motion of the particles, and a self-consistent field treatment of the electromagnetic fields generated by the plasma. The limitations and shortcomings of the hydromagnetic treatment are discussed. *E. A. Jackson* (Princeton, N.J.)

7617:

Kihara, Taro. Ion-electron relaxation of plasmas in a strong magnetic field. I. J. Phys. Soc. **14** (1959), 1751–1754.

The author discusses the temperature relaxation between ions and electrons of different temperatures, in a fully ionized plasma with a homogeneous magnetic field. In the case where the radius of gyration of the electrons is much smaller than their Debye length, a simple technique is used to derive the relaxation time: one calculates orbits, then averages over the distribution function—the resulting integrals involve a cut-off which, for a strong field, depends on the radius of gyration. *R. M. May* (Cambridge, Mass.)

7618:

Kihara, Taro; Midzuno, Yukio; Sakuma, Kyoko; Shizume, Toshio. Ion-electron relaxation of plasmas in a magnetic field. II. J. Phys. Soc. Japan **15** (1960), 684–687.

The technique used in paper I of this series (see preceding review) is refined by introducing a "force-correlation" method, and then applied to calculate the electron-ion relaxation in magnetic fields of arbitrary strength. The relaxation rate, R, defined by $d(T_2-T_1)/dt=(T_2-T_1)R$ is found to be

$$R = R_0 ln(kT_2 l_D/Ze^2) \quad \text{for } \omega_e \lesssim \omega_p,$$

$$R = R_0[ln(kT_2 l_D/Ze^2)+\tfrac{1}{2}(ln\omega_e/\omega_p)^2] \quad \text{for } \omega_e \gg \omega_p,$$

where T_1 and T_2 are the ion and electron temperatures, ω_e and ω_p the cyclotron and plasma frequencies for electrons, l_D the Debye length, and R_0 is a constant involving the mass, temperature, and density of the electrons and of the ions. *R. M. May* (Cambridge, Mass.)

7619:

Sturrock, P. A. Generalization of the Lagrange expansion with application to physical problems. J. Mathematical Phys. **1** (1960), 405–408.

One may study perturbations of particle distributions, such as electron beams and electron-ion plasmas, in terms of the displacement of the particles from their unperturbed position. Often the need arises to relate this "Lagrangian" description to the Eulerian variables that specify physical quantities (charge density, etc.) as functions of the actual position of the particles. This paper arrives at such a relation for n-dimensional space in what may be considered a generalization of the Lagrange expansion. An example of the relation gives the perturbed particle density $\bar{\rho}(x)$ in terms of the unperturbed particle density $\rho(x)$ and the displacement $\lambda\xi_r(x)$ (λ is a dimensionless expansion parameter and must be chosen small enough so that no particle "crossover" occurs)

$$\bar{\rho}(x) = \rho(x)-\lambda\frac{\partial}{\partial x_r}\{\rho(x)\xi_r(x)\}+\frac{\lambda^2}{2!}\frac{\partial^2}{\partial x_r\partial x_s}\{\rho(x)\xi_r(x)\xi_s(x)\}-\cdots.$$

Other applications of the relation give the Eulerian current density, and the equivalent surface charge and dipole layers ascribed to a rippling beam with an abrupt boundary, in terms of $\xi_r(x)$. In its six-dimensional form the method leads to a simple derivation of the Fokker-Planck equation. *H. A. Haus* (Cambridge, Mass.)

7620:

Hain, K.; Hain, G.; Roberts, K. V.; Roberts, S. J.; Köppendörfer, W. Fully ionized pinch collapse. Z. Naturforschg. **15a** (1960), 1039–1050.

Authors' summary: "A fully ionized plasma is assumed. To this plasma cylindrically-symmetric magnetic fields are applied, thus causing a pinch collapse. The plasma is treated in hydromagnetic approximation, including electric and thermal conductivity. Separate temperatures are assigned to the electrons and ions.

"Two schemes are developed for solving numerically the resulting system of six partial differential equations: the explicit scheme for rather fast pinches, where a numerical stability requirement causes the timestep to be bounded by the characteristics given by the Alfvén speed, and an implicit scheme, which consists essentially in converting the momentum equation into a second order difference equation with coefficients determined by iteration; here there is no such restriction on the timestep. These schemes were made to work on the U.K.A.E.A. IBM 704 and IBM 709.

"A run is described in which the initial state was one with uniform density, temperature and B^z field. The boundary temperatures were assumed to remain constant, while the magnetic fields at the boundary were determined by the circuits for the j^z and j^θ currents. The results of the computations are in good agreement with experimental results obtained at the Technische Hochschule München by one of the authors (Köppendörfer).

"The whole program is a joint effort between A.E.R.E. Harwell and the Max-Planck-Institut, intended to discover by comparison with experiments how good the hydromagnetic approximations are. If the agreement is satisfactory (eventually using a generalised program which includes neutral gas) it should be possible to design experiments so that specified field configurations are set up."

7621:

Yoshihara, Hideo. Motion of thin bodies in a highly rarefied plasma. Phys. Fluids **4** (1961), 100–104.

The author considers the uniform motion of a thin body in an unbounded and highly rarefied plasma with a velocity lying between the thermal speeds of ions and electrons. Due to motion, the body acquires a charge and the Coulomb interaction of this charge excites plasma oscillations in the wake of the body. In studying the resultant motion of the plasma, the author neglects the collisions between the particles and also the magnetic effects. He assumes the body so thin and induced charge on it so small that its motion results in producing a small perturbation $f'(\mathbf{x}, \mathbf{c}, t)$ in the undisturbed ion distribution function $f_0(\mathbf{c})$ which is assumed to be Maxwellian. In determining the distribution function for electrons the body is assumed to be at rest so that the electrons conform to Maxwellian distribution under electrostatic force field ϕ. The following equations determine f' and ϕ:

$$\frac{\partial f'}{\partial t}+\mathbf{c}\cdot\nabla f' = \frac{e}{m_i}\nabla\phi\cdot\nabla_c f_0,$$

$$\left(\Delta-\frac{1}{R_D^2}\right)\phi = -4\pi e\int f'\,d\mathbf{c},$$

where R_D is the Debye distance. For a conductor, ϕ assumes a prescribed value, and for a dielectric, ϕ or its normal derivative has to be prescribed. In the absence of knowledge about the nature of interaction between the surface and impinging ions, it is assumed that the reflection of the ions from the surface is specular, with the particle retaining its charge after reflection. f' is taken to vanish far upstream of the body.

The motion of plasma is found to be composed of a free molecular flow component and a fluid-like collective component. In a highly rarefied plasma the surface charge affects the entire wake, in contrast to the dense plasma where the effect of the surface charge is confined to the thin Debye boundary layer adjacent to the body.

P. L. Bhatnagar (Bangalore)

7622:

Wait, James R. Propagation of electromagnetic waves along a thin plasma sheet. Canad. J. Phys. **38** (1960), 1586–1594.

This paper shows that a thin ionized sheet in the presence of a steady magnetic field will support a surface wave. The phase velocity parallel to the sheet and the attenuations in the direction of propagation and in the direction perpendicular to the sheet are all depicted graphically as functions of the sheet thickness, for various values of the collision frequency and the gyro frequency parameters. A transition region occurs when the gyro frequency is close to the electromagnetic frequency. Notably, the attenuations drop sharply with increasing gyro frequency in the transition region, for very thin sheets. A final section considers the effect of locating the plasma sheet near and parallel to a conducting plane, in which case other modes of a waveguide type are possible, in addition to the surface wave.

J. A. Morrison (Murray Hill, N.J.)

7623:

Nekrasov, F. M. On the nonlinear theory of stationary processes in an electron plasma. Ž. Èksper. Teoret. Fiz. **38** (1960), 233–238 (Russian. English summary); translated as Soviet Physics. JETP **11**, 170–173.

This paper gives the exact, steady solutions of the coupled Poisson and collisionless Boltzmann equations for an electron plasma with a background of motionless protons. All the results appear already in Bernstein, Greene and Kruskal [Phys. Rev. (2) **108** (1957), 546–550; MR **21** #1122]. *G. E. Backus* (La Jolla, Calif.)

7624:

Jackson, J. L. Electric field distribution in a dense plasma. Phys. Fluids **3** (1960), 927–931.

The author points out the importance of Stark broadening of spectral lines as a diagnostic tool for dense plasmas. He then proceeds to develop a statistical theory to calculate the fluctuating field at the center of a sphere surrounded by a thermal plasma. To do this, he first calculates the probability of a given fluctuation in the charge density distribution using Gauss's Probability Law with the free energy. (Entropy is approximated by the zeroth, first and second power terms of its series (density) expansion.) This probability is then integrated over all density distributions corresponding to a given electric field at the center of the sphere. The many integrations are neatly handled. In the process two linear three-dimensional integral equations are solved. The required probability distribution is explicitly found to be a simple Gaussian:

$$P(E) = \frac{E^2}{(4\pi\alpha^3)^{1/2}}\, e^{-E^2/4\alpha}$$

and the constant determined.

J. E. Drummond (Seattle, Wash.)

7625:

Kelly, Donald C. Microwave conductivity of a plasma in a magnetic field. Phys. Rev. (2) **119** (1960), 27–39.

The analysis starts from the Boltzmann equation in which the contribution of the rf magnetic field is neglected. A uniform plasma, in the limit of large wavelength, is studied, in which the distribution function $f(\bar{v}, t)$ is not a function of position. The distribution function is expanded in a series of orthogonal functions in velocity space with time-dependent coefficients. An infinite set of coupled equations results, which is broken off at successively higher terms, and the convergence of the scheme is tested accordingly. This is illustrated in the case of a weak electric field, zero dc magnetic field, and a completely ionized gas, by plotting several curves of the conductivity versus frequency for different orders of approximation. The effect of electron-electron collisions is compared with that of electron-ion collisions. Conductivity curves are also plotted for a finite magnetic field to compare the relative effects of electron-electron and electron-ion collisions.

H. A. Haus (Cambridge, Mass.)

7626:

Cullen, A. L. Propagation of microwaves through a magneto-plasma, and a possible method for determining the electron velocity distributions. J. Res. Nat. Bur. Standards Sect. D **64D** (1960), 509–513.

The conductivity tensor is derived in a very simple and direct way for a uniform plasma in a uniform dc magnetic field excited by transverse electromagnetic waves propagating along the dc magnetic field lines. The analysis starts from the equation of motion of a single electron and the postulate of collisions, that the electron is brought to rest by each collision. The macroscopic conductivity is obtained by integration over all particles taking into account the Doppler frequency shift of the particles with different velocities. The result for a hot plasma does not depend upon the collision frequency to first order and thus is not affected critically by the details of the assumed collision process. This result agrees with that of Sagdeyev and Shafranov derived for a collisionless plasma. Diagnostic measurements are suggested for the determination of the velocity distribution by absorption measurements on the extraordinary wave, and the potentialities and limitations are briefly discussed.

H. A. Haus (Cambridge, Mass.)

7627:

Allis, William P. Propagation of waves in a plasma in a magnetic field. IRE Trans. **MTT-9** (1961), 79–82.

A short discussion is given of the Appleton-Hartree theory of the propagation of electromagnetic waves in a cold plasma. The dispersion equation is represented graphically in various types of diagrams. A new type of representation is introduced by plotting the wave normal surfaces essentially in a (magnetic field) vs. (density) plane. This gives a neat picture of some phenomena, such as the guiding by a magnetic field ("whistlers").

A short discussion is also given of the case of nonvanishing temperature. *R. Balescu* (Brussels)

7628:

Carstoiu, John. Analogies et différences dans les tourbillons et champ magnétique. C. R. Acad. Sci. Paris **251** (1960), 509–511.

L'analogie entre l'équation qui régit les variations du champ magnétique et l'équation de Helmholtz pour le tourbillon ω,

$$\frac{d}{dt}\frac{H_i}{\rho} = \frac{H_j}{\rho}\frac{\partial v_i}{\partial x_j}, \quad \frac{d}{dt}\frac{\omega_i}{\rho} = \frac{\omega_j}{\rho}\frac{\partial v_i}{\partial x_j} \quad (i, j = 1, 2, 3)$$

permet de mettre en évidence ou de retrouver certaines propriétés du champ magnétique; on obtient ici: (a) une formulation différentielle du théorème intégral d'Elsasser relatif à la rotation du champ magnétique; (b) un théorème de conservation du champ magnétique à force libre (force-free); (c) diverses propriétés du vecteur $\mathbf{H} \times d\mathbf{r}$.

J. Naze (Marseille)

7629:

Bremmer, Hendricus. Méthodes mathématiques appliquées dans la théorie de la propagation des microondes. La propagation par diffusion; Théories concernant les fluctuations des champs. Confer. Sem. Mat. Univ. Bari **45–46**, 23 pp. (1959).

Expository paper, reflecting the author's contribution to a mathematical seminar held at the University of Bari, Italy. It surveys recent theories of scatter and time-fluctuating fields in micro-wave propagation.

C. J. Bouwkamp (Eindhoven)

7630:

Taniuti, T. The canonical theory of motion of charged particles in external electromagnetic fields. Nuovo Cimento (10) **16** (1960), 572–575.

The motion of charged particles in an electromagnetic field is studied using a collective coordinate to describe the gyration of the particles around a common guiding centre. The adiabatic invariance of the magnetic moment now follows easily.

D. ter Haar (Oxford)

7631:

Lundquist, S. Motion of charged particles in electric and magnetic fields. Rend. Scuola Internaz. Fis. "Enrico Fermi", Corso XIII (1959), pp. 9–14. Zanichelli, Bologna, 1960.

The author first states Newton's equation of motion with Lorentz' force law and then proceeds to state results for a number of special cases. For constant electric field, $\mathbf{E}$, and zero magnetic field, $\mathbf{B}$, and adding the frictional force of coulomb collisions, $-K/v^2$, he points out some of the qualitative features of "run-away". Other special cases are $\mathbf{E}=\mathbf{0}$, $\mathbf{B}=\text{const}$ (spiral orbits), $\mathbf{E}=\text{const}$, $\mathbf{B}=\text{const}$ (drift across $\mathbf{E}$ and $\mathbf{B}$ fields), $\mathbf{E}=\mathbf{E}_0 e^{i\omega t}$, $\mathbf{B}=\text{const}$ ("complicated"), $\dot{B}/B \ll qB/m$ (derivation of adiabatic invariants), $\mathbf{B}=\mathbf{B}(\mathbf{r})$, $\mathbf{E}=\mathbf{0}$ ("magnetic potential" and centrifugal force). Finally a few comments are made about magnetic confinement. The article contains a number of algebraic errors which can be noted from dimensional considerations.

J. E. Drummond (Seattle, Wash.)

7632:

Ferrari, Italo. Un teorema di unicità per il campo di un filo percorso da corrente alternata, in prossimità di un mezzo conduttore. Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. **94** (1959/60), 77–88.

Es handelt sich um die Berechnung des elektromagnetischen Feldes, das erzeugt wird von einem geraden, fadenförmigen, wechselstromdurchflossenen Leiter, der in einem homogenen isotropen Dielektrikum eingebettet ist, das einerseits sich ins Unendliche erstreckt, andererseits von einer ebenen Begrenzung eines unvollkommenen, unendlich ausgedehnten Leiters begrenzt ist. Die Leitergrenzebene ist parallel zum Faden. Das Feld setzt sich aus einem ersten Beitrag zusammen, der vom fadenförmigen Wechselstromleiter in einem allseitig unendlich ausgedehnten Dielektrikum herrührt, und aus einem zweiten Beitrag, der von den Strömen im Leiter mit ebener Begrenzung erzeugt wird. In dieser Arbeit wird bewiesen, dass bei gegebenem erstem Beitrag stets ein einziger zweiter Feldbeitrag existiert, derart, dass die Maxwellschen Gleichungen und Grenzbedingungen erfüllt werden. Der Beweis wird indirekt geführt, indem angenommen wird, dass zwei verschiedene zweite Beiträge vorhanden wären. Es wird bewiesen, dass ihre Differenz gleich Null ist.

M. J. O. Strutt (Zürich)

7633:

Finzi, Bruno. Principio d'azione stazionaria nell'elettrodinamica dei fluidi. Ann. Mat. Pura Appl. (4) **50** (1960), 319–339.

In this paper the formal structure of the equations of relativistic fluid electrodynamics is investigated with the aid of a variational principle. The author assumes a general linear relationship between electromagnetic fields and inductions and derives both sets of Maxwell's equations, using the very interesting method proposed by him in 1952 [Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **12** (1952), 378–382, 477–480; MR **15**, 665]. The variation of the total action of the field and matter (viscosity is neglected, but the conductivity is finite) with respect to the metric tensor yields, according to the Hilbert method, the energy momentum tensor and the equations of motion (this result can also be found, essentially, in W. Gordon, Ann. der Physik, **72** (1923), 421). The paper is a model of cartesian clarity and can be read with enjoyment. The reviewer would like to point out that, contrary to what is stated at the end of paragraph 6, the Euler-Lagrange equations of a variational principle are not necessarily compatible (a trivial example: if $L(x, y, y') \equiv y$, $Ly - d\,Ly'/dx \equiv 1$).

B. Bertotti (Princeton, N.J.)

7634:

Vlasov, K. B.; Išmuhametov, B. H. Rotation of the plane of polarization of elastic waves in magnetically polarized magnetoelastic media. Ž. Èksper. Teoret. Fiz. **37** (1959), 745–749 (Russian); translated as Soviet Physics JETP **10** (1960), 531–534.

It is shown that a longitudinal and two circularly polarized magnetoelastic waves can propagate, all with different velocities, in the direction of magnetization in a uniformly magnetized elastic medium. The treatment is based on phenomenological "equations of state" including the effects of elastic moduli, magnetostrictive constants, inverse magnetic susceptibilities, and exchange interactions. Appreciable rotation of the plane of polarization of a transverse wave in a ferromagnet can be expected at frequencies above 10^9 sec^{-1}.

G. E. Backus (La Jolla, Calif.)

7635:

Thomas, T. S. E. Screening effect of a circular disk. Amer. J. Phys. **29** (1961), 37–39.

Author's summary: "A theory of the effect of a circular metal disk in screening the electric field of a pole and the magnetic field of a radio-frequency dipole (eddy current screening) on the axis is developed in terms of spheroidal functions. The screening effectiveness is defined as the

ratio of the screened to the unscreened field and curves are given showing its variation along the disk axis. In the electrical case the field is zero at a certain distance along the axis." *E. T. Copson* (St. Andrews)

7636:

Pignedoli, Antonio. Neue Untersuchungen über die Bewegung von raschen elektrisierten Teilchen in überlagerten elektrischen und magnetischen Feldern. Z. Angew. Math. Mech. **39** (1959), 461–465.

Verfasser geht von der relativistischen Bewegungsgleichung eines Elektrons aus, das sich in einem stationären, axialsymmetrischen magnetischen Feld und in einem stationären, nur von einer Koordinate entlang der Achse abhängigen elektrischen Feld befindet. Ein erstes Integral der Differentialgleichung folgt aus dem Energiesatz in relativistischer Form. Dieses erste Integral wird umgeformt und führt dann für die betrachtete Bewegung auf ein System von zwei Differentialgleichungen erster Ordnung. Dieses System wird integriert. Die Lösung wird auf den besonderen Fall angewandt, dass das magnetische Feld von einem kreisförmigen Stromleiter in einer Ebene senkrecht zur Achse erzeugt wird. *M. J. O. Strutt* (Zürich)

7637:

Zin, Giovanni. Funzioni tensoriali di un vettore e leggi elementari dell'elettromagnetismo. Ann. Mat. Pura Appl. (4) **50** (1960), 341–378.

L'auteur étudie les lois élémentaires de l'électromagnétisme et leur indétermination. Il donne un algorithme mathématique permettant d'obtenir toutes les lois élémentaires conformes à l'expérience et répondant à diverses conditions logiques.

Dans le premier chapitre, sont étudiées les propriétés générales des lois élémentaires, champ créé par un élément de courant, force et couple exercés par un élément de courant sur un autre (action électrodynamique). L'auteur donne des conditions nécessaires et suffisantes pour qu'une loi élémentaire soit équivalente à celle de Laplace. Les lois élémentaires de l'action électrodynamique sont classées et divisées en sous-classes, une sous-classe contenant les lois pour lesquelles l'action d'un circuit sur un élément de courant est la même (force et couple). La sous-classe d'Ampère comprend notamment la loi classique d'Ampère-Reynard, celle d'Ampère conforme au principe de l'action et de la réaction et la loi de Grassmann.

Le deuxième chapitre est consacré à l'étude des tenseurs du second et du troisième ordres qui interviennent dans l'indétermination des lois élémentaires dont il s'agit. Les composantes de tous ces tenseurs sont fonctions des composantes du vecteur $\mathbf{r}$ qui joint un élément de courant à un point de l'espace ou à un autre élément de courant. Les tenseurs du second ordre interviennent pour l'expression du champ magnétique élémentaire. Ceux du troisième ordre pour l'action électrodynamique élémentaire. L'auteur montre que les premiers dépendent de trois fonctions de r et les seconds de sept fonctions de la même variable.

Dans le troisième chapitre, l'auteur donne l'expression la plus générale de la loi élémentaire équivalente à celle de Laplace, loi qui dépend d'une fonction arbitraire de r.

Enfin, le quatrième chapitre est consacré à la détermination des expressions générales de lois électrodynamiques élémentaires satisfaisant à des conditions données (actions nulles d'un circuit sur un élément de circuit, actions nulles d'un circuit sur un circuit, actions élémentaires satisfaisant au principe de l'action et de la réaction, lois électrodynamiques de la sous-classe d'Ampère qui respectent ce principe).

L'exposé est clair, détaillé et de compréhension facile. Des exemples sont donnés. *L. Robin* (Paris)

7638:

Agostinelli, Cataldo. Sul moto di un elettrone veloce in un campo elettromagnetico simmetrico rispetto a un asse. Boll. Un. Mat. Ital. (3) **14** (1959), 163–174.

Verfasser geht von der relativistischen Bewegungsgleichung eines Elektrons in Vektorform in einem axialsymmetrischen magnetischen Feld aus, dem ein elektrisches Feld überlagert ist. Die magnetische Feldstärke wird von einer skalaren Potentialfunktion abgeleitet, für welche eine Wellengleichung gilt. Die elektrische Feldstärke bestimmt sich ebenfalls mit Hilfe dieser Potentialfunktion. Die drei Bewegungsdifferentialgleichungen des Elektrons in Zylinderkoordinaten werden ebenfalls mit Hilfe der genannten Funktion angeschrieben. Einige besondere Bewegungsformen werden an Hand der Bewegungsdifferentialgleichungen diskutiert. Insbesondere werden magnetische Felder betrachtet, welche sinusförmig von der Zeit abhängen. In diesen Fällen gilt für die obige Potential funktion ein Ausdruck mit Besselschen Funktionen erster Art nullter Ordnung des Radius um die Achse. Die resultierenden Elektronenbewegungen werden mit Hilfe eines ersten (Energie-) Integrals der Bewegungsgleichungen betrachtet. *M. J. O. Strutt* (Zürich)

7639:

Bouix, Maurice. Étude des sources ponctuelles électromagnétiques. Ann. Télécommun. **14** (1959), 143–150.

Verfasser geht von Lösungen der vektoriellen Wellengleichung in sphärischen Koordinaten aus, welche für den "elektrischen Feldtypus" und den "magnetischen Feldtypus" ausgeschrieben werden. Diese Lösungen werden im Koordinatennullpunkt singulär. Aus dem Studium dieser Singularität ergibt sich ein System von Oberflächenströmen auf einer kleinen Kugel um den Nullpunkt. Dieses System von Oberflächenströmen wird für eine Reihe von Spezialfällen betrachtet. Es gilt der Satz: "Das elektromagnetische Feld rührt nicht nur von elektrischen Elementarströmen her, sondern auch von den entsprechenden magnetischen Elementarströmen."

M. J. O. Strutt (Zürich)

7640:

Perucca, Eligio. Elettromagnetismo, steradianti, razionalizzazione. Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. **94** (1959/60), 237–253. (French summary)

L'auteur rappelle l'idée qu'il a exprimée en 1951, que le facteur 4π qui figure dans certaines expressions classiques, c'est-à-dire non rationalisées, de l'électromagnétisme représente la valeur exprimée en steradians d'un angle solide.

Il montre comment, si on renonce à faire appel à des notions aussi abstraites que celle d'un conducteur réduit à une ligne dans laquelle la densité de courant serait infinie,

ou celle d'un point chargé d'électricité (où la densité de charge serait infinie), on est conduit à donner des énoncés équivalent à ceux de la loi de Laplace relative au champ magnétique développé par un élément de courant, ainsi que de la loi de Coulomb relative au champ électrique d'une charge, dans lesquels apparaissent des angles solides.

Il attire l'attention sur le fait que si on admet l'angle solide comme une grandeur fondamentale au même titre que la longueur, la masse, le temps, l'intensité de courant électrique, la substitution aux formules classiques, de formules rationalisées revient à faire intervenir, au lieu de grandeurs dont l'équation aux dimensions comporte l'angle solide, des grandeurs nouvelles définies de sorte que l'angle solide ne figure pas dans leurs équations aux dimensions: ces grandeurs nouvelles sont donc, au point de vue de l'analyse dimensionnelle, d'espèce différente de celles auxquelles elles sont substituées: cela parait à l'auteur un argument de valeur à opposer à la thèse dite de la "rationalisation des unités". *L. Collet* (Paris)

7641:

Carter, G. W.; Loh, S. C. The calculation of the electric field in a sphere-gap by means of dipolar co-ordinates. Proc. Inst. Elec. Engrs. C **106** (1959), no. 9, 108–111.

A series solution for the maximum electric field between two identical conducting spheres is obtained from the (nonseparable) solution to Laplace's equation in dipolar coordinates. Numerical results are plotted for the case where the spheres have equal and opposite potentials and where one is at zero potential.

E. T. Kornhauser (Providence, R.I.)

7642:

Kapica, P. L.; Fok, V. A.; Vaĭnšteĭn, L. A. Static boundary problem for a hollow cylinder of finite length. Ž. Tehn. Fiz. **29** (1959), 1177–1187 (Russian); translated as Soviet Physics. Tech. Phys. **4** (1960), 1077–1087.

Il s'agit d'un problème d'électrostatique de valeurs données aux limites sur une portion de cylindre de révolution creux, ouvert aux deux bouts et limité par deux sections droites.

Les auteurs calculent d'abord le potentiel d'une simple couche formée d'une distribution de charges superficielles de densité donnée. Ce potentiel est développé en série de Fourier. Ils étudient ensuite le système infini d'équations intégrales linéaires du type Fredholm de première espèce, qui relient les composantes de ce potentiel aux composantes correspondantes de la densité de charge, celles-ci étant les inconnues. En représentant chaque composante du potentiel, calculé à la surface du cylindre et de la densité de charge, sous forme de séries de cosinus, le problème précédent est ramené à la résolution de systèmes infinis d'équations algébriques linéaires. Les solutions numériques de ces systèmes sont obtenues par des méthodes d'itération. Les éléments des matrices de ceux-ci sont eux-mêmes calculés par intégration d'équations différentielles linéaires du quatrième ordre ou, pour des cylindres courts, par sommation de séries. Des fonctions spéciales classiques (de Bessel et de Legendre) interviennent dans cette étude.

Les auteurs annoncent la parution prochaine d'un article donnant des résultats numériques illustrant et complétant le présent travail. *L. Robin* (Paris)

7643:

Macdonald, J. R.; Edmonson, D. E. Exact solution of a time-varying capacitance problem. Proc. IRE **49** (1961), 453–466.

Authors' summary: "By means of a new method, a closed-form solution is obtained for the harmonics generated by a sinusoidally varying capacitance in series with a fixed resistor and battery. The solution describes the behavior of the condenser microphone, the vibrating-reed electrometer, a vibrating plate contact potential measuring apparatus, and a special loudspeaker improvement. With only minor modifications the solution can also apply to the case of a sinusoidally varying resistance in series with a fixed inductance and battery; thus it may, in addition, be used to calculate the response of a carbon microphone. The present large-signal solution, which applies for any finite values of the modulation index and frequency, is compared with previous small-signal approximate results, and the dependence on modulation index and frequency is investigated for such quantities as output waveform, total harmonic distortion, harmonic amplitude and phase, and average input and output power. A very distorted waveshape is obtained for low relative frequencies and values of the modulation index near and including unity."

7644:

Collins, W. D. On the solution of some axisymmetric boundary value problems by means of integral equations. II. Further problems for a circular disc and a spherical cap. Mathematika **6** (1959), 120–133.

The method of a previous paper with the same title [Quart. J. Mech. Appl. Math. **12** (1959), 232–241; MR **21** #4743] is now shown to be applicable to axisymmetric boundary value problems in which the boundaries are composed of plane and spherical parts. The solution of such a problem is made to depend on the solution of a Volterra integral equation of the first kind, and also, in some cases, on the solution of Fredholm integral equations of the second kind. *E. T. Copson* (St. Andrews)

7645:

Sturrock, P. A. In what sense do slow waves carry negative energy? J. Appl. Phys. **31** (1960), 2052–2056.

In the small amplitude theory of electron beam tubes the concept of "small amplitude power" is frequently applied. Using a one-dimensional electron beam model it is possible to express an arbitrary excitation of an electron beam in terms of a "fast" and a "slow" wave. The "small amplitude power" of the fast wave is positive, that of the slow wave is negative. These facts led the author of this paper to search for a general criterion on the basis of which one could determine the sign of the power carried by a "slow" or "fast" wave in more general media convected at a velocity v with respect to a stationary observer. He defines "slow" and "fast" waves propagating in a one-dimensional moving medium (and observed in a stationary frame) as waves of which the phase velocity does or does not change sign, respectively, upon transforming from a frame moving with the medium to the stationary frame.

He finds the basis for such a criterion in the Lagrangian density function. In any medium of which the linearized equations of motion are derivable from a quadratic

Lagrangian function, pure waves (periodic in space and time) are shown to carry positive power, if they are "fast", and negative power, if they are "slow". This proof is accomplished under the provision that the energy density is positive for both waves in a frame moving with the medium at its convective velocity v, a condition that is generally encountered. *H. A. Haus* (Cambridge, Mass.)

7646:

Le Roux, Émile. Théorie classique de l'absorption du rayonnement par des particules. C. R. Acad. Sci. Paris **251** (1960), 1741–1743.

7647:

Phariseau, P. The diffraction of light by ultrasonic waves. Simon Stevin **33** (1959), 72–93.

The author treats the problem of the diffraction of light obliquely incident on an ultrasonic travelling wave. The same problem has been investigated by A. B. Bhatia and W. J. Noble [Proc. Roy. Soc. London, Ser. A **220** (1953), 356–368], who used an integro-differential equation. The author employs a wave equation with a periodically varying index of refraction and, assuming that the variations in the index of refraction are small, transforms it to a Mathieu equation. Perturbation theory is used to evaluate the coefficients in the solution to the Mathieu equation. The results agree with those obtained by Bhatia and Noble.

G. Sinclair (Toronto)

7648:

Williams, W. E. Diffraction of an electromagnetic plane wave by a metallic sheet. Proc. Roy. Soc. London. Ser. A **257** (1960), 413–419.

From the author's summary: "A new generalized technique is developed for the solution of the problem of the diffraction of a plane-wave incident at an oblique angle on an imperfectly conducting half plane. It is shown that the solution may be deduced directly from the known scalar solutions for the half plane. The case when the incident wave is E-polarized is considered in detail. The method of solution is applicable to the case of an H-polarized wave and also to the case when the diffracting structure consists of a finite number of parallel sheets of conducting material. The solution for an arbitrary incident wave may be obtained by superposition of the plane-wave solutions." *A. E. Heins* (Ann Arbor, Mich.)

7649:

Levy, Bertram R. Diffraction by an elliptic cylinder. J. Math. Mech. **9** (1960), 147–165.

From the author's introduction: "The construction of the field diffracted by a cylinder of arbitrary convex cross section has been carried out by J. B. Keller, who found that by introducing a new class of diffracted rays a "geometrical optics" for diffracted fields could be developed. In all cases which have been previously studied it has been found that the field constructed by the geometrical theory of diffraction agrees with an asymptotic expansion of the exact solution of the corresponding boundary value problem. In this paper the author finds the asymptotic expansion of the field diffracted by an elliptic cylinder and shows that it, too, is in agreement with the geometrical theory."

A. E. Heins (Ann Arbor, Mich.)

7650:

Millar, R. F. Scattering by a grating. I, II. Canad. J. Phys. **39** (1961), 81–103, 104–118.

Part I treats the scattering of plane waves by a transmission grating upon whose elements the wave function is assumed to vanish. The grating elements are assumed to be cylinders whose generators are parallel, but no other assumption is initially made concerning their size or shape. The main object of the paper is to present a rigorous formulation of the problem which is valid when the wavelength of the incident field is near one of the Rayleigh wavelengths.

Application of Green's theorem reduces the problem to the solution of a set of integral equations. The kernels of these equations are such that a perturbation method of solution breaks down when the number of elements is large and the exciting wavelength is near one of the Rayleigh wavelengths. The author shows that addition and subtraction to these equations of a certain series produces integral equations whose kernels are such that a perturbation method may be employed for all wavelengths. The right hand sides of these new equations consist of the original incident field and two terms corresponding to plane waves propagating tangentially, and in opposite directions, along the grating. The amplitudes of these latter plane waves are linear functions of the far field patterns, along the grating, of the individual scatterers. The solution of the integral equations is then obtained in terms of the solution with a known right hand side and the far field patterns along the grating. It is then shown that these latter quantities must satisfy a set of linear algebraic equations. The method thus requires the solution of a set of integral equations and a set of algebraic equations. In general it is very involved for a large, but finite, number of elements.

The particular case of an infinite grating of identical elements is treated in some detail; in this case periodicity considerations reduce the number of algebraic equations to two. A semi-quantitative discussion of the solution near the Rayleigh wavelengths is given for elements of arbitrary shape. A complete and detailed discussion requires the solution of an integral equation and this is only attempted for small elliptic cylinders when a perturbation technique is employed. A detailed analysis of the solution is presented in this case.

Part II consists mainly in applying the approach of Part I to the case when the normal derivative of the wave function vanishes on the grating. The case of an infinite grating of elliptic cylinders is discussed in detail. The finite grating is also briefly considered.

W. E. Williams (Liverpool)

7651:

Metz, André. La déviation des ondes lumineuses par le mouvement des milieux réfringents. C. R. Acad. Sci. Paris **251** (1960), 2132–2134.

7652:

Ryu, Norio. Multiple Rayleigh scattering. Mem. Fac. Engrg. Hiroshima Univ. 1, 185–189 (1960).

The multiple scattering of low energy gamma rays is considered in the small angle approximation, assuming that the only process responsible for deflection of the gamma ray

is Rayleigh (elastic) scattering and thus neglecting Compton scattering. A formal solution is obtained of the transport equation by applying the Fourier-Bessel transform. The transformed equation can be easily solved. The solution is evaluated for the case where the Rayleigh cross-section can be approximated by Re^{-ax}, where χ is the angle of scattering, R and a are energy-dependent constants.

H. Feshbach (Cambridge, Mass.)

7653:

Seshadri, S. R. Diffraction of a plane wave by an infinite slit in a unidirectionally conducting screen. IRE Trans **AP-9** (1961), 199–207.

Author's summary: "The scattering of a plane electromagnetic wave of wave number k, by an infinite slit (of width $2a$) formed by two unidirectionally conducting, semi-infinite coplanar screens of zero thickness, is considered. By employing an integral equation procedure, a rigorous asymptotic solution is obtained up to order $(ka)^{-3/2}$. The currents induced on the screens and the first few terms in the transmission coefficient are evaluated. The similarity between this and the corresponding problem involving perfectly conducting screens is pointed out."

7654:

Sleator, F. B. A variational solution to the problem of scalar scattering by a prolate spheroid. J. Math. and Phys. **39** (1960/61), 105–120.

When scalar plane waves are scattered by a rigid prolate spheroid, the velocity potential $\phi(P)$ at any point P outside the scatterer S is the solution of a variational problem $\delta J[\phi]=0$ where

$$J[\phi]\cdot\left[\int_S \phi(Q)\left[\frac{\partial}{\partial n}\,e^{ikz}\right]_Q dS_Q\right]^2 = \int_S\int_S \phi(Q)\phi(Q')\,\frac{\partial^2}{\partial n\,\partial n'}\,G(Q,Q')\,dS_Q\,dS_{Q'},$$

where Q, Q' are points of S, $\partial/\partial n$ is differentation along the outward normal at Q, dS_Q is the surface element at Q, and $G(P, P')$ is the free-space Green's function $e^{ik\cdot(PP')}/PP'$.

If one introduces prolate spheroidal coordinates

$$x = F\sqrt{(\xi^2-1)}\sqrt{(1-\eta^2)}\cos\varphi,$$
$$y = F\sqrt{(\xi^2-1)}\sqrt{(1-\eta^2)}\sin\varphi,$$
$$z = F\xi\eta,$$

and expands ϕ in the form $\sum_\mu A_\mu(\xi)P_\mu(\eta)$, one finds that

$$J[\phi] = \frac{\sum_{\mu\nu} A_\mu A_\nu C_{\mu\nu}}{[\sum_\nu A_\nu B_\nu]^2},$$

where $C_{\mu\nu}$, B_ν are certain definite integrals. The determination of the stationary values of $J[\phi]$ is thus reduced to an algebraic problem. The back-scattering cross-section is then

$$\sigma = 4\pi|J_0|^{-2},$$

where J_0 is a stationary value.

Numerical results for the nose-on back-scattering cross-section of a particular spheroid at a single wavelength are obtained by truncating the system of linear equations given by $\delta J=0$. The fifth order result agrees with the known exact solution within about 0.3%.

E. T. Copson (St. Andrews)

7655:

Collin, Robert E. ★Field theory of guided waves. International Series in Pure and Applied Physics. McGraw-Hill Book Co., Inc., New York-Toronto-London, 1960. xiii + 606 pp. $16.50.

As stated in the preface, the book is an attempt to bridge the gap between the usual senior-first year graduate treatment of guided electromagnetic waves and the more advanced topics which are of prime importance to the research worker in the field. By and large, the attempt is highly successful in the opinion of this reviewer. Furthermore, the book should become a valuable reference in the field of microwave theory. The presentation is largely self-contained and the perceptive reader needs little background other than elementary concepts of analysis. The emphasis is on field solutions although the final results are often interpreted in terms of equivalent circuit elements. Thus microwave circuit theory is placed in its proper perspective.

The plan of the book runs as follows. Chapter 1 contains a survey of classical electromagnetic theory which is well written and places the reader in the proper frame of mind. Chapter 2 is a concise and readable discussion of scalar, vector and dyadic Green's functions. Chapter 3 treats TEM (transverse electric waves) in isotropic and anisotropic media and chapter 4 contains an exposition of TEM transmission theory. In this latter chapter, conformal-mapping techniques and variational methods are introduced.

Chapters 5 to 9 treat propagation in hollow cylindrical waveguides and in rectangular waveguides, filled with dielectric and ferrite slabs. Discontinuity effects and aperture coupling are also discussed.

Chapter 10 is a very good introduction to integral transform and function-theoretic techniques for the solution of two-part boundary-value problems. Applications to parallel plate media, bifurcated guides, and diaphragms in rectangular guides are given as examples.

In chapter 11 an introduction is given to problems which yield continuous eigenvalue spectra. Applications are to surface waves and other open structures such as a grounded dielectric slab. Finally, the theory of artificial dielectrics is exposed in the final chapter. This contains a good survey of the author's own contributions to this particular topic.

Added features are an extensive selection of problems and a mathematical appendix.

J. R. Wait (Boulder, Colo.)

7656:

Geszti, P. O. The effect of axial conductance in the corona envelope upon the inductance of the conductor in the presence of a travelling wave. Acta Tech. Acad. Sci. Hungar. **31** (1960), 19–24. (German, French, and Russian summaries)

Author's summary: "In the case of travelling waves of great current intensity and if the diameter of the conductor is small, the direction of conductance in the corona envelope will not be purely radial, as a result of the magnetic influence of the current in the conductor, but would—though to a small extent—contain an axial component as well. This may effect the resultant inductivity and thus the speed of the travelling as well as the distortion of the wave and the magnetic measurements."

7657:

Alfandari, Roger; Pauchard, Robert. Propagation des ondes du type transversal électrique dans un guide d'ondes rectangulaire rempli de ferrite et magnétisé transversalement. C. R. Acad. Sci. Paris **251** (1960), 1738–1740.

Authors' summary: "L'application d'un champ magnétique transversal perpendiculaire au plan de polarisation d'une onde guidée en milieu ferrimagnétique transforme une onde TE_{n0} incidente en onde TE_{0p}. L'existence de ces derniers modes est vérifiée expérimentalement."

7658:

Peterson, W. C.; LaRue, J. J. The node system of equations. J. Franklin Inst. **270** (1960), 175–189.

This paper follows the recent fashion of analyzing an electrical network as if it were a graph. At first, concepts from a 0-network are introduced into a 1-network energized with currents and voltages. Then an elaborate computational mechanism is established to eliminate the destructive consequences of the 0-network concepts, which should not have been introduced in the first place.

G. Kron (Schenectady, N.Y.)

7659:

Braier, Alfred; Rosman, Hugo. Contributions à l'étude générale des circuits déphaseurs. Bul. Inst. Politehn. Iaşi (N.S.) **5 (9)** (1959), no. 1–2, 291–302. (Romanian. Russian and French summaries)

7660:

Haus, Hermann A.; Adler, Richard B. ★Circuit theory of linear noisy networks. Technology Press Research Monographs. The Technology Press of M.I.T., Cambridge, Mass; John Wiley & Sons, Inc., New York; Chapman and Hall, Ltd., London; 1959. xii+79 pp. $4.50.

This book, second of the Technology Press Research Monographs, gives a broad and comprehensive representation of linear passive networks containing uncorrelated noise sources. Consideration is confined to "single frequency" or "spot noise" performance, and the culmination is optimal imbedment of amplifiers in passive networks, i.e., for minimum spot noise measure. After definition of exchangeable power from a given source as stationary value, in generalization of the classical concept of available power, imbedment in lossless, in general non-reciprocal, networks is treated and reduced to an eigenvalue problem associated with the characteristic-noise matrix. The eigenvalues are the power invariants under lossless imbedding and lead to canonical networks; the associated permissible regions are connected with the signature character of the impedance matrix. Generalization to other than impedance representation, for instance, scattering matrix, follows and leads to the definition of the general characteristic-noise matrix. This forms the basis for the general concepts of extended gain and noise measures for n-ports with internal uncorrelated sources. A complete classification based on the signs of the relevant characteristic quantities is given and the associated eigenvalue or noise measure ranges are established. Imbedment is generalized to passive networks and leads to the basic limit theorem on the noise measure of a set of amplifier two-ports. The final chapter deals with network realization of optimum amplifier noise performance. There are three canonical types, based on signature or range of "unilateral gain", the first of which comprises the majority of practical cases. The second category can be reduced to the first by non-reciprocal transformation, but the third one, the "negative-resistance" type, is disjoint. Network optimization of all three is treated in general, with some specific examples, classical and novel, including a canonical circulator circuit for the negative-resistance amplifier.—The concepts and methods developed here have found wide acceptance. The writing is generally clear and succinct, though one notes a certain imbalance, with a tendency to detail including elementary steps in the first part and undue brevity towards the end. Illustrations and printing are excellent and misprints almost totally absent. *H. G. Baerwald* (Albuquerque, N.M.)

7661:

Pease, M. C. The quadratic invariances of a generalized network. Proc. IRE **49** (1961), 488–497.

Author's summary: "The Manley-Rowe relation, as applied in the small signal linearized approximation, may be stated as a quadratic form that is invariant under the operation of the system. It is, however, only one of the set of such forms that is invariant through a given type of system.

"It is shown that the existence of quadratic invariances is a consequence of the eigenvalues of the system operator being either of unit magnitude or else grouped in pairs such that one is the conjugate reciprocal of the other. If this condition applies, then there exists at least n such linearly independent forms, where n is the number of degrees of freedom of the system. Each form then specifies a quantity that is conserved by the system.

"Methods of determining the quadratic invariant forms from the matrix operation of the system are developed. Application is made to certain simple two-port networks to illustrate the analysis and the significance of the resulting invariances. Parametric circuits are also studied. The Manley-Rowe relation is found, as expected. Other relations, applicable to subclasses of such networks are also found.

"Finally, application is made to a lossy parametric shunt element, such as an imperfect nonlinear capacity. The quadratic invariances for such a device, for the two-frequency case, are derived."

7662:

Weiss, Gerald. A network theory for carrier-suppressed modulated systems. IRE Trans **AC-6** (1961), 54–66.

Author's summary: "The performance of linear networks in the presence of carrier-suppressed modulation is re-examined in the light of the latest advances in theory. Both analysis and synthesis methods are presented."

7663:

Langefors, B. Algebraic topology applied to network problems. Svenska Aeroplan A. B. Tech-Note TN 43 (1959), 37 pp.

The author states that "It is the aim of the present paper to give an introduction to the basic concepts of algebraic topology, using electric networks for illustration." The paper defines the homology and cohomology

groups of a simplicial complex and discusses the exactness of the homology and cohomology sequence. The treatment is very elementary and, in places, not too precise.

J. B. Giever (University Park, N.M.)

7664:

Burge, Edward J. Definitions of resonance and exact conditions for resonance in some electrical circuits. I. Definitions of resonance for series and parallel LCR circuits. Amer. J. Phys. **29** (1961), 19–23.

The author states and compares six definitions of resonance for series and three for parallel LCR circuits. The conditions for obtaining resonance according to each definition by variation of any one of four parameters are obtained and tabulated. This is a useful summary of elementary results and should promote clarity in the use of resonance concepts. There is a discussion of the relative practical importance of the various definitions and of the magnitude of the differences between the resulting conditions for resonance. In Table I, $Lp-1/Cp$ should be $|Lp-1/Cp|$.

R. Cohn (New Brunswick, N.J.)

7665:

Poincelot, Paul. Théorie du câble coaxial à charge uniformément répartie. C. R. Acad. Sci. Paris **251** (1960), 1736–1737.

The propagation constant of a multi-layer coaxial cable with an intermediate ferrite layer, separated by two dielectric layers from the core and outer sheet made of the same metal, is computed by straightforward procedure, nulling the determinant of the system of linear boundary conditions. The moduli of the cylinder functions involved are large or small compared to one, which makes approximative explicit evaluation practical.

H. G. Baerwald (Albuquerque, N.M)

CLASSICAL THERMODYNAMICS, HEAT TRANSFER

See also 7312, 7313, 7465, 7881.

7666:

Ackeret, Jakob. The role of entropy in the aerospace sciences. J. Aerospace Sci. **28** (1961), 81–95, 102.

7667:

Turner, Louis A. Comments on Buchdahl's treatment of thermodynamics. Amer. J. Phys. **29** (1961), 40–44.

The author points out that an ancillary postulate introduced in a paper of the reviewer's [Z. Physik **152** (1958), 425–439; MR **20** #4418], viz., that there exist no mutually inaccessible pairs of states of an adiabatically isolated system, is not justified if systems possessing internal adiabatic partitions be contemplated. (He adds correctly in an appendix that the arguments there were restricted to "simple systems", in which case internal adiabatic partitions are disallowed by definition.) Non-simple systems are considered, and an argument is advanced intended to show that the zeroth law is contained in the second, i.e., in the Principle of Carathéodory. {It appears to the reviewer however that a definition and discussion of the properties of the empirical entropy of non-simple systems have not been provided, the argument relying on the principle of increase of entropy of non-simple systems.}

H. A. Buchdahl (Hobart)

7668:

Bernstein, B. Proof of Carathéodory's local theorem and its global application to thermostatics. J. Mathematical Phys. **1** (1960), 222–224.

A more precise statement is given of Carathéodory's theorem on the integrability of linear differential forms, and this new theorem, called "Carathéodory's local theorem" by the author, is proved. A discussion of "global applications" of the theorem follows, but this does not seem to contain anything new.

H. A. Buchdahl (Hobart)

7669:

Kvasnica, J. A remark on the third law of thermodynamics. Czechoslovak J. Phys. **10** (1960), 883–886. (Russian summary)

In analogy with the Nernst-Planck theorem and Falk's formulation of the third law of thermodynamics [Phys. Rev. (2) **115** (1959), 249–253], the author shows that the latter may be replaced by the following: "The (absolute) minimum of energy of a thermodynamic system is unattainable."

H. A. Buchdahl (Hobart)

7670:

Turcotte, Donald L. The melting of ice in a hot humid stream of air. J. Fluid Mech. **8** (1960), 123–129. (1 plate)

The author considers the rate of melting of a hemisphere of ice at the stagnation point in the flow of incompressible, warm, humid air past the hemisphere. He assumes that the evaporation from and tangential velocity of the outer edge of the water layer are too small to affect the boundary layer in air, so that Schlichting's solution for the stagnation point boundary layer is applicable. He also assumes than the tangential heat flux in the water layer is negligible. Then the equations for enthalpy and water vapor concentration reduce to an equation solved numerically by Sibulkin. Assuming that the concentration of water vapor in the air just outside the water layer is the equilibrium concentration appropriate to the local temperature and pressure, the author obtains a numerical value for the rate of ablation of the hemisphere at its stagnation point. He has performed wind tunnel experiments and obtained very good agreement with his theory.

G. E. Backus (La Jolla, Calif.)

7671:

Okano, Kōji. Odd variables and thermodynamics. Sci. Papers Inst. Phys. Chem. Res. **54** (1960), 1–5.

Author's summary: "Based upon the invariance character of the free energy with the reversal of the sense of time, the possibility of occurrence of linear couplings between even and odd variables are discussed."

7672:

Wanner, Marcel. Équilibre adiabatique ou monothermique d'un mélange de gaz soumis, dans un récipient de volume constant, à un champ de forces extérieures

massiques dérivant d'un potentiel indépendant du temps. C. R. Acad. Sci. Paris **251** (1960), 216–218.

L'auteur développe une théorie dont il donne comme conséquence un procédé de séparation partielle des constituants d'un mélange initialement homogène de gaz, sans l'aide de parois semi-perméables. *R. Gerber* (Grenoble)

7673:

Roberts, J. K. ★Heat and thermodynamics. 5th edition revised by A. R. Miller. Interscience Publishers, Inc., New York, 1960. xx + 619 pp. $7.25.

This book was first published in 1928 [Blackie, London]. Its purpose may be seen from the preface to the first edition: "The book has been written at the request of the publishers for the students in Physics. The book in general has been written from the experimental point of view . . ." It gives the subject of both phenomenological and statistical studies of heat—thermometry, calorimetry, *P-V-T* relations, change of state, thermodynamics, and heat transfer—an adequate introduction and survey. It is not an advanced text, and requires no mathematics beyond advanced calculus. Important experiments are discussed with clarity. Theories which require higher mathematics are summarized or mentioned with proper references. The authors, in the reviewer's opinion, have achieved their aim very attractively.

The coverage is quite substantial for a book of this size. This may be seen from the chapter headings: Temperature, Quantity of Heat, The Kinetic Theory of Gases, Equation of State for Gases, The Production and Measurement of Low Temperatures, The Specific Heats of Gases, The Specific Heats of Solids and Liquids, Vaporization, Fusion, Thermal Expansion, The Transfer of Heat by Conduction and Convection, The Second Law of Thermodynamics, Thermodynamic Relations and their Use, Power Cycles, The Principle of the Increase of Entropy, The Conditions of Equilibrium of a Physical or Chemical System, Transformations of Higher Order, Chemical Equilibria, The Third Law of Thermodynamics, Radiation, Planck's Radiation Formula, The Theory of the Specific Heats of Solids, The Equation of State for Solids.

The present edition is a minor revision of the fourth edition (1951) with rewritings on discussions of the temperature scale and principle of corresponding state. Some new experimental methods are added and some old ones are condensed. New references to engineering texts and literature are added. *L. N. Tao* (Chicago, Ill.)

7674:

Wolff, T. δ-Funktion von Dirac in der Wärmeleitungstheorie. Z. Angew. Math. Mech. **40** (1960), 421–422.

Manipulations with delta symbols are employed to give brief derivations of the formulas for temperatures $u(x, y, z, t)$ in an unbounded region with distributed sources and prescribed initial temperatures, and in a rectangular parallelepiped whose boundary is kept at temperature zero. *R. V. Churchill* (Ann Arbor, Mich.)

7675:

Vuorelainen, Olavi. The temperature field produced in the ground by a heated slab laid direct on ground, and the heat flow from slab to ground. Acta Polytech. Scandinav. No. 278 (1960), 60 pp.

The paper considers a number of heat conduction problems involving specification of temperature and/or heat flux distributions over areas on the plane bounding a semi-infinite solid (the earth). These distributions correspond to conditions met with in practice in cold climates under rectangular or circular buildings or very long rectangular buildings (terraced houses). Steady state (roughly winter conditions with insulation due to snow fall) and time dependent (seasonal fluctuations) problems are treated.

The methods involve the use of single and double Fourier transforms and Hankel transforms and, in some cases, the solution of dual integral equations. The paper would make easier reading if some of the mathematical content had been included as appendixes. *J. F. Clarke* (Cranfield)

7676:

Calvet, Pierre; Donnadieu, Gérard. Une méthode de mesure des coefficients de convection dans un lit poreux. C. R. Acad. Sci. Paris **250** (1960), 2324–2326.

Un cylindre rempli de petites billes de verre est parcouru par un écoulement gazeux. L'Auteur décrit une méthode de mesure du coefficient de convection, en régime transitoire, qui ne nécessite que l'enregistrement d'une température de fluide. Des résultats expérimentaux sont donnés pour des billes sphériques de 400μ de diamètre et une porosité de 0,39. Ils sont représentés par la loi $N_u = 0{,}0128\ R_e^{1{,}75}$ ($0{,}5 < R_e < 20$; les nombres de Nusselt et de Reynolds sont rapportés aux conditions à vide).

R. Gerber (Grenoble)

7677:

Calvet, Pierre. Étude, en régime non permanent, d'un cas de convection ou de diffusion forcée unidimensionnelle. C. R. Acad. Sci. Paris **251** (1960), 846–848.

L'Auteur étudie les équations du transfert, en régime non permanent, entre un écoulement fluide et un lit poreux. Les équations se réduisent au système: $\partial T/\partial\xi = \theta - T$, $\partial\theta/\partial\tau = T - \theta$ qui est résolu pour une température d'entrée, et une température initiale, données, respectivement fonction du temps et de l'abscisse. Les solutions correspondant à un apport instantané de chaleur ont été utilisées expérimentalement par l'Auteur pour la détermination du coefficient de convection dans un lit poreux [voir l'analyse ci-dessus]. *R. Gerber* (Grenoble)

7678:

Tirskiĭ, G. A. Surface melting of a semiinfinite body in plane and axially symmetric incompressible gas flow. Dokl. Akad. Nauk SSSR **132** (1960), 785–788 (Russian); translated as Soviet Physics. Dokl. **5**, 501–504.

The Navier-Stokes and heat flow equations for incompressible fluids are re-arranged, in terms of a set of suitable dimensionless quantities, to study steady state stagnation point melting problems; (i) for a plane semi-infinite body, (ii) for an axially symmetric body. In the latter case the body may have an angular velocity about the axis of symmetry. Case (i) reduces to a non-linear set of ordinary differential equations of the 12th order, case (ii) is of 16th order.

Proposals for solutions of these equations are made, but no solutions are given. *J. F. Clarke* (Cranfield)

7679:

Bergles, Arthur E.; Kaye, Joseph. Solutions to the heat-conduction equation with time-dependent boundary conditions. J. Aerospace Sci. **28** (1961), 251–252.

7680:

Kiselev, K. A.; Lazarev, A. I. Temperature gradient of an infinite plate with variable heat loss coefficients in a medium of varying temperatures. Ž. Tehn. Fiz. **30** (1960), 616–621 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 579–584.

The problem of heat conduction in an infinite plate of uniform thickness subjected to an initial temperature distribution is studied. The expression for the temperature distribution is given in the form of integrals. Approximate expressions for the temperature distribution for small values of time are also obtained.

S. K. Chakrabarty (Howrah)

7681:

Krishnan, K. S.; Sundaram, R. The distribution of temperature along electrically heated tubes and coils. I. Theoretical. Proc. Roy. Soc. London. Ser. A **257** (1960), 302–315.

A differential equation is formulated and approximately solved for the temperature distribution in a hollow tube. The heat transport by radiation in the hollow core is shown to be approximately equivalent to an extra thermal conductivity, proportional to T^3—a result already known in principle. Various special cases are considered.

J. M. Ziman (Cambridge, England)

7682:

Minyatov, A. V. The heating of an infinite cylinder enclosed in a sheath. Ž. Tehn. Fiz. **30** (1960), 611–615 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 575–578.

An infinite circular cylinder enclosed by a thin sheath is imbedded in a medium of constant temperature t_e. Both the cylinder and sheath are initially at a constant temperature t_0 ($t_0 \neq t_e$). This unsteady, one-dimensional, linear heat conduction problem is solved by Laplace transform. Also, for a sheath of high conductivity it is further assumed that the temperature of the sheath may be taken to be constant. An approximate solution is similarly found. Numerical values of the first six eigenvalues and coefficients of the series solution are included. *L. N. Tao* (Chicago, Ill.)

7683:

Redozubov, D. V. The solution of linear thermal problems with a uniformly moving boundary in a semi-infinite region. Ž. Tehn. Fiz. **30** (1960), 606–610 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 570–574.

Consider $U_t + \nu U = a^2 U_{xx}$ ($\beta t < x < \infty$) subject to conditions $-\alpha U_x + \nu U|_{x=\beta t} = \Psi(t)$. The author shows that the general solution is expressible in the form of a Riemann-Mellin integral in the complex domain. The integration is then carried out for some specific problems.

L. N. Tao (Chicago, Ill.)

7684:

Jarre, Gianni. Le analogie fra scambi simultanei di quantità di moto, di calore e di massa. I. Gas omogenei. Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. **94** (1959/60), 432–444.

Author's summary: "In questa ricerca si deducono le analogie fra i campi di velocità, di temperatura, di concentrazione e fra gli scambi di quantità di moti, di calore, di massa, per correnti veloci di miscele gas-vapore che lambiscono superfici sulle quali il vapore può subire cambiamenti di stato.

"Nella I parte si richiamano i risultati noti per gas omogenei, soggetti a soli scambi di quantità di moto e di calore. Tali risultati sono rielaborati ed interpretati in forma adatta alla successiva estensione a miscele gas-vapore."

7685:

Nazarčuk, M. M.; Pol'skiĭ, N. I. Asymptotic behavior of solutions in heat transfer. Dokl. Akad. Nauk SSSR **129** (1959), 759–761 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1227–1229.

Un récipient contient un fluide conducteur. Des particules sont réparties en n groupes, les caractéristiques thermiques étant les mêmes à l'intérieur d'un groupe. A l'instant $t=0$, le fluide est à la température $\theta=0$, les températures x_i ($i \leq 1 \leq n$) des particules des n groupes sont $x_i=1$, et les particules sont plongées dans le fluide.

L'Auteur étudie l'évolution des températures x_i due au transfert. Ce problème conduit à un système de n équations différentielles, linéaires du premier ordre. La solution générale de ce système est formée et son allure asymptotique est étudiée. *R. Gerber* (Grenoble)

7686:

Fraeijs de Veubeke, B. La méthode des fonctions propres dans les problèmes de transmission de chaleur en regime transitoire. Bull. Soc. Roy. Sci. Liège **29** (1960), 173–195.

The temperature in a solid bounded by a closed surface A satisfies a differential equation

$$\operatorname{div}(K \operatorname{grad} T) - c\dot{T} = -Q_v,$$

where $K(x, y, z)$ is the thermal conductivity, $c(x, y, z)$ the heat capacity per unit volume, $Q_v(x, y, z, t)$ the source function, and a boundary condition $K\,\partial T/\partial n + hT = Q_a$ on A, where Q_a measures the distribution of sources on A.

If there are no sources, the solution is of the form

$$T = \sum_1^\infty u_r(0) \exp(-p_r^2 t) U_r(x, y, z),$$

where the constants p_r^2 are the eigenvalues of

$$\operatorname{div}(K \operatorname{grad} U) + \lambda c U = 0$$

under the condition $K\,\partial U/\partial n + hU = 0$. It is assumed that the eigenvalues are discrete and distinct. It is shown that (except in the case $h=0$ when $\lambda=0$ is an eigenvalue), the corresponding eigenfunctions are orthogonal.

In the non-homogeneous case when there are sources within or on A, it is necessary to add a particular integral $T_q(x, y, z, t)$ which is expressed in the form

$$\sum_1^\infty v_s(t) U_r(x, y, z).$$

The results are applied to an infinite plate heated by an infinite number of parallel equidistant wires embedded in the plate. *E. T. Copson* (St. Andrews)

7687:

Berlad, A. L.; Yang, C. H. A theory of flame extinction limits. Combustion and Flame **4** (1960), 325–333.

Authors' summary: "A theory of flame extinction limits is formulated, which includes consideration of heat losses by conduction and radiation at every point of the temperature profile of the flame. The proper selection of boundary conditions is examined. In addition to the prescription of characteristic flame burning velocities (as in the adiabatic theory) this theory predicts flame extinction limits. Calculation of extinction limits are made for propane–oxygen–nitrogen flames."

7688:

Lovachev, L. A. The theory of flame propagation in branched and unbranched chain systems. Combustion and Flame **4** (1960), 357–367.

Author's summary: "An approximate method for the solution of a set of energy conservation and active centre diffusion equations, describing steady-state flame propagation by an unbranched one-centre chain reaction, is proposed. Calculated values for burning velocities and the distribution of active centres as a function of temperature are in agreement with those obtained by numerical integration. An approximate solution for the problem of flame propagation in an unbranched chain system involving two active centres is given. Theoretical burning velocity values for chlorine-hydrogen flames calculated from approximate formulae show a good fit with experimental results. Approximate formulae for burning velocities in unbranched chain systems are derived. The application of the formulae for the determination of elementary branched chain reaction rate constants from burning velocities as a function of temperature is illustrated by hydrogen flames."

7689:

Chen, Tze-Ning; Toong, Tau-Yi. Structure and propagation of laminar flames near a heat sink. Combustion and Flame **4** (1960), 313–323.

Authors' summary: "This paper presents the results obtained in a theoretical investigation of the problem of laminar flame propagation near a heat sink, the temperature of which may be either high or low as far as the chemical reaction rate is concerned. The results presented consist of two parts: first, effects on the burning velocity of an adiabatic laminar flame due to a change in the temperature of the unburnt mixture; and secondly, effects on the structure and the burning velocity of a laminar flame when the rate of heat loss per unit volume inside the flame in the direction normal to the mixture velocity and the rate of heat loss per unit area at the hot boundary in the direction parallel to the flow are prescribed by arbitrary functions. In all cases, the burning velocity is found to vary approximately with the temperature of the combustion product at the hot boundary according to an expression of the following form $S_u \sim \exp(-T_c/T_b)$ where S_u is the burning velocity, T_b is the temperature of the combustion product, and T_c is a characteristic temperature, the value of which depends on the specific case under consideration. Some of these observations have also been found to agree with the experimental results. For the special case where heat loss occurs only at the hot boundary in the direction parallel to the flow, there is a lower limit for the temperature of the combustion product, below which no steady-state solution for the burning velocity can be found. This limit is in addition to the one deduced from stability considerations."

7690:

Kinbara, Tosiro; Akita, Kazuo. An approximate solution of the equation for self-ignition. Combustion and Flame **4** (1960), 173–180.

Authors' summary: "A method is described for obtaining an approximate solution of the equation for self-ignition for the case when temperature increases with time. The method has been used in investigating the self-ignition of spherical sawdust heated in a uniform temperature field for several ambient temperatures and various radii. Good agreement is reported between theory and observation for the qualitative behaviour of temperature rise with time but results fell short of expectations with respect to the magnitude of the ignition time, which may be due to consumption of material by chemical reaction for which no allowance was made."

7691:

Menkes, J. A contribution to the theory of laminar flames with radial symmetry. Combustion and Flame **4** (1960), 1–7.

Author's summary: "The differential equations of motion of a flame with radial symmetry are transformed into an integral equation. The integral equation is solved numerically by a method of successive approximations. The dependence of the mass flow per unit area on the flame thickness and flame radius as well as on the temperature of the unreacted gas is investigated. It is demonstrated that the rate of change of the mass flow is determined by any two of the three parameters. The results of the calculations are presented graphically. The implications of the findings for the stability of combustion waves are discussed."

7692:

Eisen, C. L.; Gross, R. A.; Rivlin, T. J. Theoretical calculations in gaseous detonation. Combustion and Flame **4** (1960), 137–147.

Authors' summary: "Extensive Chapman-Jouquet, strong and weak detonation properties have been obtained for a five atom (C, H, O, N, A) system. A digital computer programme has been developed which determines detonation properties in steady one-dimensional equilibrium flow. The equations and method of solution are set forth. Equilibrium and frozen sound speeds are calculated for the hot product mixtures. The effect of initial conditions and comparisons with experiment are given."

7693:

Spalding, D. B. The theory of steady laminar spherical flame propagation: equations and numerical solution. Combustion and Flame **4** (1960), 51–58.

Author's summary: "A combustible gas is supposed to

emerge steadily from a point source immersed in an atmosphere of adiabatic combustion products. The differential energy equation is solved numerically, and it is shown that the effective radius of the spherical flame is larger than that predicted from the assumptions that the flame is very thin and that it has the same burning rate per unit area as a plane flame. The error increases as the rate of injection and the laminar flame speed of the mixture decrease. The thermal conductivity of the gas is assumed constant in the numerical analysis, but it is shown how the resulting data can be used in cases for which conductivity depends upon temperature."

7694:

Westenberg, A. A.; Favin, S. The theory of a spherical premixed laminar flame. Combustion and Flame **4** (1960), 161–171.

Authors' summary: "The general equations governing the behaviour of a low velocity, steady state, premixed, laminar flame are given for the case of spherical symmetry, and the differences from the plane flame pointed out. Numerical solutions obtained on the IBM-650 computer for the simple $A \rightarrow B$ flame with Lewis number unity are discussed. Two innovations introduced to this type of calculation are: (a) the integrations were performed by starting at the cold instead of the hot boundary, which was simpler in this case, and (b) the flameholder temperature was regarded as the eigenvalue instead of the mass flow rate. The definition of burning velocity is discussed, and its dependence on radius of curvature is clearly shown by the results. The relation between flameholder heat loss and stability is examined in the light of a recent discovery by Hirschfelder and co-workers. The advantages of the spherical flame for flame structure studies are described."

7695:

Brown, E. A., Jr. Explosive decompression of water. Proc. 1960 Heat Transfer Fluid Mech. Inst., pp. 135–149. Stanford Univ. Press, Stanford, Calif., 1960.

From the author's summary: "The problem of explosive decompression of water was considered theoretically in the simplified one-dimensional form, and the results were compared with preliminary shock tube measurements. A thermodynamic equilibrium type expansion was considered. This led to the usual centered rarefaction wave with an isentropic change of state. An order of magnitude argument was used to show why such a process was unlikely to occur. A two step expansion process was analyzed wherein the liquid expands into a metastable state then relaxes into an equilibrium two phase mixture. The limiting case of zero entropy change was considered in detail. This resulted in a maximum value for the metastable liquid pressure and a description of the pressure-time history in the shock tube."

QUANTUM MECHANICS

See also A6715, 7351, 7439, 7457, 7638, 7185, 7821.

7696:

Brittin, Wesley E.; Downs, B. W. (Editors). ★Lectures in theoretical physics. Vol. II. Lectures delivered at the Summer Institute for Theoretical Physics, University of Colorado, Boulder, 1959. Interscience Publishers, New York-London, 1960. vii + 483 pp. $9.00.

The syllabi of the lectures that are condensed in this book have very different length and style but are vaguely related by their topics. Salam's and Sakurai's articles deal with the symmetry laws and invariance properties that govern the behavior of elementary particles. A substantial review of the many body problem in quantum mechanics is given by Hugenholtz. Dresden surveys modern developments in abstract quantum field theory. The rest of the lectures deal with more specific topics. Rohrlich gives an account on modern aspects for classical electron theory. Hypernuclei and the Λ-nucleon interaction are discussed by Downs. The present status of our knowledge on beta decay is summarized by Jensen. The theory of masers is covered by Lamb's article.

Although several of the articles are essentially rough lecture notes and even minor errors do occur, this collection will be a useful aid for many graduate students and also for advanced research workers who wish to enlarge their knowledge in these fields and get acquainted with relatively recent developments. [Reviews of 7 individual articles will be published separately].

P. Roman (Boston, Mass.)

7697:

Wigner, Eugene P. Phenomenological distinction between unitary and antiunitary symmetry operators. J. Mathematical Phys. **1** (1960), 414–416.

The physical predictions of a quantum theory may be stated in terms of transition probabilities $|(\phi, \psi)|^2$, ϕ and ψ being vectors in the Hilbert space of states. The present note shows how unitary and anti-unitary symmetry operators can be distinguished by their effects on transition probabilities. Consider two orthogonal states φ_1 and φ_2, both invariant under a symmetry operator T. The state vectors whose transition probability is $\frac{1}{2}$ into these states are of the form $\psi_\alpha = 2^{-1/2}(\varphi_1 + e^{i\alpha}\varphi_2)$. If T is unitary $|(\psi_\alpha, T\psi_\alpha)|^2$ is independent of α, while if T is anti-unitary $|(\psi_\alpha, T\psi_\alpha)|^2 = \frac{1}{2}(1 + \cos 2\alpha)$. This criterion is generalized to the case of n orthogonal invariant states.

A. S. Wightman (Princeton, N.J.)

7698:

Broyles, A. A. Derivation of the Fermi-Thomas equation. Amer. J. Phys. **29** (1961), 81–83.

Author's summary: "A derivation of the Fermi-Thomas equation for a neutral atom is presented to show it as an approximation to the usual Hartree and WKB approximations."

7699:

Enz, C. P.; Thellung, A. Nullpunktsenergie und Anordnung nicht vertauschbarer Faktoren im Hamiltonoperator. Helv. Phys. Acta **33** (1960), 839–848. (English summary)

A heuristic discussion of the mathematical and physical significance of the zero-point energy for a hamiltonian given as a function of canonical variables p and q. The authors' thesis is that measurable effects, in the cases examined, are unaffected by the order of the factors, although the resulting hamiltonian is non-unique and the zero-point energy can have physical significance.

I. E. Segal (Cambridge, Mass.)

7700:

Fényes, I. Über das Verhältnis des wellenmechanischen Energieeigenwertproblems zur klassischen Mechanik. Acta Phys. Acad. Sci. Hungar. **9** (1958/59), 245–259. (Russian summary)

Following up his earlier investigation of the limiting process $\hbar \to 0$ leading from wavemechanics to classical mechanics in the WKB method [same Acta **4** (1954), 133–147; MR **19**, 651], the author investigates the case of stationary states for which the limiting process is not trivial, since these states have no classical analogue.

L. Tisza (Cambridge, Mass.)

7701:

Fišer, Ya. [Fischer, Jan]; Čulli, S. [Ciulli, S.] Recurrent construction of angular operators. Ž. Èksper. Teoret. Fiz. **38** (1960), 1740–1750 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1256–1262.

The authors propose an elementary recursive method for finding the angular momentum operators for any process in terms of known operators for simpler processes. The method avoids the use of Clebsch-Gordon and Racah coefficients. Angular momentum operators are defined by

$$\mathcal{J}(J, i, f) = \sum_{M=-J}^{M=J} |J, M, i\rangle\langle f, J, M|,$$

where i, f are other quantum numbers. The problem is to find the complete set of operators $\mathcal{J}$ of a complicated process if we know the complete set of operators L for a simpler process. For example, L might be the process $s_1 + s_2 + \cdots \to s_1' + s_2' + \cdots$ and $\mathcal{J}$ the process $f_1 + s_2 + \cdots \to f_1' + s_2' + \cdots$, where f means a particle with spin $\frac{1}{2}$ and s a particle with spin 0. This example is treated in detail, and it is shown that the results for $\pi + N \to \pi + N$, and $\pi + N \to \pi + \pi + N$ agree with those obtained by the usual methods.

R. F. Streater (Princeton, N.J.)

7702:

Hellman, Olavi. On the Schrödinger eigenvalue problem. Mat.-Fys. Medd. Danske Vid. Selsk. **32**, no. 4, 24 pp. (1960).

Transcendental equations are set up for discrete eigenvalues of $R'' + 2r^{-1}R' - l(l+1)r^{-2}R + k(\lambda - V(r))R = 0$, where $V(r) = 0$ outside $[0, L]$; if $l = 0$, V has to be integrable within the well, but if $l \neq 0$ it has to be expressible as a power series there. For a closely-related previous paper of the author see Ann. Acad. Sci. Fenn. Ser. A. VI No. 11 (1958) [MR **20** #7525].

F. V. Atkinson (Toronto)

7703:

Barut, A. O.; Leiser, M. Note on gauge transformations in quantum mechanics. Amer. J. Phys. **29** (1961), 24–26.

A discussion of the way in which the gauge enters into expressions for observables of a charged particle moving in an external field.

P. W. Higgs (Edinburgh)

7704:

Дирак, П. А. М. [Dirac, P. A. M.]. ★Принципы квантовой механики. [The principles of quantum mechanics]. Translated from the 4th English ed. by Yu. N. Demkov and G. F. Drukarev; edited and with a preface by V. A. Fok. Gosudarstv. Izdat. Fis.-Mat. Lit., Moscow, 1960. 434 pp. 13.50 r.

This translation of the fourth edition of Dirac's classic book has footnotes by the editor (V. Fok) and translators to help the reader over the rough spots. There is an appendix by Fok explaining the formal correspondence between canonical transformations in classical mechanics and quantum mechanics. There have not been many changes from the third edition [Clarendon, Oxford, 1947; MR **9**, 319]. The last chapter on quantum electrodynamics has been rewritten; it does not attempt to treat renormalization theory, with which the author is dissatisfied.

A. S. Wightman (Princeton, N.J.)

7705:

Lamb, W. E., Jr. Quantum mechanical amplifiers. Lectures in theoretical physics (Boulder, Colo., 1959), pp. 435–483. Interscience, New York, 1960.

The article contains a discussion of the basic theory of quantum mechanical amplifiers, now commonly called masers. The classical and quantum theory of emission and absorption is briefly summarized, then the coherent excitation and the thermodynamics of two-level systems is discussed. The operation of the first working maser is explained. The small- and strong-signal theory is developed. The maser's action as an amplifier is analyzed. The theory of three-level masers is worked out. The article concludes with a thorough bibliography.

P. Roman (Boston, Mass.)

7706:

Hori, Shoichi. On the wave functions of higher spin particles. Progr. Theoret. Phys. **21** (1959), 613–624.

An orthogonal set of $2s+1$ wave functions for a spin s particle is obtained with the help of spin $\frac{1}{2}$ and spin 1 wave functions for particles at rest. The rules for vector addition of angular momentum and a subsequent Lorentz transformation to a moving system complete the construction.

C. G. Bollini (Buenos Aires)

7707:

Winogradzki, Judith. Sur le calcul des huit spineurs du second rang dont les composantes sont invariantes par rapport aux transformations du groupe de Lorentz général. C. R. Acad. Sci. Paris **251** (1960), 1983–1985.

Author's summary: "On expose une méthode qui permet de déterminer les spineurs du second rang à composantes invariantes sans résoudre explicitement les équations d'invariance."

7708:

Demkov, Yu. N. Variational principles for nonstationary quantum mechanical problems and perturbation theory. Ž. Èksper. Teoret. Fiz. **38** (1960), 1879–1886 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1351–1355.

The non-invariance with respect to addition of a function of time, of the formulae for the transition probability of a quantum mechanical system between two states represented by non-orthogonal wave functions, is eliminated by means of a variational principle. Assuming that the functions $f_1\phi_1$, $f_2\phi_2$ provide a better approximation than ϕ_1, ϕ_2 to the exact solutions of Schrödinger equation of a perturbed system, the author finds differential equations

for the arbitrary, time dependent, multipliers f_1, f_2, which correspond to the stationary transition probability. These are solved easily. The results are applied to the problem of charge exchange in fast collisions, where results obtained previously by D. R. Bates [Proc. Roy. Soc. London. Ser. A. **247** (1958), 294–301; MR **20** #7540] are rederived.

A. H. Klotz (Newcastle-upon-Tyne)

7709:

Lyubarskiĭ, G. Ya. ★The application of group theory in physics. Translated from the Russian by Stevan Dedijer. Pergamon Press, New York-Oxford-London-Paris, 1960. ix+380 pp. $10.00.

Translation of *Teoriya grupp i ee primenenie v fizike* [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1957; MR **21** #5441].

7710:

Jeszenszky, F. Einige Bemerkungen zum quantenmechanischen Begriff der physikalischen Grössen. Acta Phys. Acad. Sci. Hungar. **11**, 217–224 (1960). (Russian summary)

Verfasser untersucht drei Fälle quantenmechanischer Systeme: (1) freie Systeme, (2) ein in einem Raumbereich eingeschränktes System, (3) Massenerscheinungen. In allen drei Fällen wird das Problem der allgemeinsten Aussage untersucht, die jeweils über die kanonisch-konjugierten Grössen gemacht werden kann. Die Untersuchung wird mit den von J. von Neumann geschaffenen mathematischen Hilfsmitteln durchgeführt. Dementsprechend wird die Verwendung der Dirac'schen δ-Funktion vermieden und auch die Methode der Einführung von Distributionen als mathematisch unzureichend angesehen. Da der Ort der Potentialwände (im Fall (2)) nicht scharf angegeben werden kann, fallen auch die Impulswerte des in einem Raumbereich eingeschränkten Systems unscharf aus. Sieht man von dieser Idealisierung ab, so führt die Theorie zum Wellenpaketbild, das sich auch ergibt, wenn man im Falle einer Potentialwand von endlicher Höhe den Tunneleffekt berücksichtigt. Bei den Massenerscheinungen berücksichtigt die Theorie das Verhalten eines makroskopischen Systems (Meßgerät). Quantenmechanische Systeme in Wechselwirkung mit Meßgeräten rufen Ereignisse hervor, die nicht eindeutig bestimmt sind. Dabei erweisen sich die Unterschiede zwischen den Unschärferelationen eines auf einen Raumbereich beschränkten Systems gegenüber Massenerscheinungen als recht erheblich. *M. Pinl* (Cologne)

7711:

Brillouin, L. Poincaré and the shortcomings of the Hamilton-Jacobi method for classical or quantized mechanics. Arch. Rational Mech. Anal. **5**, 76–94 (1960).

Dans l'application de la méthode de Hamilton-Jacobi par les physiciens tels que M. Born, des difficultés se produisent quand apparaît une condition de dégénérescence: relation à coefficients entiers entre les fréquences. Un point de dégénérescence peut provoquer un changement brusque de définition des variables "angulaires" et des variables "d'action"; un exemple précis est donné à ce sujet (on remarque d'ailleurs que la dégénérescence peut être le fait non seulement de l'hamiltonien mais aussi des conditions initiales). L. Brillouin attire l'attention sur l'importance en tout cela du théorème célèbre de Poincaré d'après lequel il n'y a pas en général d'autre intégrale analytique uniforme que l'intégrale d'énergie; il observe que la démonstration de ce fait repose sur ce que, dans tout domaine des x si petit soit-il, il y a une infinité de points où est réalisée la condition $\sum m_i \partial F_0/\partial x_i = 0$ où les m sont des entiers non tous nuls. L'article se termine par une suite de remarques judicieuses sur la différence des points de vue du mathématicien et du physicien: en particulier les questions d'irrationalité ou de commensurabilité ne se posent pas pour le physicien. D'autre part il souligne que la méthode B.K.W. qui permet de faire la transition entre la Mécanique ondulatoire et la Mécanique classique n'a été discutée que pour des problèmes à "variables séparées"; dans le cas général de nombreuses questions restent jusqu'ici sans réponse. *M. Janet* (Paris)

7712:

Henderson, M. G.; Scherr, Charles W. Helium wave function in momentum space. Phys. Rev. (2) **120** (1960), 150–152.

Authors' summary: "Approximate solutions to the integral Schrödinger equation in momentum space are obtained. The iteration scheme of Svartholm is used to obtain the first iterated wave function and the half-iterated energy. A wave function of the type

$$\phi = \sum C_{ij}[\exp(-\alpha_i p_1^2 - \alpha_j p_2^2) + \exp(-\alpha_j p_1^2 - \alpha_i p_2^2)]$$

is employed to start the iteration procedure. The best energy value computed using a wave function with three nonlinear parameters is -2.8915 atomic units. This energy is to be compared with the result of a conventional variational calculation using the same wave function in coordinate space, -2.85112 atomic units."

A. H. Klotz (Newcastle-upon-Tyne)

7713:

Tietz, T. Über Eigenwerte und Eigenfunktionen der Schrödinger-Gleichung für das Thomas-Fermische Potential. Acta Phys. Acad. Sci. Hungar. **11**, 391–400 (1960). (Russian summary)

The author approximates to the Thomas-Fermi potential of the free neutral atom by the function $\varphi = (1 + Ar^2)^{-1}$, $A = 0.64301\ Z^{1/3}$, and studies the eigenvalues and eigenfunctions of Schrödinger's equation for two different kinds of spherically symmetric potential $V(r)$, namely,

$$-\frac{(Z-1)e^2}{r}\varphi - \frac{e^2}{r}, \quad -\frac{Ze^2}{r}\varphi.$$

In either case, the radial eigenfunctions are expressible as an elementary function times an infinite series of hypergeometric functions, in which the coefficients satisfy a three-term linear recurrence relation. Hence the continued-fraction technique for numerical calculations of eigenvalues is applicable. The author compares his numerical results with those obtained previously by Hartree and by Latter. Also, an analytic expression is given for the zero- and first-order terms of the eigenvalues.

C. J. Bouwkamp (Eindhoven)

7714:

Sasakawa, Tatuya. The wave packet interpretation of the scattering. Progr. Theoret. Phys. Suppl. No. 11 (1959), 69–116.

The author considers the time dependent description of the scattering process. A wave-packet (the S state only is considered) is scattered from an interaction region of radius R. The interaction is simulated by logarithmic boundary condition. The time dependence of the scattered wave is analyzed into hard sphere and resonant components. The time dependence of the latter is considered in its dependence on the parameters ΔE (deviation from exact resonance), Γ (width of the resonance) and finally the time required by the wave packet to travel a distance equal to its size. The author next considers the resonant scattering from many levels, employing the "uniform" model. Here the important parameter is $\pi\Gamma/2D$ where D is the distance in energy between levels. He shows that if $\pi\Gamma/2D \ll 1$, then the scattering may be broken up into two parts: one in which there is no time delay inside the nucleus, the delay being of the order of $\hbar/\Gamma$, the other involving the minimal time required for the packet to traverse the scattering region. When $\pi\Gamma/2D \gg 1$ these two aspects cannot be separated. The implication of these results for the optical model of nucleon-nucleus scattering is discussed. *H. Feshbach* (Cambridge, Mass.)

7715:

Urban, P.; Zingl, H. Streuung schneller Elektronen. Fortschr. Physik **7** (1959), 641–674.

This is a review of theory and experimental results of scattering of fast (relativistic) electrons by atomic nuclei, in particular also by deuterons and protons. Formulae for elastic scattering, in Born-approximation, and in terms of phase shifts, are reviewed, and the conclusion about nuclear charge densities discussed. Inelastic scattering is then mentioned, in particular excitation of rotational levels, electro-disintegration, and meson production. The final sections deal with the scattering by protons, the determination of its charge and magnetic moment distribution (Form factors F_1 and F_2), and the analysis of electron-deuteron collisions.

The paper surveys, in 30 pages, a great amount of theoretical and experimental material, in a sketchy way, but useful as a first introduction into the field.

F. Villars (Cambridge, Mass.)

7716:

Swan, P. An improved approximation for scattering problems. II. Nuclear Phys. **21** (1960), 233–236.

An approximation proposed previously by the author [Nuclear Phys. **18** (1960), 245–270; MR **22** #6489] is improved for the case of long tailed potentials.

J. L. Gammel (Los Alamos, N.M.)

7717:

Smith, Felix T. Lifetime matrix in collision theory. Phys. Rev. (2) **118** (1960), 349–356.

The collision lifetime is defined as the limit as $R \to \infty$ of the difference between the time spent by the particles within a distance R of each other and the time they would spend there in the absence of interaction. For a one-dimensional elastic collision the relations $Q = \hbar d\eta/dE = -i\hbar S^* dS/dE$ are proved where η is the phase shift, E the energy and $S = e^{i\eta}$ is the scattering matrix for this simple case. In three dimensions for elastic collisions Q_{ll} is similarly defined for each angular momentum l. A lower bound is established for the energy derivative of the phase shift. For inelastic collisions a matrix Q_{ij} is defined and $Q = i\hbar S dS^*/dE$. This is related to Eisenbud's [unpublished dissertation, Princeton, June, 1948] delay-time matrix. It is shown that the average delay experienced by a particle injected in the ith channel is Q_{ii}. The transformation properties and eigenvalues of Q are discussed. The wavefunctions which diagonalize Q, and its eigenvalues q_{ii}, which are the associated lifetimes, are useful in the discussion of collision and decay processes.

C. Strachan (Aberdeen)

7718:

Serdobol'skiĭ, V. I. Dispersion formulae for overlapping levels. Ž. Èksper. Teoret. Fiz. **38** (1960), 1903–1906 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1368–1370.

The author assumes that the S-matrix (for a nuclear reaction), and not the R-matrix, is the sum of a uniform, slowly varying term and a linear sum of resonance terms, like a dispersion relation. The resonance energies E_λ and half-widths γ depend on the incident energy, as in the Kapur-Peirels theory, to which the present theory is related. The half-widths are complex now, but the number of free parameters in the theory is reduced by the unitary condition on the S-matrix, which determines the phases of the half-widths.

Cases are considered when an isolated level corresponding to compound nucleus formation lies in the optical resonance region, or when two levels of the compound nucleus overlap or a set of weakly overlapping levels exists in the optical resonance region. The approximations used do not work if the levels of the compound nucleus are so broad that they overlap the nuclear reaction thresholds.

R. F. Streater (Princeton, N.J.)

7719:

Keil, E.; Zeitler, E.; Zinn, W. Zur Einfach- und Mehrfachstreuung geladener Teilchen. Z. Naturforschg. **15a** (1960), 1031–1038.

Authors' summary: "Die auf das Wentzelsche Verfahren zurückgehenden Ansätze von Molière werden für das Gebiet der Einfach- und Mehrfachstreuung (mittlere Stoßzahlen von 0 bis 20) ausgewertet.

"Für sehr kleine Stoßzahlen können die Winkelverteilungen direkt nach dem statistischen Verfahren von Wentzel berechnet werden, da der Anteil der genau zweifach gestreuten Teilchen sich noch berechnen läßt und der Anteil der drei- und mehrfach gestreuten Teilchen vernachlässigbar ist.

"Für Stoßzahlen zwischen 1 und 20 wird eine für die Durchführung der Auswertung auf einer elektronischen Rechenmaschine besonders geeignete Approximation für ganze Stoßzahlen benutzt.

"Die Ergebnisse sind in drei Tabellen zusammengestellt, aus denen man die Winkelverteilungen, die integrierten Winkelverteilungen und die über die Schichtdicke gemittelten Winkelverteilungen für mittlere Stoßzahlen zwischen 0 und 20 entnehmen kann."

7720:

Fleischmann, Hans. Zur Kleinwinkeltheorie der Vielfachstreuung. Z. Naturforschg. **15a** (1960), 1090–1096.

7721:

Murota, Toshiyuki. On radiative corrections due to soft photons. Progr. Theoret. Phys. **24** (1960), 1109–1117.

Under certain conditions the contribution due to soft photons to a cross section can be written exactly as a multiplicative factor. These conditions are exhibited and the factor in question is derived in perturbation theory but without recourse to the classical current approximation. Instead, the T-product form of the scattering matrix is suitably approximated to yield the desired result.

F. Rohrlich (Iowa City, Iowa)

7722:

Biswas, S. N.; Gupta, V. Effective range for K^+-nucleon interaction from dispersion relations. Nuclear Phys. **21** (1960), 137–141.

Authors' summary: "An exact formula for the effective range for $K^+ - P$ interaction, in the S-wave approximation, has been deduced using dispersion theoretic techniques. It is shown that if the assumption of zero effective range, which is generally used for a phenomenological fit of $K-N$ data, is valid then our analysis clearly indicates that the K-meson-hyperon relative parity must be odd (the $\Sigma - \Lambda$ relative parity being even). The contribution of the dispersion integral over the unphysical region has been fully taken into account in the present analysis. A general discussion of the nature of the coupling is given for a non-vanishing but small effective range. The advantage of the use of the effective range formula to determine the sign of the relative parities is also pointed out."

7723:

Janković, Z. On the inelastic scattering by deformed nuclei. Nuovo Cimento (10) **17** (1960), 281–287. (Italian summary)

Author's summary: "A more general treatment of the nuclear inelastic scattering on even-even target nuclei is given particularly by introducing other collective target states besides rotational ones, the deformed diffuse nuclear and spin-orbit potentials and the Coulomb potential for the deformed target nucleus. In the first order approximation explicit expressions for the differential cross-section and for the total cross-section for the transition $0 \rightarrow 2$ are deduced."

7724:

Hilgevoord, Jan. ★Dispersion relations and causal description: An introduction to dispersion relations in field theory. Thesis, Univ. of Amsterdam. Series in Physics. North-Holland Publishing Co., Amsterdam, 1960. vi+140 pp. $4.00.

Quantum field theory suffers from a number of serious difficulties. However, in view of the great success of renormalization theory in quantum electrodynamics, it seems likely that the starting points of field theory are reasonable, but that the actual forms of field equations and interactions are not formulated properly and that perturbation theory is not applied in an appropriate manner. In recent years this realization has lead to a search for and a formulation of the basic assumptions of quantum field theory. One is then interested to see whether these assumptions lead already to consequences that can be checked by experiment. One main line of research along these lines is to derive dispersion relations for interacting quantized fields. Since a detailed knowledge of the interaction is not needed for their derivation, the study of dispersion relations fits well into the above mentioned approach of "abstract" field theory. Dispersion relations provide a possibility for a check of the basic assumptions of quantum field theory. Also, in some cases, they might provide a method to deal with strong interactions where perturbational methods are useless.

One of the basic assumptions used to derive dispersion relations is the causality condition. The main purpose of the present monograph is to work out this point clearly.

There seems to exist some confusion about the strict meaning of causality conditions. Therefore, the first chapter aims at clarifying and illustrating this concept. The second chapter discusses the causal propagation of the solutions of wave equations, together with an analysis of the various propagators and their meaning. Chapter 3 is even more mathematical in its character: it is devoted to the mathematical theorems which relate causality conditions to dispersion relations. In chapter 4 dispersion relations are derived for scattering problems of the unquantized wave equations. The causality condition is stated here in two different ways: first, as a condition on the scattering of wave packets, and then as a condition on the propagation function. This propagator method is applied in a manner which is formally very similar to the treatment in field theory. The last (and longest) chapter begins with a review of the postulates of relativistic quantum field theory, then proceeds with deriving an expression for the elements of the S-matrix in terms of the extrapolating fields. Finally, the methods for deriving dispersion relations are presented and discussed. An Appendix gives the derivation of the Jost-Lehmann-Dyson representation of expectation values of certain commutators.

This book will certainly fill in a serous gap existing in the coherent treatment of a very important field of contemporary research and should be seriously recommended for scholars interested in these new developments.

P. Roman (Boston, Mass.)

7725:

Trigg, George L. Virtual binding and its relation to resonance scattering. Amer. J. Phys. **28** (1960), 711–715.

Resonance scattering is usually recognised as due to virtual binding, and this article serves the very useful didactic purpose of bringing out clearly the relationship between the two phenomena. After giving a unified treatment of the two topics, the connection between them is shown directly by means of three explicit formulae. The last of these indicates that the true resonance energy is displaced from the virtual energy by an amount depending on the width and location of the virtual state.

B. S. Madhavarao (Poona)

7726:

Beck, G.; Nussenzveig, H. M. On the physical interpretation of complex poles of the S-matrix. I. Nuovo Cimento (10) **16** (1960), 416–449. (Italian summary)

Although the title of this paper suggests that it deals with a subject which is specific to quantum mechanical theory, this is not the case. The problem treated is a central one in applied mathematics and is formulated incorrectly

in numerous papers and books in the literature of physics and engineering. Stated roughly, it attempts to give a direct interpretation to the use of "eigenfunctions" corresponding to complex "eigenvalues" for self-adjoint linear partial differential operators in the solution of exterior boundary value problems. The authors do not attempt to formulate or treat this question in mathematical generality, but restrict themselves to the detailed discussion of three typical examples: (1) a harmonic oscillator coupled to a semi-infinite string; (2) electromagnetic oscillations on a perfectly conducting sphere; and (3) the wave mechanical theory of scattering by an impenetrable sphere.

In each case the formal solution of the given combined initial-value and boundary-value problem is found by standard methods. This solution is reformulated as an integral solution with an appropriate Green's function, which is allowed to have singularities of the delta-function type. These kernel functions are then separated into parts representing "propagators" for incoming and outgoing systems of waves. In addition to delta-functions, these propagators contain members which are linear superpositions of "eigenfunctions" corresponding to complex eigenvalues.

The authors show in a satisfactory manner that the solutions which they obtain do not suffer from the defects of other formal solutions of similar problems as given in the literature. {In this respect their results are covered by the more general analysis given by Titchmarsh [*Eigenfunction expansions associated with second-order differential equations, Vol. 1, 2*, Clarendon, Oxford, 1946, 1958; MR 8, 458; **20** #1065] in which the complex eigenfunctions appear as an intermediate stage in the proof of expansion theorems for functions in L_2-space.}

Apart from the question of improvement of mathematical methodology in the solution of initial-value problems, one question of physical importance remains. The authors point out, as has been done by others, that the time-decay characteristic of such solutions is not usually of the exponential form of the radioactive decay law. Since the latter law underlies the concept of transition probabilities in quantum theory, it is important to know under what conditions it may fail to hold. {At present no definitive answer can be given to this question, and it may be doubted whether the mathematical model is truly representative of the physical facts.}

{Despite its restriction to very special problems, the special character of the physical interpretations offered, and its avoidance of modern work in functional analysis, the reviewer feels that this is a paper of significant importance for the clarification of a wide class of problems in theoretical physics.} *E. L. Hill* (Minneapolis, Minn.)

7727:

Bang, Jens. On the angular distribution of the scattered particles in Coulomb excitation. Mat.-Fys. Medd. Danske Vid. Selsk. **32**, no. 5, 16 pp. (1960).

The angular distribution of scattered particles in Coulomb excitation is calculated for the case of electric quadrupole excitation and vanishing energy transfer; numerical values are given for a number of scattering angles and incident energies. Furthermore, an expression for the cross section at a deflection angle equal to zero is derived, valid also for finite energy transfer. *T. Regge* (Turin)

7728:

Chang, T. S. Remarks on Chew-Low equations. Sci. Record (N.S.) **3** (1959), 628–634.

7729:

Cabibbo, N.; Gatto, R.; Zemach, C. A theorem on the elimination of contact muon-electron interactions. Nuovo Cimento (10) **16** (1960), 168–174. (Italian summary)

A generalization of the theorem of N. Cabibbo and R. Gatto [Phys. Rev. (2) **116** (1959), 1334–1338; MR **22** #2410], G. Feinberg, P. Kabir and S. Weinberg [Phys. Rev. Lett. **3** (1959), 527–530]. Utilizing orthogonality of the projection operators $\frac{1}{2}(1 \pm \gamma_5)$ the authors prove that the Lagrangian of the muon-electron interaction $-\bar{\psi}[\gamma\mu(\partial\mu - ieA\mu)(A + \gamma_5 B) + C + i\gamma_5 D]\psi$, where $\psi = \begin{pmatrix} e \\ \mu \end{pmatrix}$ (8-component spinor), can be transformed into

$$-\bar{\psi}'[\gamma\mu(\partial\mu - ieA\mu) + M]\psi'$$

by a unitary matrix in the case of the energy operator being positive definite, and they interpret the meaning.

N. Kumasawa (Tokyo)

7730:

Charap, J. M.; Fubini, S. P. The field-theoretical definition of nuclear potential. II. Nuovo Cimento (10) **15** (1960), 73–86. (Italian summary)

The work of a previous paper in Nuovo Cimento (10) **14** (1959), 540–559 [MR **22** #6478] determining a potential to reproduce the low energy scattering amplitude given by field theory for two nucleons is extended from the case of scalar nucleons scattering through neutral mesons to scattering through charged mesons (when one gets exchange forces). The isotopic spin formalism is used to separate out the different isotopic spin states; an effective potential valid in each of these is determined from a Khuri dispersion relation [Phys. Rev. (2) **107** (1957), 1148–1156; MR **22** #6476] relating the potential to the scattering amplitude. *A. Herzenberg* (Manchester)

7731:

Federbush, Paul G.; Johnson, Kenneth A. Uniqueness property of the twofold vacuum expectation value. Phys. Rev. (2) **120** (1960), 1926.

The authors show under general assumptions that if at equal times the two point vacuum expectation value of a given field coincides with that of a free field, then the given field is equivalent to that free field.

O. W. Greenberg (Cambridge, Mass.)

7732:

Jauch, J. M. On Pauli's transformation. Nuovo Cimento (10) **16** (1960), 1068–1072. (Italian summary)

It is shown that Pauli's transformation which mixes particles with anti-particles is canonical but not unitary. This is an important result. The author further shows that it is possible, in a simple way, to generalize the transformation in such a way that it becomes unitary. However no generalization can be found for which the resulting unitary transformation can be written as local transformation in the field operators. *A. Salam* (London)

7733:

Kazes, E. A soluble model in field theory. I. Nuovo Cimento (10) **14** (1959), 815–826. (Italian summary)

Author's summary: "The Lee model has been modified by giving the V-particle a more complicated structure, and still leaving the $\mathfrak{N}$-θ scattering amplitude exactly soluble. This modification also yields a wider variety of soluble processes. The same limitation that applies to the cut-off size in the Lee model is reproduced in this model. The calculation is performed by using the Hamiltonian formalism as well as dispersion methods."

K. Johnson (Cambridge, Mass.)

7734:

Kazes, E. A soluble model in field theory. II. Unstable particle and bound state description. Nuovo Cimento (10) **15** (1960), 537–550. (Italian summary)

Author's summary: "Examining a model presented earlier we performed a calculation of unstable particle production and decay without separating these two processes. The usual separation of production and decay is seen to be valid for a time t, such that $\hbar/\mu_3 c^2 \ll t \ll 2\tau \ln 2\pi\tau t^2 Q^3$, $Q \gg 1/\tau$, where Q is the energy above threshold for unstable particle production and τ is the lifetime. When the above conditions are valid the unstable states possess a simple enough time development to be characterized with a particle label. Our model also predicts bound state production for suitable cut-off functions. The Nishijima-Zimmermann formalism for handling bound states in conjunction with dispersion relations reproduces direct calculations." *K. Johnson* (Cambridge, Mass.)

7735:

Venables, H. A. Maxwell's equations and matrix elements in quantum electrodynamics. Canad. J. Phys. **39** (1961), 141–144.

It is well known that Maxwell's equations can be written as two 2-component spinor equations. These can be combined to one first order equation for the 4-component spinor ϕ representing the electromagnetic field. In conjunction with the Dirac equation for the electron field ψ the author constructs a first order equation involving ϕ and ψ. A formal solution for $\phi\psi$ in terms of a suitably chosen Green function yields the correct matrix elements of second order processes. *F. Rohrlich* (Iowa City, Iowa)

7736:

Weinberg, Steven. High-energy behavior in quantum field-theory. Phys. Rev. (2) **118** (1960), 838–849.

Author's summary: "An attack is made on the problem of determining the asymptotic behavior at high energies and momenta of the Green's functions of quantum field theory, using new mathematical methods from the theory of real variables. We define a class A_n of functions of n real variables, whose asymptotic behavior may be specified in a certain manner by means of certain 'asymptotic coefficients.' The Feynman integrands of perturbation theory (with energies taken imaginary) belong to such classes. We then prove that if certain conditions on the asymptotic coefficients are satisfied, then an integral over k of the variables converges, and belongs to the class A_{n-k} with new asymptotic coefficients simply related to the old ones. When applied to perturbation theory this theorem validates the renormalization procedure of Dyson and Salam, proving that the renormalized integrals actually do always converge, and provides a simple rule for calculating the asymptotic behavior of any Green's function to any order of perturbation theory."

O. Hara (Duluth, Minn.)

7737:

Dresden, M. Aspects of abstract field theory. Lectures in theoretical physics (Boulder, Colo., 1959), pp. 366–434. Interscience, New York, 1960.

The general purpose of this article is to provide, for a person familiar with the ideas of conventional quantum field theory, a way to become acquainted with a number of methods and results of some abstract versions of quantum field theory. Since, at the moment, there is no coherent treatise available in this field, the present article is greatly welcome. The style and exposition is clear and the precise logical interrelations between various points are worked out in detail. The generally informal character of the original lectures has been largely retained. This is partly an advantage, but at the same time a source of occasional looseness and of a number of misprints.

The article begins with a concise yet most useful summary of quantum electrodynamics. Due attention is paid to pointing out the various mathematical inadequacies. The next section discusses the asymptotic formulation of quantum field theory, and is based on the works of Lehman, Symanzik and Zimmerman. Then, as a main application of these general ideas, an explicit expression for matrix elements of the S-matrix in terms of t-functions is derived and the relations between the t-functions are discussed. The following section is concerned with the derivation of the Low equations, which are obtained by an analysis of the recursion formulae. The main ideas for establishing dispersion relations (for forward scattering) are then presented, and the article concludes with a brief summary of the theory of spectral representations of vacuum expectation values and Wightman's program to construct an axiomatic formulation of field theory based on the analytic properties of n-fold vacuum expectation values. *P. Roman* (Boston, Mass.)

7738:

Ahiezer, I. A.; Peletminskiĭ, S. V. Use of the methods of quantum field theory for the investigation of the thermodynamical properties of a gas of electrons and photons. Ž. Èksper. Teoret. Fiz. **38** (1960), 1829–1839 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1316–1322.

The thermodynamic potential of a gas of electrons, positrons and photons is analyzed by the method of Feynmann diagrams, taking into account terms up to order $e^4 \log e^2$. Divergences which appear in the high momentum region of the virtual particles are removed by renormalization. The thermodynamic potential is split into the exchange part $\Delta\Omega_e$, proportional to e^2, and the correlation part $\Delta\Omega_c$, containing charge in higher powers than the second. The formula for $\Delta\Omega_c$ reduces in the non-relativistic limit to the form obtained by Fradkin [same Ž. **36** (1959), 1286–1289; MR **21** #6750]; also the form for high temperatures is considered. Finally, the corrections to the energy of black radiation due to the interaction between the photons and electron-positron pairs are obtained. *R. F. Streater* (Princeton, N.J.)

7739:

Taylor, J. G.; Warburton, A. E. A. Complex singularities of partial-wave amplitudes in perturbation theory. Phys. Rev. (2) **120** (1960), 1506–1507.

This paper establishes the interesting property that the partial wave dispersion relations derived from the fourth order scattering amplitude continue to hold even when the Mandelstam representation ceases to be true. It is not known whether this is so in any higher order.

The poof is incomplete in one respect, in that the argument presented to exclude coincident singularities in the $\cos\theta$ integration is invalid. It is easy to put this right for the amplitude in question, but the important corollary that it is only necessary to establish forward and backward dispersion relations in order to prove the general case of partial wave dispersion relations is not established.

J. C. Polkinghorne (Cambridge, England)

7740:

Szász, Levente. Über die Berechnung der Korrelationsenergie der Atomelektronen. Z. Naturforschg. **15a** (1960), 909–926. (English summary)

Author's summary: "To calculate the correlation energy of an atom with N electrons we suggest the wave function

(a) $\psi =$

$$\frac{1}{\sqrt{N!}}\tilde{A}\{\varphi_1(1)\,\varphi_2(2)\cdots\varphi_N(N)[1+\sum_{j=1}^{N}\sum_{k=j+1}^{N}W_{jk}(j,k)]\},$$

where $\tilde{A}$ is the antisymmetrizer operator, $\varphi_1, \varphi_2, \cdots, \varphi_N$ are one electron wave functions, and W_{jk} are correlation functions of the following form:

(b) $$W_{jk}(1,2)=\sum_{m,n,l}c_{jk}^{m,n,l}(r_1-r_2)^{2m}(r_1+r_2)^{n}r_{12}^{l},$$

where the constants $c_{jk}^{m,n,l}$ are variational parameters. The function (a) is a generalization of the wave function of Hylleraas for He. After a discussion of the properties of our function, an energy expression is derived. Numerical calculation is made for the ground state of the Be atom with the function

(c) $$\psi_{Be}=\frac{1}{\sqrt{4!}}\tilde{A}\{\varphi_1(r_1)\varphi_2(r_2)\varphi_3(r_3)\varphi_4(r_4)[1+c_1r_{12}+c_2r_{34}]\},$$

where φ_1 and φ_2 are $1s$ wave functions, φ_3 and φ_4 are $2s$ wave functions, r_1, r_2, r_3 and r_4 are the radial coordinates of the four electrons, r_{12} and r_{34} are the distances between the corresponding electrons, and c_1 and c_2 are variational parameters. Using the one electron wave functions calculated by Roothaan et al. with the Roothaan procedure, we got the energy value $E=-14.624$ a.u. while the Hartree-Fock and experimental values are $E_{H.F}=-14.570$ a.u. and $E_{exp}=-14.668$ a.u. respectively. Thus the function (c) gives about one-half of the correlation energy of the Be atom."

7741:

Walsh, Peter; Borowitz, Sidney. Application of wave functions containing interelectron coordinates. II. Approximate energy levels of atoms. Phys. Rev. (2) **119** (1960), 1274–1283.

[For part I see Phys. Rev. (2) **115** (1959), 1206–1215; MR **22** #2377.]

Following the ideas of Pluvinage (1950) the Hamiltonian for an atomic system is split into an unperturbed part, which is separable and which contains the interelectron potentials as well as the electron-nuclear potentials, and into a perturbing term which is always finite and vanishes whenever an electron is far from the nucleus. The zero order energies obtained in this way are surprisingly accurate. However, the difficulties associated with the use of the zero-order wave functions as the basis for a first order or variational calculation are formidable.

A. C. Hurley (Melbourne)

7742:

Breene, R. G., Jr. Analytic wave functions. III. The spin-orbit, spin-other orbit, and spin-spin interactions. Phys. Rev. (2) **119** (1960), 1615–1618.

Wave functions obtained by the methods described in the previous two papers [Phys. Rev. (2) **111** (1958), 1111–1113; **113** (1959), 809–813; MR **22** #3487, 3488] are used to obtain perturbation expressions for the three interactions mentioned; a second approximation to the spin-orbit interaction is also calculated. The results are given for the ground state configurations of the oxygen atom.

D. F. Mayers (Oxford)

7743:

Alder, Kurt; Winther, Aage. On the theory of multiple Coulomb excitation with heavy ions. Mat.-Fys. Medd. Danske Vid. Selsk. **32**, no. 8, 72 pp. (1960).

Author's summary: "The present paper contains formulae and tables for the evaluation of multiple Coulomb excitation cross section of rotational and vibrational states. For other cases, general calculational procedures have been developed and these are illustrated through examples. For the larger part of the work, the collision time is assumed to be short compared to the nuclear period. The investigation is furthermore simplified by an approximate treatment of the dependence of the cross section on the deflection angle of the projectile. The accuracy of the approximation is also discussed."

T. Regge (Turin)

7744:

Abrahamson, Adolf A.; Hatcher, Robert D.; Vineyard, George H. Interatomic repulsive potentials at very small and intermediate separations. Phys. Rev. (2) **121** (1961), 159–171.

Authors' summary: "Using a minimal and a maximal principle, respectively, two approximate expressions for the interaction potential between atoms are given such that their mean, $\bar{U}(R)$, differs from the exact value, U_0, in the Thomas-Fermi-Dirac (TFD) approximation, by not more than 4% for the case of two-center system; and by not more than 14%, in the Thomas-Fermi (TF) approximation, for the case of a three-center system. The respective limits of applicability of these potentials are discussed, and some of their applications are pointed out."

7745:

Brueckner, Keith A.; Gammel, John L.; Kubis, Joseph T. Calculation of single-particle energies in the theory of nuclear matter. Phys. Rev. (2) **118** (1960), 1438–1441.

7746:

Gammel, J. L.; Thaler, R. M. Spin-orbit coupling in the proton-proton interaction. Phys. Rev. (2) **107** (1957), 291–298.

Authors' summary: "The proton-proton interaction is examined at high energies. A phenomenological potential is derived from the scattering data. In the triplet state, this potential has a strong short-range ($\sim 0.4 \times 10^{-13}$ cm) repulsive core region, outside of which a long-ranged tensor force and extremely short-range spin-orbit ($\mathbf{L} \cdot \mathbf{S}$) force are effective. Good fits can be obtained to all the p-p data below 310 Mev. More important, at 310 Mev the calculated phase shifts are very close to the phase shifts which give the best fit found in a phase-shift analysis of the 310-Mev data, and at all lower energies precision fits to the experimental data can be obtained by very slight (if any) changes in the calculated phase shifts."

7747:

Gammel, J. L.; Thaler, R. M. Spin-orbit coupling in the neutron-proton interaction. Phys. Rev. (2) **107** (1957), 1337–1340.

Authors' summary: "It is shown that if the nucleon-nucleon interaction is charge-independent, then the n-p scattering and polarization data in the energy range 0–310 Mev require the presence of a spin-orbit term in the triplet even-parity potential of the same short range as (but of somewhat less depth than) the spin-orbit term found previously for the triplet odd-parity potential."

7748:

Clark, John W. Effective spin-orbit potential in correlated heavy nuclei. Ann. Physics **11** (1960), 483–500.

Spin orbit splittings in the region of Pb^{208} resulting from recent phenomenological potentials [J. L. Gammel and R. M. Thaler, #7746, 7747 and P. S. Signell, R. Zinn, and R. E. Marshak, Phys. Rev. Lett. **1** (1958), 416–418] are estimated using correlated nuclear wave functions of the Jastrow type [R. Jastrow, Phys. Rev. (2) **98** (1955), 1479–1484]. The results are in agreement with conclusions based on experimental data.

J. L. Gammel (Los Alamos, N.M.)

7749:

Sugie, Atsushi. The imaginary part of the optical potential. Progr. Theoret. Phys. **21** (1959), 681–695.

The form of the imaginary part of the optical potential, as an operator for the finite nucleus, is derived by an approximate reduction of the Schrödinger equation. It is essentially a second order calculation, but this is supported by the intermediate coupling model and the assumption that the non-diagonal elements of the interaction matrix have random signs. The imaginary part thus obtained is a non-local but almost separable potential. The form allows the physical interpretation that the imaginary part corresponds to the process in which the incident nucleon jumps to an unoccupied single particle state below the incident energy and excites the target nucleus making the total energy nearly conserved. The relation to C. Bloch's theory [Nuclear Phys. **3** (1957), 137–152] is also discussed.

G. L. Walker (Providence, R.I.)

7750:

Балдин, А. М. [Baldin, A. M.]; Гольданский, В. И. [Gol'danskiĭ, V. I.]; Розенталь, И. Л. [Rozental', I. L.]. ★Кинематика ядерных реакций. [Kinematics of nuclear reactions]. Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1959. 296 pp. 8.60 r.

Ce livre s'adresse en particulier aux physiciens expérimentaux s'intéressant aux réactions nucléaires et aux particules étranges.

Une partie est consacrée à l'exposé des cinématiques classiques et quantiques des réactions nucléaires.

Des méthodes de construction graphique pour les cinématiques des réactions et d'analyses des réactions à très hautes énergies sont données.

Deux appendices sont consacrés à des tables et graphiques de relations cinématiques pour de nombreuses réactions ainsi que des coefficients de Clebsch, Gordan et Racah, Z, Z_γ et X.

P. Chevallier (Strasbourg)

7751:

Werntz, Carl. Three-body nuclear problem with repulsive core forces. Phys. Rev. (2) **121** (1961), 849–853.

A variational calculation of the binding energy of the triton has been carried out using the Gartenhaus potential. The result is that the triton is unbound by about 8 Mev. This result is attributed to the even parity tensor potential which is relatively large compared to the even parity central potential. It is asserted that since this property is also characteristic of the Signell-Marshak potential that it would also lead to an unbound triton.

Such calculations are full of pitfalls. Contrary to the assertion in the first paragraph of the paper that Blatt finds that the Gammel-Thaler potential gives an unbound triton (by 2 Mev), Blatt now claims that this potential gives too much binding (10 Mev instead of 8 Mev). According to private correspondence with the reviewer, Blatt does not fully understand why his previous result was wrong; he only knows that as he increased the complexity of his trial function the binding energy increased.

The trial functions in the present paper are very simple. The reviewer is not inclined to believe the result.

J. L. Gammel (Los Alamos, N.M.)

7752:

Emery, V. J.; Sessler, A. M. Energy gap in nuclear matter. Phys. Rev. (2) **119** (1960), 248–250.

The energy gap is calculated for nuclear matter at various densities. This gives some insight into the situation in real nuclei, which have varying densities in the surface region. The equation for the energy gap is linearized for numerical solution. The Gammel-Thaler potential is used. The energy gap is found to depend very strongly on the effective mass and on density, and it seems to go to zero at densities a little above the normal density nuclear matter. The energy gap in real nuclei might be expected to be considerably larger than the gap in nuclear matter at equilibrium density, since the average density is lower. It is shown that this strong dependence of the energy gap on density is related to the small value of the phase shift for momenta near the Fermi momentum.

D. J. Thouless (Birmingham)

7753:

Šmelev, V. P. Polarization of the hydrogen atom in the ground state by the field of a point charge. Ž. Èksper. Teoret. Fiz. **38** (1960), 1528–1533 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1102–1105.

By a suitable modification of the method used by earlier workers to solve the problem of two centres in the case of the hydrogen molecule, the author derives the dipole moment of a hydrogen atom in the ground state, induced by a proton, as a function of the distance between this and the atomic nucleus. Since, however, the proton and the hydrogen atom in this state do not form a stable system, he considers whether such a system could be obtained if the proton be replaced by a positron. For this purpose, he uses the variational method earlier employed by him [same Ž. **37** (1959), 458–466] to calculate the state of the quasimolecule system (pe^-e^+) which can dissociate into a proton, and a positronium atom (e^+e^-), and investigates whether a state of the same system exists which could dissociate into a positron and a hydrogen atom. His conclusion is that dissociation into such a stable system is highly improbable, unless the mass of the positron were four times greater. He further indicates that this result confirms an earlier one obtained by Ore [Phy. Rev. (2) **73** (1948), 1313–1317] that the ionised hydrogen molecule H_2^+ could exist in a stable state if the mass of one of its protons were decreased to 4.5 times the mass of an electron. While the Schrödinger equation technique of non-relativistic quantum mechanics is certainly valid when applied to problems relating to the hydrogen molecule, it appears necessary to justify the use of the same when dealing with a system like the (pe^-e^+) quasi-molecule.

B. S. Madhavarao (Poona)

7754:

Scarfone, L. M.; McKinley, W. A. Ghost states and pair effects in the Lee model. Nuovo Cimento (10) **17** (1960), 678–686. (Italian summary)

Authors' summary: "Pauli and Källén's treatment of the one particle V-states in the ordinary Lee model is applied to J. S. Goldstein's extension of the Lee model which includes pair effects in the θ-particle. Contrary to an earlier conclusion, it is shown that there is simultaneously a ghost state in the V-spectrum with energy greater than the normal V energy as well as a ghost in the θ-spectrum for negative values of one of the renormalization constants."

7755:

Jean, Maurice. Note sur l'interprétation collective des deux premiers niveaux 2+ des noyaux pairs. (English summary). Nuclear Phys. **21** (1960), 142–156.

Author's summary: "It is shown that the qualitative agreement with the experimentally observed ratio of electromagnetic de-excitation probabilities of the first 2+ levels of even nuclei which is exhibited by the theoretical predictions of the Davydov and Filippov asymmetric rotor model, can equally well be obtained for a more conventional model effecting a gradual transition between the quadrupole oscillations of spherical nuclei and the vibrations and rotations of spheroidal ones."

7756:

Sawicki, J.; Moszkowski, S. A. Imaginary part of the optical model potential in nuclear matter. Nuclear Phys. **21** (1960), 456–461.

Authors' summary: "The imaginary part of the optical model potential in nuclear matter has been calculated for various values of the energy, assuming Gammel-Thaler nucleon-nucleon interactions. The calculations were made using the separation method in which the interaction is separated into a short range and long range part, the former giving zero phase shift. To a good approximation, only the long range component contributes to the imaginary part of the optical potential."

7757:

Sengupta, S. Coulomb energies of mirror nuclei. Nuclear Phys. **21** (1960), 542–554.

From the author's summary: "A semiclassical expression for Coulomb energies of the mirror nuclei has been derived. The exchange part in the above expression is derived on a statistical model which gives in addition to the usual term, a pairing effect of magnitude 0.33. Inconsistencies in some of the earlier expressions have been pointed out. The radius parameter r_0 from this formula agrees very well with the electron scattering values."

7758:

Feshbach, Herman. Unified theory of nuclear reactions. Ann. Physics **5** (1958), 357–390.

A formal description of nuclear reactions in terms of a generalized optical model is given in this paper. It is shown that this potential has the properties necessary to describe resonant elastic scattering as well as a non-resonant part which can be associated with potential scattering. The author has also treated inelastic scattering. He shows it is possible to derive a generalized optical potential in this case as well, and can identify the terms giving rise to both resonant and direct nuclear reactions.

N. S. Wall (Cambridge, Mass.)

7759:

Vojta, Günter. Die Entropie von Spinsystemen bei der magnetischen Kernresonanz. I. Ann. Physik (7) **6** (1960), 31–43.

The entropy of different spin systems is evaluated explicitly and it is shown that one must use the general formalism of quantum statistics rather than elementary considerations. A discussion is given of the concepts of temperature, entropy, and the principle of minimum production of entropy for the case of spin systems.

D. ter Haar (Oxford)

7760:

Griffith, T. C.; Power, E. A. (Editors). ★Nuclear forces and the few-nucleon problem. Proceedings of the International Conference held at the Physics Department, University College, London on 8–11 July 1959. 2 volumes. Pergamon Press, New York-Oxford-London-Paris, 1960. xiii + vii + 712 pp. $30.00.

In recent years great strides have been made toward the classical problem of understanding the atomic nucleus in the light of our knowledge of the interactions between

nucleons. This two volume report on the July 1959 conference presents a very complete description of the status of the experimental data and its theoretical interpretation and significance of that data. Unfortunately conference proceedings a year and a half old are valuable, not as an encyclopedia of the current problems and achievements, but as a review and compendium of facts and opinions which have influenced the work since the conference. In this sense the review papers read by Marshak, Brueckner, Chew, R. Wilson, Dixon, Massey, and Emery serve to introduce and educate the readers in a most effective manner to the various aspects of the fields these authors have treated.

There are only a few main themes which were very thoroughly discussed. Probably first and foremost are the experimental aspects and phenomenological interpretation of nucleon-nucleon scattering and photo-disintegration. Secondly there is the theoretical understanding of the phenomenological nucleon-nucleon potentials based upon the meson theory of nuclear forces. There is also a recurrence of these two themes in problems involving the deuteron, triton, He^3 and He^4. However, there was relatively little discussion of three- (or more) body forces. Finally the question of deducing nuclear properties based on the understanding of the nucleon-nucleon interaction was discussed.

Many of the controversial questions raised at the conference have long since been settled to everyone's satisfaction. For example the controversy between the Harvard and Harwell data (fig. 8 on page 54) has now been resolved in favor of the Harvard data. Not only has that controversy been settled, but many modifications of the phenomenological potentials used to interpret those particular experimental results have been made based upon the exchange terms suggested by meson theories of the nucleon interaction.

The long delay in publishing the proceedings has caused them to be outdated in many other areas as well. The finite nuclear calculations of Blatt and his coworkers have been published in recent months. In the 90–150 Mev. and 210 Mev. energy range many new experiments both on the P-P system and the N-P system have been completed and in fact very recently N-P phase shifts have become available.

One can see by comparing the proceedings of the 1960 Kingston Conference on Nuclear Structure (they were published about six weeks after the conference) that though the London Conference proceedings have an improved appearance, format, and structure to the papers, their value as a report is greatly diminished by the delay.

N. S. Wall (Cambridge, Mass.)

7761:

Hartmann, Hermann. Zur Theorie der π-Elektronensysteme. Z. Naturforschg. **15a** (1960), 993–1003.

Author's summary: "Das Hückelsche zweite Näherungsverfahren wird durch Mitberücksichtigung der höheren Atomzustände erweitert. Dabei ergibt sich die Erklärung für das Scheibesche Phänomen. Der Begriff 'theoretische Sonderenergie (Resonanzenergie)' wird richtiggestellt. Die bekannten Schwierigkeiten, die sich beim Vergleich spektroskopischer und kalorischer Energiewerte im Rahmen der Einelektronentheorie der π-Elektronensysteme bisher immer ergeben haben, verschwinden."

7762:

Tadokoro, Hiroyuki. Normal vibrations of the polymer molecules of helical configuration. J. Chem. Phys. **33** (1960), 1558–1567.

A method for numerical calculations of the infrared- and Raman-active vibration frequences of helical molecules is described and applied to a simple model of polyoxymethylene. *P. W. Higgs* (Edinburgh)

7763:

Maes, Serge. Les corrections du troisième ordre à l'énergie de vibration-rotation des molécules polyatomiques. Cahiers de Phys. **14** (1960), 125–208. (English summary)

Da man nach dem heutigen Stande der Messtechnik so die reinen Rotationsspektren von mehratomigen Molekülen im Mikrowellengebiet, wie die Rotationsschwingungsspektren von solchen Molekülen im nahen ultraroten Gebiet mit grosser Präzision ausmessen kann, so setzt sich der Verfasser das Ziel die dritte Näherung der Wechselwirkungsenergie von Oszillation und Rotation sehr genau zu berechnen. Die zweite Näherung dieser Energie wurde erstmals von H. H. Nielsen [Rev. Mod. Phys. **23** (1951), 90–136] berechnet. Mit der Auswertung der dritten und der vierten Näherung haben sich mehrere Autoren beschäftigt. Diese Arbeiten bilden den Ausgangspunkt der Berechnungen des Verfassers.

In den gewohnten Bezeichnungen der Quantenmechanik hat man $H\psi = E_{VR}\psi$, wo $H = T + V$ der Hamiltonsche Operator ist und die E_{VR} die Rotationsschwingungsniveaus eines mehratomigen Moleküls bedeuten. ψ ist eine Funktion der für die Oszillationen eingeführten Normalkoordinaten Q_{s_σ} und der drei Eulerschen Winkeln θ, φ und χ. Nach der gewohnten Methode kann dann H in eine Reihe entwickelt werden: $H = H_0 + H_1 + H_2 + H_3 + \cdots$. H_0 ist einfach die Hamiltonsche Funktion des starren Rotators und der harmonischen Schwingungen. Als Rotationsquantenzahlen werden die bekannten Symbole J, K und M und als Oszillationsquantenzahlen v_s und l_s eingeführt. Die Operatoren H_n sind dann alle in J und M diagonal, H_0 auch in v_s und l_s. Durch eine erste Berührungstransformation $H' = THT^{-1}$, wo T ein unitärer Operator und ausserdem $T = e^{iS}$ ist (S ist hermitisch), wird erreicht, dass jetzt $H_0' + H_1'$ auch bezüglich v_s diagonal wird. Eine analoge weitere Transformation bewirkt, dass auch $H_0^+ + H_1^+ + H_2^+$ diagonal in dieser Quantenzahl wird. Das ganze Problem ist dann die säkulare Determinantengleichung der quantenmechanischen Störungstheorie bis zu dem dritten Gliede in H^+ zu lösen. In den H müssen noch die Glieder nach den Transformationen umgeordnet werden, nach dieser Operation wird die Hamiltonsche Funktion mit h bezeichnet. Es folgt, dass $h_0^+ = h_0 = H_0$ und $h_1^+ = h_1' = h_1$ ist. Weiter werden dann im ersten Teil der Arbeit die Matrizenelemente von h_3' angegeben, im zweiten werden einige Fälle von Resonanzen dritter Ordnung besprochen und endlich werden im dritten Teil die erhaltenen Resultate mit der Erfahrung verglichen. Als wichtigstes Resultat des ersten Teiles erhält der Verfasser die Formel

$$(1)\quad E_{VR}^{(3)} = hc \sum l_t K \left[\eta_{t,T} J(J+1) + \eta_{t,K} K^2 + \eta_t + \sum_s \eta_{t,s} \left(v_s + \frac{g_s}{2} \right) \right].$$

Die in (1) stehenden Koeffizienten werden explizit angegeben. (1) ist nur solange gültig, bis die in l_i und K nichtdiagonalen Elemente zu dem Resultat nichts beitragen. Im zweiten Teil erhält man noch eine ziemlich einfache Formel für linear dreiatomige Moleküle. In der Arbeit werden besonders die ganz asymmetrischen Moleküle, dann solche welche die Symmetrien C_{3v} und D_{3h} (in den Schoenfliesschen Bezeichnungen) besitzen und endlich die linearen mehratomigen Moleküle besprochen.

T. Neugebauer (Budapest)

7764:

Salem, L. The calculation of dispersion forces. Molecular Phys. **3** (1960), 441–452.

Approximate estimates of the dispersion forces for various pairs of interacting atoms and molecules are given. The importance of including correlation effects, first pointed out by Vinti [Phys. Rev. (2) **41** (1932), 813–817], is stressed. *A. Dalgarno* (Belfast)

7765:

Henry, Lucien; Amat, Gilbert. Sur les coefficients d'interaction entre la vibration et la rotation dans les molécules polyatomiques. II. Cahiers de Phys. **14** (1960), 230–256. (English summary)

[For part I see Amat and Henry, Cahiers de Phys. **12** (1958), 273–286; MR **21** #1171.]

Authors' summary: "The nonvanishing coefficients $\zeta^{\alpha}_{\sigma\sigma'\sigma''}$ (Coriolis coupling coefficients) and $a_{\sigma\sigma'}{}^{\alpha\beta}$ (variation coefficients of moments of inertia with respect to normal coordinates) are given for any asymmetric or axially symmetric molecule and relations are established between these nonvanishing coefficients. By using standard rules for the orientation of the twofold degenerate normal coordinates it is possible to obtain the former relations in a general form for any molecule belonging to a given symmetry group." *A. C. Hurley* (Melbourne)

7766:

Baudet, Jean; Cabaret, Françoise; Tillieu, Jacques; Guy, Jean. Table d'intégrales à deux centres. II. J. Phys. Radium **21** (1960), 105–111. (English summary)

Authors' summary: "Some two-centre integrals occurring in the study of molecular properties (especially of magnetic susceptibilities), are tabulated in a condensed form, allowing fast numerical calculations.

"This table is the continuation of a preceding one by Tillieu, Baudet, and Guy published in same J. **18** (1957), 455–458."

7767:

Adams, William H. On the solution of the Hartree-Fock equation in terms of localized orbitals. J. Chem. Phys. **34** (1961), 89–102.

Author's summary: "The Hartree-Fock method is discussed with emphasis placed on the transformation properties of the Hartree-Fock equation. It is emphasized that the Hartree-Fock equation may be solved in terms of nonorthogonal one-electron functions, and that in some cases it may be more convenient to choose such solutions. Equations are developed which define the localized one-electron functions and it is shown how these equations may be solved. For a system of closed shell atoms or ions, it is suggested that the localized orbitals of each atom or ion can be expanded in terms of functions centered on its nucleus. This suggestion is based on the success of the ionic theory of crystals. Due to the symmetry of a crystal, it is suggested that use of the localized orbitals could lead to expressions for the first order, Hartree-Fock density matrix and the Hartree-Fock energy of a crystal, i.e., one could obtain the solution of the Hartree-Fock equation for a crystal."

7768:

Clinton, William L. Sum rule for transition probabilities. J. Chem. Phys. **34** (1961), 273–275.

Author's summary: "A sum rule is derived for the spontaneous emission transition probability. In particular it is shown that for an atom $\sum_m{}^{\infty} W_{nm} = Z\rho_n(0)$, where W_{nm} is the spontaneous emission transition probability, Z is the nuclear charge, and $\rho_n(0)$ is the electron density of the nth electronic state evaluated at the nucleus."

7769:

Dupont-Bourdelet, Françoise; Tillieu, Jacques; Guy, Jean. Sur le calcul variationnel des propriétés moléculaires. J. Phys. Radium **22** (1961), 9–18. (English summary)

Authors' summary: "A general formalism is set up for the variational calculation of molecular properties (especially, electric and magnetic) in order to account for several simultaneous perturbations. Energies are explicitly calculated up to the fourth order and it appears that the formulae only contain the perturbed functions of the first and second orders. By means of certain hypotheses, the equations and matrix terms can be separated (often in a simply additive form) either to different perturbating operators or to different independent groups of particles (e.g., electrons and nuclei)."

7770:

Singer, K. The use of Gaussian (exponential quadratic) wave functions in molecular problems. I. General formulae for the evaluation of integrals. Proc. Roy. Soc. London. Ser. A **258** (1960), 412–420.

It is shown that for wave functions formed of linear combinations of products of polynomials and generalised Gaussian functions, all potential energy integrals may be reduced to simple quadratures. It is also possible to include without difficulty correlation effects and nuclear oscillations. *D. F. Mayers* (Oxford)

7771:

Longstaff, J. V. L.; Singer, K. The use of Gaussian (exponential quadratic) wave functions in molecular problems. II. Wave functions for the ground states of the hydrogen atom and of the hydrogen molecule. Proc. Roy. Soc. London. Ser. A **258** (1960), 421–430.

The method described in the previous paper [see preceding review] is applied to two simple systems. An extensive discussion is given comparing the results with other work; results show the improvement resulting from the inclusion of correlation parameters, and the use of variable centres. *D. F. Mayers* (Oxford)

7772:

Bazley, Norman W. Lower bounds for eigenvalues with application to the helium atom. Phys. Rev. (2) **120** (1960), 144–149.

A method is described for finding lower bounds to the eigenvalues of the Schrödinger equation. A base problem is chosen, having an exact solution with eigenvalues lower than those of the given problem. Intermediate problems are then defined having eigenvalues between those of the base problem and the given problem. These intermediate problems are solved by means of algebraic matrix calculations, giving a sequence of lower bounds to the eigenvalue. The method is applied to the two lowest states of the helium atom. *D. F. Mayers* (Oxford)

7773:

Greenwood, H. H.; Hayward, T. H. J. Properties of the self-consistent field treatment of conjugated molecules. Molecular Phys. **3** (1960), 495–509.

The polarizabilities of π-electron systems in the self-consistent field method of Pople are examined. Certain symmetry properties are established for substituted hydrocarbons. Numerical values of the polarizabilities for benzene, naphthalene, anthracene and phenanthrene are compared with chemical properties and values determined from the simple Hückel theory.

A. C. Hurley (Melbourne)

7774:

Wulfman, Carl E. Approximate electronic energy surfaces from cuspless wave functions. J. Chem. Phys. **33** (1960), 1567–1576.

The eigenfunctions of a molecular Hamiltonian in which all Coulomb potentials have been replaced by Hooke's law potentials are determined. Their electronic parts are Gaussian wave functions originating at the centre of nuclear charge. These wave functions are used to discuss the stability and shape of a number of approximately homonuclear molecules, especially 21–26 electron triatomics. *A. C. Hurley* (Melbourne)

7775:

Mulckhuyse, Jacob Joan. ★Molecules and models: Investigations on the axiomatization of structure theory in chemistry. Thesis, University of Amsterdam, 1960. vi + 67 pp.

The classical structural theory of organic chemistry is formalized by the use of symbolic logic. At first attention is confined to hydrocarbons and for these the formalization is divided into two parts: an abstract part (theory A) which deals only with the sets of individuals and their relations (the usual valence rules), and a geometrical part (theory A′) which deals with the spatial distributions of the individuals, and can be considered as an extension of the abstract theory. The formalization is carried out as far as possible using only elementary (first-order) logic. For this reason the geometrical axiom system of Tarski [Proc. Internat. Sympos. (Univ. of Calif., Berkeley, Dec. 26, 1957–Jan. 4, 1958), pp. 16–29, North-Holland, Amsterdam, 1959; MR **21** #4919] is employed in theory *A′*. However in order to give an exhaustive description of structure theory it is found necessary to appeal to second order logic. In later chapters extensions of the theory are considered which incorporate steric hindrance (by the introduction of van der Waals radii), ionic structures and atoms other than carbon and hydrogen. The theory remains essentially geometric in character, however, and wave-mechanical considerations such as resonance are outside its scope.

It is claimed that, as well as clarifying the logical foundations of structure theory, the formalized theory may be used as the basis for a more systematic nomenclature for organic molecules. *A. C. Hurley* (Melbourne)

7776:

Furui, Shin-ya; Sakuma, Tetsuro. On the test of Global Symmetry. Progr. Theoret. Phys. **24** (1960), 18–26.

An analysis is made of the $K^- p$ reactions at low energy (only s-waves), under the assumptions of invariance under time reversal, i-spin conservation and global symmetry. Comparison with experiment shows no agreement.

S. A. Wouthuysen (Amsterdam)

7777:

Shirokov, Yu, M. Types of symmetry of elementary particles. Nuclear Phys. **15** (1960), 13–15.

A classification is made, on grounds of relativistic invariance only, of the types of symmetry of elementary particles under spatial reflection, time-inversion and space-time-inversion. *S. A. Wouthuysen* (Amsterdam)

7778:

Lundby, Arne. Weak interactions. Experiments on parity, charge conjugation and time reversal symmetries. Progress in elementary particle and cosmic ray physics, Vol. 5, pp. 1–96. North-Holland, Amsterdam; Interscience, New York; 1960.

This is easily the best survey of the subject of weak interactions available at present. The theoretical part reviews the steady development of theoretical ideas inspired by the discovery of the breakdown of parity and fostered by the brilliant experiments characteristic of this exciting period, culminating in the universal V-A interaction. While none of the "highbrow" theoretical calculations are discussed, the crucial roles of the simpler but more significant calculations and their comparison with experiments (and vice versa!) are clearly set forth. The section on experimental techniques reviews the various methods of polarizing nuclei, the measurement of circular polarization of gamma rays and the determination of electron and positron helicities. Attention is called in the concluding section to the poor state of knowledge of the weak interactions of strange particles, particularly the non-leptonic modes. *E. C. G. Sudarshan* (Rochester, N.Y.)

7779:

Katsumori, Hiroshi. Electromagnetic mass difference of elementary particles. Progr. Theoret. Phys. **24** (1960), 35–38.

The possibility that the mass differences between the members of isotopic multiplets may be of electromagnetic origin is discussed. A simple cutoff procedure is employed for making divergent self-energy integrals and the calculation is carried out to second order in the electromagnetic interaction. The strong interactions are taken phenomenologically through form factors for the charge and magnetic

moment type interactions. The self-energy contributions depend critically on the high momentum behaviour of the form factors which are not known from the electron scattering type experiments; and a class of form factors for nucleons fitted to both the electron scattering data and the mass differences is exhibited. Most of the specific calculations have been made in one form or another by various people (including the author) but are here collected together; the "explanations" are plausible but are hardly more than that; and the same comment applies to the treatment of "strong interaction corrections".

E. C. G. Sudarshan (Rochester, N.Y.)

7780:

Ioffe, B. L. Limits of applicability of the weak-interaction theory. Ž. Èksper. Teoret. Fiz. **38** (1960), 1608–1614 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1158–1162.

The corrections to lowest order calculations using a local four-fermion interaction Lagrangian are all infinite unless some kind of cut-off of the momenta of the virtual particles is introduced. This author assumes that "weak interactions preserve their form up to momenta $\sim\Lambda$" so that all integrations over virtual momenta are carried out up to this magnitude of the four-momenta; and he attempts to determine Λ by comparing the higher order corrections to: (1) the ratio of muon decay and beta decay effective coupling constants; (2) radiative decay of the muon; and (3) three-electron decay of the muon, with experimental data. By a plausible but oversimplified calculation the upper limits to Λ are found to be several hundred Bev except for the comparison with the muon radiative decay; the latter gives an upper limit of about 50 Bev. These numbers can only be trusted as an order of magnitude estimate.

E. C. G. Sudarshan (Rochester, N.Y.)

7781:

McConnell, J. Theory of antinucleons. Progress in elementary particle and cosmic ray physics, Vol. 5, pp. 205–256. North-Holland, Amsterdam; Interscience, New York; 1960.

An elementary summary of antinucleon production and annihilation selection rules and of the predictions of field theory, of various statistical models, and of specific nuclear models. The treatment rather unc tically includes the results of radiation damping, perturbation theory and Johnson-Teller calculations, that would seem to be of principally historic interest, along with theories that explain more realistically the large cross section and pion multiplicity in annihilation processes.

S. Bludman (Berkeley, Calif.)

7782:

Sayasov, Yu. S.; Mel'nikov, V. K. Theory of the capture of particles into the synchronous regime of acceleration taking into account the nonconservation of the equations of motion. Ž. Tehn. Fiz. **30** (1960), 656–664 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 618–626.

Le but des auteurs est de faire le calcul systématique des limites de la région de capture des particules, et ce, en tenant compte de la non-conservation des équations de mouvements.

Le résultat semble être d'un grand intérêt technique.

P. Chevallier (Strasbourg)

7783:

Calogero, F.; Zemach, C. Particle creation in electron-electron collisions. Phys. Rev. (2) **120** (1960), 1860–1866.

Authors' summary: "Pair production in high-energy electron-electron collisions is studied with special attention given to pion pair production. A method of calculation is formulated which yields results with reasonable directness in the relativistic limit. The orders of magnitude of counting rates for various experimental settings are ascertained. A complete result is obtained for the case in which two pions emerge with equal energies and opposite momenta."

7784:

Bosco, B.; De Alfaro, V. Three-pion contribution to the electromagnetic structure of the nucleon. Phys. Rev. (2) **115** (1959), 215–219.

Authors' summary: "In this paper the contribution of three pions to the scalar electromagnetic form factors of the nucleon are computed by using the fixed source meson theory without rescattering corrections. The $(\gamma, 3\pi)$ interaction is taken phenomenologically as a point interaction. It is shown that for those values of the $(\gamma, 3\pi)$ coupling constant compatible with photoproduction experiments, the experimental charge distribution could be roughly fitted with a cutoff in the dispersion integral of the order of 7 pion masses."

7785:

Gol'fand, Yu. A. On the introduction of an "elementary length" in the relativistic theory of elementary particles. Ž. Èksper. Teoret. Fiz. **37** (1959), 504–509 (Russian); translated as Soviet Physics. JETP **10** (1960), 356–360.

Author's summary: "A momentum space of constant curvature is introduced into the theory in place of the pseudo-Euclidean momentum space. The Feynman diagram technique is suitably generalized. Finite results are obtained in the lowest order perturbation theory approximation for the fermion and boson self-energy."

P. Roman (Boston, Mass.)

7786:

Bincer, Adam M. Electromagnetic structure of the nucleon. Phys. Rev. (2) **118** (1960), 855–863.

Author's summary: "Dispersion relations are proved for the electromagnetic and mesonic nucleon vertex functions considered as a function of the nucleon mass. The results are used to express the isotopic scalar and the isotopic vector electromagnetic form factors of the nucleon in terms of pion electroproduction (or photoproduction) and pion-nucleon scattering amplitudes in the $J=\frac{1}{2}$, $T=\frac{1}{2}$ state."

O. Hara (Duluth, Minn.)

7787:

Jensen, J. Hans D. Present status and problems in the theory of beta decay. Lectures in theoretical physics (Boulder, Colo., 1959), pp. 331–365. Interscience, New York, 1960.

The following topics are reviewed. The beta spectrum, angular correlations, recoil experiments, parity violation and the general treatment of polarization of a beam of Fermions; calculation of matrix elements in beta decay; Fermi transitions with parity violation for the free nucleon

and for complex nuclei, Gamow-Teller transitions; relative phases; the decay of the free neutron; symmetry properties of the beta interaction; renormalization problems and the conserved current hypothesis; the effect of the nuclear Coulomb field on the wave function.

P. Roman (Boston, Mass.)

7788:

Lee, T. D.; Yang, C. N. Implications of the intermediate boson basis of the weak interactions: existence of a quartet of intermediate bosons and their dual isotopic spin transformation properties. Phys. Rev. (2) **199** (1960), 1410–1419.

Authors' summary: "Assuming that all weak interactions are transmitted through an intermediate boson field W, it is shown that the observed $|\Delta \mathbf{I}| = \frac{1}{2}$ rule and the small observed mass difference between K_1 and K_2 lead to the conclusion that there exist four W particles: $W^{\pm}$, W^0, and $\overline{W}^0$. Furthermore, a natural assignment of the isotopic spin transformation property of these W particles follows a dual scheme in which the W's behave sometimes as $I=\frac{1}{2}$ and sometimes as $I=1$ particles. Various experimental implications are discussed, including neutrino capture experiments, strong collisions exhibiting apparent nonconservation of strangeness, and strong collisions with apparent lepton production."

G. Feinberg (New York)

7789:

Katayama, Yasuhisa. On the weak interactions. An. Acad. Brasil. Ci. **32** (1960), 195–205.

From the author's summary: "The study of the weak interactions seems to be one of the important trends for the understanding of the natural laws in the realm of elementary particles. This paper is devoted to summarize the present situation in that branch and also to discuss attempts to solve some important problems which are still open, but which may become essential in the near future."

7790:

Iwata, Kenzo. Universality of coupling constants and individual Γ_5-invariance. Progr. Theoret. Phys. **24** (1960), 308–316.

7791:

Jouvet, Bernard. Invariance de jauge, masse du photon et forme asymptotique du propagateur du photon. C. R. Acad. Sci. Paris **251** (1960), 1119–1121.

Author's summary: "On expose une solution au vieux paradoxe de l'incompatibilité de l'invariance de jauge avec l'existence, certaine, d'une masse nue du photon; on en déduit une intégrale première des équations de l'électrodynamique quantique, s'exprimant comme une identité faisant intervenir la constante de structure fine (α) et que doit satisfaire le propagateur du photon (Δ_F'). Cette équation détermine la forme asymptotique de Δ_F' à tout ordre en α."

7792:

Deutsch, Claude. Calcul de l'état fondamental du deutéron à symétrie sphérique. C. R. Acad. Sci. Paris **251** (1960), 1459–1461.

Author's summary: "On établit une théorie phénoménologique du deutéron basée uniquement sur l'échange d'un méson Π^- entre le proton et le neutron. On calcule l'état fondamental à symétrie sphérique à l'approximation non relativiste. On trouve, en négligeant les forces tensorielles, $-2{,}17 \pm 0{,}108$ MeV. On trouve une distribution exponentielle de charge confirmée par diffraction électronique, en ajustant le facteur de structure avec $k = 8{,}25$. On détermine un potentiel neutron-proton en 'cœur dur' répulsif aux très courtes distances."

7793:

Deutsch, Claude. Calcul relativiste de l'état fondamental du deutéron. C. R. Acad. Sci. Paris **251** (1960), 1609–1611.

Author's summary: "On généralise le calcul phénoménologique du deutéron établi dans une Note précédente (#7792). On considère les nucléons comme des sources ponctuelles, sans recul, du champ mésique. On obtient une théorie invariante par conjugaison de charge. On en déduit l'indépendance de charge des forces nucléaires. On dédouble le champ complexe en deux champs scalaires neutres. On voit alors que le potentiel de Yukawa n'a de sens physique que d'un point de vue relativiste. La constante d'interaction proton-neutron est donnée par $g^2/4\pi = e^2$."

7794:

Araújo, J. M. Collective vibrations of closed-shell nuclei. Nuclear Phys. **13** (1959), 360–381.

This is an attempt to determine the frequencies of collective vibrations in nuclei by means of a time-dependent Hartree-Fock method. The self-consistent (H-F)-potential is assumed to be a square well with radius

$$R(\theta, \varphi, t) = R_0(1 + \sum_{lm} \alpha_{lm}(t) y_{lm}(\theta\varphi)).$$

Time dependent eigenfunctions of this well are constructed, and finally the H-F-energy determined. It contains a time dependent term,

$$\delta E = \tfrac{1}{2} \sum_{lm} (C_l(\omega)|\alpha_{lm}(t)|^2 + B_l(\omega)|\dot{\alpha}_{lm}(t)|^2);$$

C and B depend on the assumed frequency ω and are given by

$$C_l(\omega) = F_l(\omega) - \tfrac{1}{2}\omega F_l'(\omega),$$

$$B_l(\omega) = -\tfrac{1}{2}F_l(\omega)/\omega = C_l(\omega)/\omega^2.$$

This gives $F_l(\omega) = 0$ as the equations for the frequency. In contrast to this, the adiabatic approximation gives $(C_l)_{\mathrm{Ad}} = C_l(0)$, $(B_l)_{\mathrm{Ad}} = -\frac{1}{2}F_l''(0)$.

Numerical results for octupole vibrations in O^{16} and quadrupole vibrations in Si^{28} are presented.

F. Villars (Cambridge, Mass.)

7795:

Peretti, J. Remark on dilute Bose systems. Phys. Fluids **3** (1960), 68–71.

The equivalence between the method of Bogolyubov [Acad. Sci. USSR. J. Phys. **11** (1947), 23–32; MR **9**, 168] and that of Lee, Huang, and Yang [Phys. Rev. (2) **106** (1957), 1135–1145; MR **19**, 479] is exhibited.

O. Penrose (London)

7796:

Kumar, Kailash. Validity of the two-particle approximation in the many-body problem. Nuclear Phys. **21** (1960), 99–105.

The coupled integro-differential equations for the correlation functions of the many-body Schrödinger equation derived by W. Brenig [Nuclear Phys. **4** (1957), 363–374; MR **19**, 711] are shown to be equivalent to those obtained in the method of superposition of configurations [R. K. Nesbet, Phys. Rev. (2) **109** (1958), 1632–1638; MR **19**, 1135]. With this connection the author is able to give expressions by which corrections may be calculated to the two-particle approximation of Brenig's formalism. The magnitude of energy corrections is discussed.

J. R. Klauder (Murrary Hill, N.J.)

7797:

Iwamoto, Fumiaki. Inclusion of hole motions in Brueckner theory. Progr. Theoret. Phys. **23** (1960), 871–881.

The equations for small oscillations of a many-fermion system about an equilibrium state are modified to take account of particle-particle and hole-hole couplings; the approximation called the generalized random phase approximation by other workers is obtained. The condition that all oscillations should be stable determines the superfluid ground state. The ground state energy is expressed as a sum of the zero-point energies of the oscillations. An expression for the ground state vector is obtained.

D. J. Thouless (Birmingham)

7798:

Möbius, P. Treatment of special translationally invariant three-body problems by adapted coordinates. Nuclear Phys. **18** (1960), 224–244.

The purpose of this paper seems to be to separate the center of mass motion, rotational motion, spin function and isobaric spin functions from the total wave function in the nuclear many-body problem.

The center of mass motion is separated. The author assumes that the interaction depends only on particle coordinates. Of course, then, we can separate the spin and isobaric spin wave functions. The rotational motion of the whole system is not separated. (It should be possible because the interaction is invariant under the rotation of the whole system.) The adapted coordinates are introduced and the Pauli principle is taken into account. The resulting formula becomes very complicated, even in the three body problem.

T. Sasakawa (Cambridge, Mass.)

7799:

Hugenholtz, N. M. Many body problem in quantum mechanics. Lectures in theoretical physics (Boulder, Colo., 1959), pp. 269–330. Interscience, New York, 1960.

Various methods for calculating properties of the ground states and low lying excited states of large systems of interacting particles are reviewed. The resolvent method and the Green's function method for Fermi particles are worked out in great detail. The limitations and difficulties of these methods are discussed and some basic applications are given. A brief treatment of Bose particle systems concludes this systematic survey.

P. Roman (Boston, Mass.)

7800:

Smith, Felix T. Generalized angular momentum in many-body collisions. Phys. Rev. (2) **120** (1960), 1058–1069.

The kinematics of three- and N-body collision is described in terms of a grand angular momentum tensor Λ_{ij} ($i, j = 1, \cdots, 3(N-1)$). The kinetic energy of the relative motion of N particles appears then in the form

$$T = \frac{1}{2\mu}\left(p_r^2 + \frac{\Lambda^2}{r^2}\right),$$

μ being a reduced mass, r and p_r a radial distance and associated momentum, and $\Lambda^2 = \frac{1}{2}\sum_{ij}\Lambda_{ij}^2$.

The three body problem is discussed in some detail; the relation of this approach to the more conventional description of the 3-body motion is established. For the purpose of a quantum mechanical description of 3-body collision processes, sets of commutating operators (derived from Λ_{ij}) are offered, in terms of whose eigenvalues initial and final states may be described.

F. Villars (Cambridge, Mass.)

7801:

Yokota, Toshio. Expansion theorem of density matrix, virial expansion and new formula of multiple scattering. J. Phys. Soc. Japan **15** (1960), 779–794.

Author's summary: "Expansion formulas for density matrices are derived with the use of the calculus of ordered exponentials. Using the formulas, the expressions for virial coefficients are explicitly obtained. The expression of the third virial coefficient is calculated to obtain its expansion formula in terms of $\hbar^3\beta/M$. The expansion formulas for density matrices can be used to derive a new formula of multiple scattering, which involves Luttinger and Kohn's formula as a special case. The expansion formula for the normalized density matrix is also given. Expansion formulas which are to be applied to irreversible processes and relaxation phenomena are also given."

A. Klein (Philadelphia, Pa.)

7802:

Migdal, A. B. Superfluidity and the moments of inertia of nuclei. Nuclear Phys. **13** (1959), 655–674.

The author presents a method for computing the effective moment of inertia of a finite system of Fermions (a nucleus), based upon Gorkov's Green's function formalism in the theory of superconductivity. The results are dependent upon the average potential assumed for the nucleus; the experimental data lie between the theoretical curves for the oscillator well and the square well, closer to the former. Although it is reasonable to expect abnormally low moments of inertia in a superfluid system, the reviewer feels that the various approximations introduced at different points of the calculation need more detailed discussion; similar computations based on weakly interacting Fermi gases are highly sensitive to approximations. See: Amado and Brueckner [Phys. Rev. (2) **115** (1959), 778–784; MR **21** #6252] and G. Wentzel [Phys. Rev. Lett. **4** (1960), 349–351].

J. M. Blatt (Sydney)

7803:

Fisher, J. C. Alternative superconducting ground states. Austral. J. Phys. **13** (1960), 446–450.

The possibility of a pairing of electrons with the same

spin, instead of electrons with opposite spins, in a superconductor is considered. It is shown that the two possible pairing schemes lead to identical thermodynamics, and that ultrasonic attenuation and nuclear spin relaxation are similar in the two schemes. The spin paramagnetism vanishes at zero temperature if opposite spins are paired, but is large if parallel spins are paired. The author does not consider the fact that parallel pairing involves antisymmetric matrix elements of the interaction, and this would appear to lead to a vanishing of the energy gap in some directions. He concludes that either pairing scheme is compatible with the evidence.

D. J. Thouless (Birmingham)

RELATIVITY

See also 7351.

7804:

Arzeliès, Henri. ★Milieux conducteurs ou polarisables en mouvement. Avec la collaboration de J. Henry. Études Relativistes. Gauthier-Villars, Paris, 1959. xliv+347 pp. 58 NF; $12.12.

Cet ouvrage, d'un caractère très original, est en même temps d'une lecture attrayante, instructive et de nature à stimuler la réflexion et l'esprit critique. Ainsi que le dit M. Arzelies, le sujet traité nécessite une bonne connaissance de l'électromagnétisme et de la relativité. Or si, de nos jours, il convient de fonder l'électromagnétisme relativiste sur les équations de Maxwell, la transformation de Lorentz et la relativité générale, il importe aussi de fixer le sens physique des symboles qui entrent dans les équations, c'est-à-dire de relier les termes mathématiques, d'une part aux définitions, d'autre part aux résultats des mesures; à ce propos, l'auteur insiste sur la signification de quatre sortes de courants: les courants de conduction, de déplacement (ou de polarisation), de convection (ou de Rowland), et de Röntgen, qui apparaissent dans les effets du mouvement des milieux conducteurs ou polarisables. Des références bibliographiques abondantes, ainsi que de nombreuses descriptions d'expériences, anciennes ou récentes, minutieusement critiquées, illustrent l'exposé théorique.

Le livre comprend deux parties et six appendices. La première partie se rapporte au mouvement rectiligne et uniforme, et fait intervenir la relativité restreinte, tandis que la deuxième partie concerne le mouvement accéléré, en particulier le mouvement circulaire uniforme, et s'appuie sur la relativité généralisée. Dans les appendices, on trouve divers développements mathématiques dont il est fait usage au cours du livre: applications du calcul tensoriel, du calcul matriciel, des fonctions spéciales, etc.

Nous parlerons brièvement de la préface, où l'auteur expose des thèses philosophiques que nous ne pouvons juger ici; toutefois, du point de vue strictement scientifique, le lecteur peut y trouver une critique subtile de la connaissance en physique.

P. M. Poincelot (Issy-les-Moulineaux)

7805:

Durell, Clement V. ★Readable relativity. Foreword by Freeman J. Dyson. Harper Torchbooks/The Science Library. Harper & Brothers, New York, 1960. xi+146 pp. $1.25.

This book was first published in England in 1926 [G. Bell & Sons, London] and now a quarter of a century later is still one of the best books on relativity written for the layman who has a knowledge of an elementary algebra and coordinate geometry. These readers will find the book very 'readable' indeed. The fundamental principles of the theory are clearly stated and there are numerous examples. Almost the whole of the book is devoted to the special theory and in the short account of the general theory only gravitational and not electromagnetic phenomena are mentioned. At the conclusion of each chapter there are a number of questions of which the answers are given at the end of the book. Although the book may be recommended as a trustworthy guide to relativity theory, the reviewer does not agree with the statement that the progress of modern astronomy "is due entirely to the successive aids that inventions have given in supplementing the powers of the naked eyes and naked hands" (p. 8).

The American edition contains a foreword by Mr. F. J. Dyson who is a former pupil of Mr. Durell. In this a brief account is given of the author's ability as a teacher and writer.

G. L. Clark (London)

7806:

Štepa, N. I. Motion of a relativistic charged particle in the magnetic field produced by a constant cylindrical current of a rarefied plasma. Ž. Tehn. Fiz. **29** (1959), 1346–1353 (Russian); translated as Soviet Physics. Tech. Phys. **4** (1960), 1237–1243.

Détermination du mouvement d'une particule électrisée dans un champ magnétique constant, à l'approximation de la Relativité restreinte, valable pour des accélérations faibles. Le champ magnétique est produit par une nappe de courants constants, dont la répartition est la suivante: à l'intérieur d'un cylindre circulaire, la densité de courant est uniforme et les filets de courant sont parallèles à l'axe; à l'extérieur du cylindre, le courant est nul. L'auteur décrit le mouvement et la trajectoire. Différents cas sont possibles suivant la valeur de la vitesse initiale et son orientation.

P. M. Poincelot (Issy-les-Moulineaux)

7807:

Figueras, Henri. Géométrie différentielle des trajectoires des particules chargées. C. R. Acad. Sci. Paris **250** (1960), 2143–2145.

L'auteur étudie l'existence et la position des deux- et trois-plans osculateurs en un point d'une trajectoire d'une particule d'épreuve chargée. Il calcule aussi les première et deuxième courbures. Les discussions sont faites en fonction du rapport de la charge à la masse de la particule.

J. Charles-Renaudie (Montpellier)

7808:

Bertotti, Bruno. Significato delle trasformazioni spazio-temporali di Clauser. Ist. Lombardo Accad. Sci. Lett. Rend. A **92** (1957/58), 473–478.

Let

$$ds^2 = g_{00}(x^1, x^2, x^3)(dx^0)^2 - \{(dx^1)^2 + (dx^2)^2 + dx^3)^2\}$$

be a static quadratic differential form; the coordinate transformations keeping the static character invariant have been determined by E. Clauser [Atti Accad. Naz.

Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **13** (1952), 116–120; MR **14**, 807]. The author proves that these transformations can be interpreted as rectilinear uniformly accelerated movements of an observer with respect to the original system. *E. Bompiani* (Rome)

7809:

Dingle, Herbert. Relativity and electromagnetism: an epistemological appraisal. Philos. Sci. **27** (1960), 233–253.

It is argued that the rules of correspondence between physical quantities and the theoretical terms (space and time measurements) in special relativity are inconsistent. The reviewer disagrees. The equations at the foot of p. 246 are not interpreted in special relativity according to the rules which the author uses.

C. W. Kilmister (London)

7810:

Sokolov, A. A. The clock paradox in the motion of charged particles in a magnetic field. Dokl. Akad. Nauk SSSR **131** (1960), 75–77 (Russian); translated as Soviet Physics. Dokl. **5**, 287–290.

The author considers the acceleration arising in connection with the clock paradox as produced by an electromagnetic field, for example on an electron. In this way he avoids appeal to general relativity. In addition to elucidating certain general questions associated with the clock paradox, he points out that the formulae obtained can be tested experimentally, for example, in studies of the lifetimes of spontaneously decaying charged particles.

G. J. Whitrow (London)

7811:

Sherwin, C. W. Some recent experimental tests of the "clock paradox". Phys. Rev. (2) **120** (1960), 17–21.

The author shows that recent experiments on the temperature dependence of the Mössbauer effect in Fe^{57} provide direct experimental confirmation of the assumption, often made in discussions of the clock paradox, that acceleration (as distinct from velocity) has no influence on clock-rate, and also confirmation of second-order velocity-dependent effects on clock-rate. *G. J. Whitrow* (London)

7812:

van Oosterhout, G. W. The clock paradox in so-called relativity theory. Discussion by H. J. Groenewold and B. R. A. Nijboer. Nederl. Tijdschr. Natuurk. **27** (1961), 43–50. (Dutch)

7813:

Kowalski, K. L. Relativistic reaction systems and the asymmetry of time scales. Amer. J. Phys. **28** (1960), 487–491.

The author discusses in detail a particular example of the clock paradox without appealing explicitly to a force field and argues that a time dilatation of arbitrary magnitude is due to accelerations alone, besides the usual effect due to uniform relative motion. *G. J. Whitrow* (London)

7814:

Bradbury, Ted Clay. Relativistic theory of the behavior of clocks. Amer. J. Phys. **28** (1960), 443–446.

Assuming that the reference frame in the vicinity of an accelerated particle is momentarily equivalent to an inertial frame, relative to which its velocity is zero, the author argues that an appeal to general relativity is not necessary when discussing the clock paradox so long as a single clock in the accelerator frame is considered. This appeal is necessary, however, in his opinion, if two clocks separated by an appreciable distance are introduced into the accelerator frame. A solution for this case is given.

G. J. Whitrow (London)

7815:

Datzeff, Assène. Sur le sens physique de la relativité restreinte. Cahiers de Phys. **14** (1960), 99–108.

L'auteur propose de revenir à la conception d'un éther ou subvac (substance of the vacuum), qui n'entraînerait pas les difficultés de l'ancien milieu, et serait compatible avec la Relativité. Son but est de donner un support matériel aux champs, et de tenter d'en approfondir la nature physique. Dans cet exposé, fait sous la forme d'une conférence, l'auteur se montre prudent et demeure volontairement assez vague.

D'un point de vue objectif, indépendamment de toute hypothèse, on peut remarquer que des équations aux dérivées partielles, telles que celles de Maxwell ou de M. Schrödinger, conduisent à l'équation de continuité de l'hydrodynamique, qui confère à l'espace certaines propriétés d'un fluide. En outre, ces relations montrent quelle place tient la mécanique des fluides dans toute la physique.

P. M. Poincelot (Issy-les-Moulineaux)

7816:

Pham Mau Quan. Sur la dynamique analytique du point en Relativité restreinte. C. R. Acad. Sci. Paris **251** (1960), 639–641.

Étude du mouvement d'un point matériel dans un champ de forces statique, dérivant d'un potentiel scalaire, à l'approximation de la Relativité restreinte. Partant de l'expression tensorielle de la force généralisée, l'auteur écrit l'équation aux dérivées partielles du mouvement, sous une forme covariante; puis il définit une fonction de Lagrange et généralise les développements classiques de la Mécanique d'Hamilton et de Jacobi, pour un espace doté d'une métrique riemanienne; il formule les expressions généralisées du principe de Maupertuis, des équations canoniques d'Hamilton et de l'équation d'Hamilton-Jacobi. *P. M. Poincelot* (Issy-les-Moulineaux)

7817:

Tzou, Kuo-Hsien. Étude des champs tensoriels sans condition supplémentaire. Ann. Physique (3) **2** (1957), 778–826.

There exist two general methods of approach to the study of systems of linear partial differential equations (field equations) which are invariant under the Lorentz group. The first is to construct equations which transform according to specified irreducible (or reducible) linear representations of this group [M. A. Naĭmark, *Lineĭnye predstavleniya gruppy Lorenca*, Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1958; MR **21** #4995; chapter 4]; the other is to consider families of equations formulated in some desired way, and then to examine the representations under which they transform. In physical terms, the first procedure specifies the spins of the particles described by

the field in advance, while the second examines the types of particles to be represented by the field. The author follows the latter procedure for systems of field equations such as are used currently in quantized field theories. The major concern is with the use of conditions needed for the identification and specification of the spins of the particles associated with various types of fields. The field equations studied include interaction terms, so that they are capable of describing creation and annihilation of particles of different spins. *E. L. Hill* (Minneapolis, Minn.)

7818:

Ahmavaara, Y. Chirality invariance and the Lorentz group. Ann. Acad. Sci. Fenn. Ser. A. VI No. 48 (1960), 23 pp.

"A unique extension of an analytic Lie group is accomplished by asking for those transformations of the space of the generic parameters onto itself which permute the classes of the group." This technique is used to construct invariant extensions of the proper orthochronous homogeneous Lorentz group and in addition to the inversions the extension also contains the chirality transformation [A. S. Eddington, *Fundamental theory*, University Press, Cambridge, 1949; MR **11**, 144; S. Watanabe, Phys. Rev. (2) **106** (1957), 1306–1315; MR **19**, 925]. Incidentally, this paper gives a leisurely but elegant introduction to the homogeneous Lorentz group; a corresponding discussion of the inhomogeneous Lorentz group is promised.

E. C. G. Sudarshan (Rochester, N.Y.)

7819:

Berenda, Carlton W. Temporal reversal of events in restricted relativity. Amer. J. Phys. **28** (1960), 799–801.

According to the special theory of relativity, when the interval between events in an inertial system S is space-like, the temporal order of these events can be reversed in some other inertial system S'. This paper discusses an idealised experiment which demonstrates this 'time reversal', when the process defining the events in S, or S', has velocity greater than the velocity of light.

C. Gilbert (Newcastle-upon-Tyne)

7820:

Hillion, Pierre; Vigier, Jean-Pierre. Espaces vectoriels invariants sous le groupe des rotations tridimensionnelles complexes conjuguées. Cahiers de Phys. **14** (1960), 219–229.

The authors associate with each Lorentz transformation a rotation in a complex three dimensional Euclidean space and the complex conjugate of this rotation. Explicit representations of the Lorentz group are constructed as functions of the complex Euler angles associated with the complex rotations. Examples of spinor and vector representations are discussed in some detail.

A. H. Taub (Urbana, Ill.)

7821:

Costa de Beauregard, O. Vers la réconciliation des formalismes de la relativité générale et des quanta. Vers la quantification du champ de gravitation. Rev. Questions Sci. (5) **21** (1960), 473–484.

A non-technical account of some of the difficulties in quantising general relativity, and of the progress made by Lichnerowicz and his fellow-workers in overcoming them.

C. W. Kilmister (London)

7822:

Fok, V. A. The Einstein statics in conformal space. Ž. Èksper. Teoret. Fiz. **38** (1960), 1476–1485 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1067–1072.

Let $g_{\mu\nu}$ be the metric of a static or stationary space-time ($\mu, \nu = 0, 1, 2, 3$) with coordinate system adapted so that the (time-like) Killing vector is $\xi^\mu \stackrel{\bullet}{=} \delta_0{}^\mu$. It is usual to study such space-times in terms of the (definite) 3-space metric

$$a_{ij} = g_{ij} - g_{0i}g_{0j}/g_{00} \quad (i, j = 1, 2, 3).$$

The author proposes (at least for static spaces) instead to base the study on the metric $h_{ij} = g_{00}a_{ij}$. Among the consequences, developed in the paper, of this proposal are (i) if the 4-space coordinates are harmonic, so are the 3-space coordinates; (ii) the second derivatives of g_{00} appear in G_{ij} (components of the Einstein tensor) only through h_{ij} and not explicitly. A gravitational stress tensor is identified, and several examples, as well as a possible generalization, are discussed. *F. A. E. Pirani* (London)

7823:

Israel, W. Evolution in general relativity. Nuovo Cimento (10) **18** (1960), 397–399. (Italian summary)

It is shown that "no matter-filled cosmological model satisfying $0 < p < \frac{1}{3}\rho$ can evolve from a flat space-time". Indeterminacy in the evolution of cosmological models is discussed briefly in terms of an equation of state [compare W. B. Bonnor, J. Math. Mech. **9** (1960), 439–444; MR **22** #3552; p. 441]. *F. A. E. Pirani* (London)

7824:

Géhéniau, M. J. Sur les identités attachées à une densité. Colloque sur la théorie de la relativité 1959, 89–91. Centre Belge Rech. Math., 1960.

$\mathscr{F}$ est une densité fonction de tenseurs y^A et de leurs dérivées des deux premiers ordres; une hypothèse de variance de $\mathscr{F}$ par rapport à un groupe de changements de coordonnées engendré par des transformations infinitésimales conduit à des identités satisfaites par les dérivées de $\mathscr{F}$. Des applications de ces formules à la relativité générale sont mentionnées. *J. Renaudie* (Montpellier)

7825:

Papapetrou, A.; Treder, H. Zur Frage der Existenz von singularitätsfreien Lösungen der allgemein-relativistischen Feldgleichungen die Teilchenmodelle darstellen konnten. II. Ann. Physik (7) **6** (1960), 311–327.

The content of this paper consists of a further generalization of the theorem that there is no metric field which (1) tends at infinity to the Minkowskian value, (2) is periodic in time, (3) satisfies Einstein's empty space equations. [The theorem was proved by the authors, same Ann. (7) **3** (1959), 360–372; MR **21** #7054.] The generalization concerns the possibility of the existence of asymptotical null-surfaces which are bounded in the sense that they are confined in finite space-like regions of the Riemannian space. The paper uses geometrical methods of thinking rather than the more computational approach.

J. Plebański (Warsaw)

7826:

Blohinčev, D. I. Fluctuations of space-time metric. Nuovo Cimento (10) **16** (1960), 382–387.

The effects of turbulent motions of matter on the metric tensor and the space-time interval are estimated by calculating the root-mean-square deviations of these quantities from their average values. The statistical effect on the metric in the microworld is estimated for oscillations of the energy of the field particles.

C. Gilbert (Newcastle-upon-Tyne)

7827:

Arnowitt, R.; Deser, S.; Misner, C. W. Interior Schwarzschild solutions and interpretation of source terms. Phys. Rev. (2) **120** (1960), 321–324.

This is an examination of a class of spherically symmetric interior solutions of the Einstein-Maxwell equations which in a certain limit go over into the point-source solutions discussed previously by the authors [Phy. Rev. (2) **120** (1960), 313–320]. The model used is a sphere of incoherent charged fluid which is instantaneously everywhere at rest at $t=0$. It is shown that, if the spatial coordinates are chosen to be isotropic, the entire metric is everywhere nonsingular at $t=0$ and for a finite time subsequently, and that, as the initial radius of the sphere is made to tend to zero, the metric becomes static and remains nonsingular except at the origin.

P. W. Higgs (Edinburgh)

7828:

Forward, Robert L.; Zipoy, David; Weber, J.; Smith, Stewart; Benioff, Hugo. Upper limit for interstellar millicycle gravitational radiation. Nature **189** (1961), 473.

The limit to the Riemann tensor power spectrum, in the vicinity of lc./hr., is 10^{-75}/cm^4 (rad./sec). It was derived from data obtained from a Benioff strain seismograph at Isabella, Calif. during a quiet period when the earth's natural vibrational modes did not contribute to the strains [cf. Weber, Phys. Rev. (2) **117** (1960), 306–313; MR **22** #1426].

F. A. E. Pirani (London)

7829:

Venini, Carlo. Massa di un corpuscolo elettrizzato nella seconda approssimazione dell'ultima teoria unitaria einsteiniana. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **27** (1959), 362–367.

En théorie de la gravitation la variation de la masse d'une particule en mouvement a été déterminée par Einstein et Infeld : termes ajoutés à la masse au repos. La détermination est faite ici au second ordre près de la masse d'une particule matérielle électrisée soumise au champ gravitationnel et electromagnétique d'une autre particule : elle est invariante en première approximation comme en relativité générale ; en seconde approximation on doit ajouter à la masse au repos les termes considérés par Einstein et Infeld et un autre pour un champ électromagnétique non nul ; ce dernier terme est proportionnel aux charges des deux particules et à l'inverse de leur distance. L'expression trouvée peut être généralisé au cas de plusieurs particules. *J. Charles-Renaudie* (Montpellier)

7830:

Bergmann, Otto. Problem of Rainich for two-component spinors. J. Mathematical Phys. **1** (1960), 172–177.

In the "already unified field theory" of the gravitational and electromagnetic fields [e.g., C. W. Misner and J. A. Wheeler, Ann. Physics **2** (1957), 525–603; MR **19**, 1237] the usual Einstein-Maxwell equations are replaced by equations which involve only the Ricci tensor R_{kl} and its covariant derivative, i.e., in which the Maxwell tensor f_{kl} does not appear explicitly. The present paper is intended to be a first attack on the problem of constructing an analogous "already unified theory" of the combined electromagnetic and neutrino fields. In other words, the ultimate aim is to replace the combined set A of Einstein equations with neutrino source and Weyl's equations by a set B of equations involving R_{kl} and its covariant derivative alone, in such a way that A and B are mutually equivalent. Only results incidental to the central problem appear to have been presented. *H. A. Buchdahl* (Hobart)

7831:

Latrémolière, Claude. Sur l'induction électromagnétique en Relativité générale. C. R. Acad. Sci. Paris **250** (1960), 4114–4116.

Dans le cadre de la relativité générale sont étudiées les propriétés de deux formes H et G liées au champ électromagnétique (notations de A. Lichnérowicz). A l'approximation linéaire on a $G_{\alpha\beta}=\frac{1}{2}\varepsilon_{\alpha\beta}{}^{\lambda\mu}H_{\lambda\mu}$; l'auteur montre que la doible deux-forme ε peut être construite à partir des variables de champ. Dans le cas statique le tenseur d'impulsion-énergie est explicité en fonction de G, H, ε.

Les surfaces au travers des quelles les dérivées des deux premiers ordres des champs subissent des discontinuités sont mises en évidence ; elles sont définies à l'aide de cônes caractéristiques du quatrième ordre en général. Dans le cas du vide et pour un front purement électromagnétique on retrouve l'équation de propagation mise en évidence par A. Lichnérowicz [Ann. Mat. Pura Appl. (4) **50** (1960), 1–95; MR **22** #6554]. *J. Charles-Renaudie* (Montpellier)

7832:

Latrémolière, Claude. Sur la radiation pure en présence d'induction électromagnétique. C. R. Acad. Sci. Paris **251** (1960), 338–339.

Cette note fait suite à celle-là dans l'analyse précedénte. L'auteur étudie ici le cas de la radiation pure. Pour ce cas existe en chaque point une direction unique R^a de propagation de l'énergie. L'auteur définit une radiation pure de type intégrable ; elle entraîne $\mathscr{L}(R)H_{\alpha\beta}=0$ où $\mathscr{L}$ désigne une dérivée de Lie. Il est établi que les R^a ont les propriétés classiques des rayons. *J. Charles-Renaudie* (Montpellier)

7833:

Hlavatý, Václav. Aperçu général de la théorie du champ unifié d'Einstein. Colloque sur la théorie de la relativité 1959, 9–23. Centre Belge Rech. Math., 1960.

L'auteur fait ici une étude géométrique de la variété espace-temps X_4 qui lui permettra ensuite une identification entre tenseurs et équations de structure, avec les champs gravifiques et électromagnétiques et les équations qui les régissent.

Les principes de base et les propriétés géométriques qui en résultent ont été exposés aussi par l'auteur dans *Geometry of Einstein's unified field theory* [Noordhoff, Groningen 1957 ; MR **20** #5067].

Principe A : la théorie unifiée est déterminée par 16

potentiels. Principe B: ces potentiels déterminent la courbure et la torsion de X_4. Principe C: ils sont solutions d'équations différentielles du champ imposant des conditions à la courbure et la torsion de X_4; l'identification des champs gravitationnels et électromagnétiques avec les potentiels doit être tirée des équations du champ.

A conduit à l'introduction d'un tenseur quelconque $g_{\alpha\beta}$ dont l'auteur étudie les propriétés géométriques en distinguant trois classes selon les valeurs propres de $g_{[\alpha\beta]}$ (antisymétrisé) par rapport à $g_{(\alpha\beta)}$ (symétrisé). Une interprétation géométrique est obtenue en associant à X_4 l'espace projectif P_3.

B entraîne l'existence d'équations linéaires entre la connexion $\Gamma_{\alpha\beta}{}^{\lambda}$ et le tenseur g et ses dérivées du premier ordre. La résolution en est étudiée pour les trois classes mises précédemment en évidence.

L'étude de C se divise en trois: une étude de la structure de X_4 imposée par les équations déduites de A et de B et en adoptant le dernier système des équations d'Einstein; elle a été faite dans l'ouvrage cité.

Une identification entre tenseurs géométriques et tenseurs de champ: le champ électromagnétique est déterminé d'une manière unique en imposant au tenseur électromagnétique d'être une fonction tensorielle des $g_{\alpha\beta}$ seuls et de satisfaire aux équations de Maxwell; le champ gravitationnel, le tenseur d'impulsion-énergie et la fonction de gravitation sont identifiés par une étude de la forme des équations.

Les équations du mouvement obtenues respectent la loi unifiée d'Einstein: une particule soumise au champ unifié $g_{\lambda\mu}$ décrit une ligne autoparallèle de la connexion.

Des détails de cette étude ainsi que diverses applications sont donnés dans l'ouvrage précité.

J. Charles-Renaudie (Montpellier)

7834:

Debever, M. R. Champs électromagnétiques et champs gravifiques. Colloque sur la théorie de la relativité 1959, 79–88. Centre Belge Rech. Math., 1960.

Dans un espace riemannien de métrique $ds^2=g_{ij}dx^idx^j$ hyperbolique normale est défini le champ électromagnétique par la deux-forme extérieure F. Dans le cas non singulier F peut se décomposer d'une manière unique en $F=Ea+H{}^*a$ où E et H sont deux scalaires invariants de F et a une deux-forme qui définit un deux-plan.

Une condition nécessaire et suffisante pour que à a soit associée une forme F (équations de Maxwell-Rainich) s'exprime sous forme d'équations du second ordre par rapport à a. A une forme a donnée correspond alors F dépendant de deux constantes arbitraires.

Dans la deuxième partie est étudiée la correspondance entre le champ de deux-plans défini par a et les connexions métriques par rapport auxquelles il est un champ de deux-plans parallèles. Deux connexions particulières de ce type sont considérées; on les caractérise à l'aide des composantes irréductibles de leurs tenseurs de torsion S. Alors les équations de Maxwell-Rainich se traduisent resp. par $dS=0$, $d*S=0$. Les conditions d'existence de a associée à une connexion donnée sont établies. L'auteur remarque que les deux connexions doivent être introduites pour définir les champs gravitationnel et électromagnétique, ce qui ne conduit pas à un schéma unitaire très satisfaisant. Deux résultats relatifs aux espaces statiques à symétrie sphérique sont signalés.

J. Charles-Renaudie (Montpellier)

7835:

Debever, Robert; Cahen, Michel. Champs électromagnétiques constants en Relativité générale. C. R. Acad. Sci. Paris **251** (1960), 1160–1162.

Assumptions: The Maxwell-Einstein equations, with a Maxwell field $a_{\lambda\mu}$ whose covariant derivative is a (vector) multiple of itself. Results: (1) Non-singular fields: a and its dual $*a$ have no common direction, and each is covariant constant. Space-time must be conform-Minkowskian. The solutions are those of Bertotti [Phys. Rev. **116** (1959), 1331–1333; MR **22** #1401] and Robinson [Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **7** (1959), 351–352; MR **21** #7070]. (2) Singular fields: a and $*a$ have a common null direction. The Weyl tensor is Petrov type III or lower. The Maxwell-Einstein equations reduce to a single partial differential equation, of which solutions, including all the conform-Minkowskian solutions, are exhibited. The vacuum solutions include plane-fronted waves.

F. A. E. Pirani (London)

7836:

Huyen-Dang-Vu. Le théorème des courants dans la théorie d'Einstein-Schrödinger. C. R. Acad. Sci. Paris **251** (1960), 47–48.

7837:

Kichenassamy, S. Sur une nouvelle forme des identités de conservation en théorie unitaire. C. R. Acad. Sci. Paris **251** (1960), 1349–1351.

Space-time is endowed with a tensor field $g_{\alpha\beta}$, an affine connection $L^{\lambda}{}_{\mu\nu}$ with zero torsion, and a vector field Γ_{ρ}. Known conservation identities are derived from variational principles, without recourse to equations of linear displacement, for a variety of unified theories.

F. A. E. Pirani (London)

7838:

Chau, Nguyen Phong. Sur une généralisation des équations du champ unifié asymétrique. C. R. Acad. Sci. Paris **250** (1960), 3125–3127.

The author examines a generalization of the Einstein-Schrödinger aysmmetric unified field theory, that has been proposed by Tonnelat [same C. R. **250** (1960), 2327–2329; MR **22** #3554]. He shows that some specializations of the numerical coefficients lead to formerly investigated theories. He suggests another possible specialization which leads to relatively simple field equations. [The latter were subsequently discussed in further publications of the same author; see reviews #7839, 7840, 7841.]

A. Peres (Haifa)

7839:

Chau, Nguyen Phong. Sur les identités de conservation d'une généralisation de la théorie du champ unifié asymétrique. C. R. Acad. Sci. Paris **250** (1960), 3579–3581.

By means of the Noether theorem, the author derives the explicit form of the conservation laws that hold in the asymmetric unified field theory proposed in a previous publication [see preceding review]. *A. Peres* (Haifa)

7840:

Chau, Nguyen Phong. Les équations approchée d'une généralisation de la théorie du champ unifié d'Einstein-Schrödinger. C. R. Acad. Sci. Paris **251** (1960), 44–46.

Pour surmonter des difficultés rencontrées en théorie classique d'Einstein-Schrödinger l'auteur propose l'adjonction de termes au lagrangien habituel et forme les équations associées qui sont explicitées à l'approximation du second ordre. L'identification avec les équations de Maxwell conduit au choix de constantes introduites précédemment dans le lagrangien.

J. Charles-Renaudie (Montpellier)

7841:

Chau, Nguyen Phong. Sur l'invariance projective d'une généralisation de la théorie du champ unifié asymétrique. C. R. Acad. Sci. Paris **251** (1960), 207–209.

Une transformation de la connexion $\Gamma_{\alpha\beta}^{\lambda}$ ou $\tilde{\Gamma}_{\alpha\beta}^{\lambda}=\Gamma_{\beta\alpha}^{\lambda}$ de l'espace est dite projective si elle est du type $\Gamma_{\alpha\beta}^{\lambda}\rightarrow\Gamma_{\alpha\beta}^{\lambda}+\delta_{\alpha}^{\lambda}\Lambda_{\beta}$ ou $\Gamma_{\alpha\beta}^{\lambda}\rightarrow\Gamma_{\alpha\beta}^{\lambda}+\delta_{\beta}^{\lambda}\Lambda_{\alpha}$. On considère a lors des équations d'Einstein généralisées déduites d'un lagrangien combinaison linéaire des différents tenseurs de courbure que l'on peut former avec Γ et $\tilde{\Gamma}$ et de $\Gamma_{\alpha}\Gamma_{\beta}$. La condition d'invariance des équations par rapport à une combinaison de deux transformations projectives conduit au choix des coefficients introduits dans le lagrangien.

J. Charles-Renaudie (Montpellier)

7842:

Schmutzer, Ernst. Axiomatik der metrischen Spintensoren in der projektiven Relativitätstheorie. Z. Naturforschg. **15a** (1960), 831–835.

Generally covariant spinor algebra in 5-space is developed from assumptions of hermiticity and of product structure for the (5-dimensional) connecting quantities $\sigma_{\mu j\beta}$. Projection into space-time yields results agreeing with the author's earlier work [same Z. **15a** (1960), 355–362; MR **22** #603]. *F. A. E. Pirani* (London)

ASTRONOMY

See also A6669, 7351, 7611.

7843:

Baker, Robert M. L., Jr.; Makemson, Maud W. ★An introduction to astrodynamics. Academic Press, New York-London, 1960. xiv+358 pp. $7.50.

About half the book is taken up with celestial mechanics as the term is ordinarily understood. The remainder is concerned with other problems related to the motion of natural and artificial bodies in the solar system. In the theoretical portions, the emphasis is on results and on the relation to observational and engineering problems.

In the first of two sections, there is an introduction, in which the equations of the two-body problem in the plane are derived, followed by historical and observational chapters on comets and minor planets. The problem of the orbit in space is next treated, in conjunction with that of space coordinate systems; the first section is closed by an important chapter on the constants of the solar system and the atmosphere of the earth.

In the second, more advanced section, the principal methods of orbit determination are outlined in the first chapter, with emphasis on vector treatment and the Gibbs method. Next the general n-body problem is stated, including lift and drag forces; and in the following two chapters, methods of special perturbations (numerical integration of the equations of motion) with emphasis on Encke's method and the variation of parameters, and methods of general perturbation (integration by trigonometrical series) are described.

The last three chapters discuss drag forces, low thrust forces (such as are expected from ion rockets) and other non-Newtonian effects; the methods of observation and tracking; and the application of astrodynamics to the planning of interplanetary orbits.

There is a glossary of terms (27 pages) a glossary of symbols, and 10 pages of exercises.

The authors are members of the Herrick school of thought both in approach and theory. The book is aimed at the engineer who needs the results of dynamical calculations in space. It will also interest the general reader because the authors state the motivation of their calculations with emphasis and vigor.

J. A. O'Keefe (Chevy Chase, Md.)

7844:

Von Hoerner, Sebastian. Die numerische Integration des n-Körper-Problemes für Sternhaufen. I. Z. Astrophys. **50** (1960), 184–214. (English summary)

The author sets out to approach the problem of stellar dynamics by integration of the equations of motion of a limited number n of mass-points ($n=4$, 8, 12, and 16) approximating a small cluster. The numerical results reveal that, after the lapse of relaxation lines agreeing closely with Chandrasekhar statistical prediction, the internal structure is obtained which shows an increasing excess density due to the escape of mass particles from the centre; while near the centre the density distribution approximates that of an isothermal polytropic gas sphere within 25%. The velocity distribution is Maxwellian within 17%, but shows excess of particles with higher velocities.

Z. Kopal (Manchester)

7845:

Suharev, L. N. The simultaneous determination of latitude and clock correction from observations of four stars at the same altitude. Astr. Ž. **37** (1960), 555–566 (Russian); translated as Soviet Astr. AJ **4**, 528–538.

Description of methods for choosing stars from a catalog in such a way that they will be grouped in triples or quadruples having the same altitude, or nearly so, at a given instant, and will be well-distributed in azimuth. The choice depends on formulas of spherical trigonometry which are represented by nomograms.

J. A. O'Keefe (Chevy Chase, Md.)

7846:

Pariĭskiĭ, N. N. The influence of earth tides on the secular retardation of the earth's rotation. Astr. Ž. **37** (1960), 543–549 (Russian); translated as Soviet Astr. AJ **4**, 515–522.

7847:

Leimanis, E. Qualitative methods in general dynamics and celestial mechanics. Appl. Mech. Rev. **12** (1959), 665–670.

Survey article.

7848:

Lindblad, Olof. On tidal interaction between galaxies. Stockholms Obs. Ann. **21**, no. 3, 38 pp. (1960).

The motion of a test particle under the gravitational attraction of two bodies, one of which (satellite galaxy) is considered to be a point-mass and moving in a circular orbit around the other (primary galaxy), is discussed to show the possibility of the formation of an intergalactic bridge arising in a double galaxy as a consequence of tidal interaction between the components. Assuming that both the satellite galaxy and the test particle move in the plane of symmetry of the primary, the problem is analogous to that of the main problem of the lunar theory except that the primary is not assumed a point-mass but has the mass distribution of the Schmidt model.

In an earlier part of the paper an analytical theory of the first order is developed on the assumption that the test particle moves in an orbit nearly circular around the primary and with radial and tangential deviations from it as dependent variables. The equations of motion, of the Gyldén-Lindstedt type, are treated by the method of characteristic exponents with emphasis on the resonance effect which arises when the mean motion of the apse of the test particle is close to that of the circular motion of the satellite galaxy. The characteristic exponent, γ, and the mean motion of the apse, $\kappa-\omega$, depend on the position angle ϑ of the apocenter referred to the satellite:

$$\gamma = \frac{\mu}{2\omega} P \sin 2\vartheta, \quad \kappa^2-\omega^2 = \mu P \cos 2\vartheta,$$

where κ is the frequency of radial oscillation, ω the mean motion of the position angle of the particle in a general solution of the associated homogeneous differential equations, P a quantity depending on the distance of the satellite from the primary and the radius of the reference circular orbit of the particle as well as ω, and μ a parameter depending on the distance only. All these quantities are constant in the first order theory. Maximum instability arises at $\vartheta=\pi/4, 3\pi/4$; that is, in the resonance case $\kappa=\omega$.

When the effects of the second and higher orders are considered, some of these parameters cease to be constant and depend on the amplitude of the oscillation. The shift of values of γ is particularly important for the discussion of the development of the orbit. The remaining part of the paper is devoted to study the behavior of the motion over a long interval of time. The equations of motion in rotating cartesian coordinates with the mean motion of the satellite as the angular velocity are numerically integrated with use of electronic computers.

Based on the preliminary analytical study, the following four cases of the motion are investigated as being of interest.

1. Resonance case. Proper values of the initial position and velocity components are given by formulas derived in the preliminary study. These depend on the initial position angle of the particle, θ_0, and ϑ. Assigning $\pi/4$ to ϑ, several examples with different values of θ_0 are studied. The time interval of integration covers several billion years. By these numerical examples several types of development of orbits are shown. The expansion of orbits due to the factor $\exp(\gamma t)$ is found in the direction $\theta\approx 65°$ instead of 45° which is expected from the linear theory.

2. Scanning through the region of instability. The region of maximum instability is studied by computing a series of orbits for various values of the radius of the reference circle and the possibility of ejection of matter to large distances is demonstrated.

3. Counter-tide. The case $\vartheta=3\pi/4$ is examined. Orbits expand in the direction $\theta\approx 3\pi/4$ as expected from the linear theory. The results show the possible existence of counter-tide.

4. The case when the distance of the satellite from the primary is large. The mean motion of the apse changes rapidly during the first several revolutions and reduces to the case of simple resonance, which may be consistent with a theorem of Hagihara about stability of Gyldén-Lindstedt type of equations.

Finally the effect of a close passage by a field galaxy is examined in a numerical example.

Gen-ichiro Hori (New Haven, Conn.)

7849:

Lindblad, Olof. The development of spiral structure in a galaxy approached by numerical computations. Stockholms Obs. Ann. **21**, no. 4, 73 pp. (1960), (3 plates).

The development of spiral structure in a galaxy is discussed, based on the behavior of a dynamical system consisting of several dozens of mass-points arranged initially along concentric rings around the center of the galaxy. The mutual attractions of the particles are considered as well as a central force due to the potential of the Schmidt model.

The development of a single ring is first discussed. The initial configuration of the particles along a slightly elongated circle (dispersion ring) is determined by the theory of dispersion orbits. The orbit of a particle under the attraction of the central force is, in a rotating coordinate system with the angular velocity ω',

$$\xi = c_1 + c\cos n(\theta-\theta_0),$$

where ξ is the differential radius, θ the longitude; n a function of ω', κ (the frequency of ξ in the epicyclic motion), and ω (the circular angular velocity appropriate for the Schmidt model). Then, n is a function of ω' and c_1, the latter through κ and ω. If ω' is so chosen that n is stationary with the first order variation of c_1, the above epicyclic motion is a dispersion orbit. A cloud of particles with somewhat different values of c_1 will disperse due to the differential rotation but along the dispersion orbit. Important cases are when n is an integer; then the dispersion orbit is closed. Under the Schmidt model of the attraction, $n\approx 2$ in an inner region (up to the central distance 10 kpc.) and $n\approx 1$ in the outer region. The latter is expected because the central force is reduced to inverse square law in the outer region of the Galaxy. After choosing a dispersion orbit with proper angular velocity of the rotating coordinates, particles are arranged along the orbit; the configuration is a dispersion ring. Balancing of the ring against mutual attractions of the particles is taken into account.

The development of the dispersion ring is traced by numerical integration with use of electronic computers. In order to avoid the singularity of collision, a lower limit is assumed to the mutual distance of particles within which the attraction is taken to be proportional to the distance. The effect of this discontinuity of the derivative of the force is considered by adopting different values for the limiting distance. The mass of the particle is of the order $10^7 M_\odot$, the size of the dispersion rings are several kpc., the

limiting distance is about 1 kpc, and the integration covers a billion years. The dispersion rings are found to be stable.

Next, the perturbation of a stable dispersion ring on a surrounding circular ring or rings is discussed. The surrounding rings are gradually deformed to reveal an arm-like structure in time interval covered by the integration. The discussion is followed by the interactions between a dispersion ring and massive circular rings. They result in the development of a tightly wound spiral structure with leading arms that, at a later stage, tend to shift over into trailing ones.

Finally, as an alternative initial deviation from rotational symmetry, a circular ring is provided with a bisymmetrical density variation along its circumference with approximately the angular velocity of circular motion. The integration reveals the development of bar-type structure. *Gen-ichiro Hori* (New Haven, Conn.)

7850:

Wellmann, P. Die Abteilung der Elemente photometrischer Doppelsternsysteme mit Hilfe einer Elektronenrechenmaschine. Math.-Tech.-Wirtschaft **7** (1960), 55–58.

7851:

Agekyan, T. A. General features of the evolution of rotating systems of gravitating bodies. Astr. Ž. **37** (1960), 317–326 (Russian. English summary); translated as Soviet Astr. AJ **4**, 298–307.

Description qualitative des systèmes stellaires. L'auteur introduit la notion de champ regulier pouvant être stationnaire et de champ irrégulier localement et non stationnaire. Le mécanisme de l'evasion est décrit de façon très simplifiée et purement qualitative. Un grand nombre d'idées exprimées ici sont très ingénieuses mais méritent vérification quantitative. *E. Schatzman* (Paris)

7852:

Cook, G. E.; King-Hele, D. G.; Walker, Doreen M. C. The contraction of satellite orbits under the influence of air drag. I. With spherically symmetrical atmosphere. Proc. Roy. Soc. London. Ser. A **257** (1960), 224–249.

The authors study the effect of air drag on the orbits of artificial Earth satellites by perturbation methods, under the assumptions that: (1) the terrestrial atmosphere is spherically symmetrical and at rest with respect to the surface of the Earth; (2) the air density diminishes exponentially with height, and the resulting head-drag is proportional to the square of the velocity of the satellite; (3) the eccentricity of the satellite's orbit is small ($e \leqq 0.2$); (4) gravitational perturbations arising from the terrestrial oblateness as well as from the attraction of the sun and the moon are ignorable.

Under these conditions; the equations for the perturbations of the elements are integrated by quadratures; and the results disclose the way in which the perigee distance and the orbital period of the satellite diminish with the time, as a result of the gradual dissipation of kinetic energy due to air resistance. *Z. Kopal* (Manchester)

7853:

Cook, G. E.; Plimmer, R. N. A. The effect of atmospheric rotation on the orbital plane of a near-earth satellite. Proc. Roy. London. Ser. A **258** (1960), 516–528.

The authors integrate by quadratures the equations for the perturbations of the orbital elements of an artificial Earth satellite caused by atmospheric drag, under conditions identical with those adopted previously by Cook, King-Hele, and Walker [#7852], and obtain explicit expressions for the secular perturbations causing the orbital plane to rotate about the Earth's axis, and affecting its inclination. *Z. Kopal* (Manchester)

7854:

Kozai, Yoshihide. Effect of precession and nutation on the orbital elements of a close earth satellite. Astr. J. **65** (1960), 621–623.

The motion of an artificial earth satellite is usually studied assuming that the equatorial plane is fixed in inertial space. This is not strictly true, however, since the earth undergoes precessional and nutational motions which will introduce several small time-dependent terms in the expression for the earth's gravitational potential. The author considers both a fixed and moving coordinate system and suggests that, for precise orbital studies, the inclination and the argument of perigee should be referred to the equator of date, and the longitude of the node should be measured from a fixed point along a fixed plane and then along the equator of date. The orbital elements given in Smithsonian Astrophysical Observatory Special Reports are now referred to this system.

G. E. Cook (Farnborough)

7855:

Geyling, F. T. Satellite perturbations from extra-terrestrial gravitation and radiation pressure. J. Franklin Inst. **269** (1960), 375–407.

The author sets out to integrate the Hamiltonian equations governing the motion of an artificial Earth satellite, in a moving system of coordinates consisting of an orthogonal triad whose origin is located on an elliptical orbit which the satellite would describe in the absence of perturbations; in this way the actual positions (rather than osculating elements of the orbit) are obtained for practical applications. Calculations have been carried out for the effects of perturbations of the initial conditions of the satellite motion, as well as those due to the terrestrial oblateness, the lunar and solar attraction, and the solar radiation pressure. *Z. Kopal* (Manchester)

7856:

Sobolev, V. V. The theory of stellar evolution. Astr Ž. **37** (1960), 387–395 (Russian); translated as Soviet Astr. AJ **4**, 372–379.

Étude de l'hypothèse que l'équation d'énergie n'est plus stationnaire. Le temps d'évasion du rayonnement depuis l'intérieur de l'étoile est de l'ordre de GM^2/LR, c'est-à-dire, de l'ordre de l'échelle de temps de Kelvin-Helmholtz. Il en résulte que si ce temps est plus long que la durée de vie d'une source nucléaire d'énergie, il faut utiliser l'équation d'énergie non stationnaire. *E. Schatzman* (Paris)

7857:

Gilvarry, J. J. Escape of planetary atmospheres. I. Escape layer. Phys. Fluids **4** (1960), 2–7.

L'auteur établit une rélation entre l'altitude de la couche d'évasion et les paramètres physiques de la couche atmosphérique immédiatement sous-jacente. La vérification avec les données numériques de Nicolet est médiocre, en raison du caractère simplifié du modèle d'atmosphère utilisé. Cependant l'auteur obtient une estimation raisonnable de l'altitude d'évasion dans l'atmosphère terrestre (450 km.). L'auteur examine également la diffusion thermique d'une espèce peu abondante dans l'atmosphère.

E. Schatzman (Paris)

7858:

Gilvarry, J. J. Escape of planetary atmospheres. II. Lifetimes of minor constituents. Phys. Fluids **4** (1961), 8–12.

Une correction importante est apportée à l'évaluation de Spitzer, mais le désaccord subsiste entre la température estimée pour les besoins de l'évasion de l'hélium, et la température déduite des vols de fusée et de l'échelle de hauteur des couches *F*. *E. Schatzman* (Paris)

7859:

Sen, K. K. On the estimate of the asymptotic value of optical thickness of a spherically symmetric stellar atmosphere in the non-conservation case in the second approximation. Indian J. Theoret. Phys. **7** (1959), 45–52.

Author's summary: "The asymptotic value of the optical thickness of a spherically symmetric stellar atmosphere has been calculated in the second approximation for non-conservative isotropic scattering case. Wick-Chandrasekhar method has been used for solving the integro-differential equation of transfer appropriate to the problem."

7860:

Wilson, P. R. The constant flux problem in non-uniform exponential media. Austral. J. Phys. **13** (1960), 461–469.

The paper gives an approximate solution of the radiative transfer equation

$$\Delta J = \frac{1}{\chi} \nabla J \cdot \nabla \chi,$$

where J denotes the total intensity and χ the attenuation coefficient derived by Giovanelli [R. G. Giovanelli, Austral. J. Phys. **12** (1959), 164–170; MR **21** #7081] under the assumption that the medium is in radiative equilibrium and that the flux across planes parallel to the surface is constant. The medium is assumed to occupy the semi-infinite space $z \leq 0$ and to be isotropic. The attenuation coefficient is given by

$$\chi = \chi_0 e^{-\alpha z}(1 + \alpha \cos lx)$$

so that it increases exponentially with depth below the surface of the medium and varies sinusoidally in one direction parallel to the surface. The author determines the emission of radiation and the total and the directional intensity at the surface, correct to the first power of α. The solution is applicable in some of the astrophysical problems like granulation in the solar photosphere and structure of umbra and penumbra of sunspots and is a generalization of the work of Giovanelli which does not take into account the variation of the physical quantities with depth. *P. L. Bhatnagar* (Bangalore)

7861:

Kahalas, Sheldon L. Magnetohydrodynamic wave propagation in the ionosphere. Phys. Fluids **3** (1960), 372–378.

The dispersion relations for the propagation of magneto-hydrodynamic waves in a compressible medium under conditions appropriate to the ionosphere are reviewed; in particular it is assumed that the Alfvén velocity is much greater than the sound velocity. The dependence of attenuation of the three modes on the direction of propagation is shown for the case of large finite conductivity, and the effects of the electron pressure and Hall current are shown to be negligible.

Finally, the transmission of electromagnetic energy across a sharp plane boundary between an incompressible perfectly conducting plasma and a vacuum is investigated for waves travelling along the magnetic field, which is normal to the boundary. The coupling coefficient is exhibited as a function of the frequency; in the ionosphere it is estimated as about 1 per cent. *K. C. Westfold* (Clayton)

7862:

Prendergast, Kevin H. The motion of gas streams in close binary systems. Astrophys. J. **132** (1960), 162–174.

An approximate solution of the equations of hydrodynamics has been obtained for the flow of gas in the vicinity of a close binary system (in which, for the assumed density of 10^{10} particles per cc, the mean free path is of the order 10 km and, therefore, small in comparison with the dimensions of the system), under the assumption that the pressure terms in the equations of motion are small in comparison with the gravitational and centrifugal forces. The velocity component perpendicular to the plane of the binary orbit is ignored, and so are its components perpendicular to the Lagrangian surface of zero-velocity in the gravitational dipole of the revolving stars. It is shown that the Jacobi integral of the restricted problem of three bodies admits of a hydrodynamic analogue, and the existence of the integral makes it possible to obtain an elementary solution of the equation of continuity.

It appears that the gaseous envelope surrounding the binary system is this unless the mean-square turbulent velocity is comparable with the orbital velocity of the two stars. The velocity has been computed for the mass-ratio $m_1/m_2 = 3$ of the two finite bodies; and the flow pattern thus found exhibits several features in common with the empirical models which have been constructed to account for the spectroscopic observations of such eclipsing systems as RW Tauri or RZ Scuti. *Z. Kopal* (Manchester)

7863:

Cole, K. D. Hydromagnetics and stellar structure. Nature **189** (1961), 31–33.

7864:

Laird, M. J. Magneto-hydrostatics of stellar atmospheres. I. The stability of the axially symmetric case. Monthly Not. Roy. Astr. Soc. **121** (1960), 197–200.

Calcul de l'énergie potentielle dans le cas particulier d'un champ magnétique ayant la symétrie axiale et compte tenu de la gravité. La valeur propre de l'équation différentielle du mouvement apparaît alors comme le quotient de deux intégrales. Le résultat, analogue aux autres résultats classiques maintenant sur l'équilibre d'un plasma en magnéto-hydrodynamique, conduit à une condition suffisante de stabilité. *E. Schatzman* (Paris)

7865:

Bondi, H. Magneto-hydrostatics of stellar atmospheres. II. The axially symmetric equilibrium configurations. Monthly Not. Roy. Astr. Soc. **121** (1960), 201–207.

Étude de l'équation aux dérivées partielles de l'équilibre dans le cas axi-symmétrique. La solution dépend de deux fonctions $N(V)$ et $L(V)$, détérminées par les conditions aux limites, et de deux fonctions de la pression $P(p)$ et du rayon $S(R)$ que l'on pourrait détérminer si l'on connaissait mieux les processus dans la couronne. À défaut, l'auteur envisage des cas particuliers. Il est possible que les lignes de force dans certaines conditions se détachent de la surface de l'étoile, ce qui introduit une condition de stabilité. *E. Schatzman* (Paris)

7866:

Laird, M. J. Magneto-hydrostatics of stellar atmospheres. III. The axially symmetric equilibrium configurations (continued). Monthly Not. Roy. Astr. Soc. **121** (1960), 208–212.

Intégration de l'équation d'équilibre dans un cas plus général que celui, envisagé par Bondi [#7865]. Étude d'un exemple. Extension au cas d'une étoile en rotation avec des hypothèses plus restrictives sur la structure de l'atmosphère. *E. Schatzman* (Paris)

7867:

Bonnor, W. B. The relativistic model of the steady-state universe. Monthly Not. Roy. Astr. Soc. **121** (1960), 475–481.

McCrea [Proc. Roy. Soc. London. Ser. A **206** (1951), 562–575; MR **12**, 866] has proposed a general relativity interpretation of the steady-state theory. It is now pointed out that the interpretation depends on the equation of state $p+\rho=0$, p being the pressure, and ρ the density, of the material in the model universe. This equation of state implies that the velocity of the material in the model universe is completely undetermined. Another consequence is that matter with this equation of state is indifferent to the presence of a gravitational field. McCrea has also suggested a Newtonian interpretation of the steady state cosmology, in which, however, the analogue of the mass of a unit element of fluid is taken to be $p+\rho$. Since, in ordinary units, the symbol p stands for the pressure divided by the square of the velocity of light, this involves a modification of Newtonian theory because terms of this kind tend to zero on passing from relativity theory to Newtonian mechanics. Bonnor argues that, since $p+\rho=0$, the Newtonian equation of motion of a small fluid element gives no determinate acceleration for the element. He concludes that the motion of matter is also indeterminate in the Newtonian model of the steady-state universe. *G. C. McVittie* (Urbana, Ill.)

7868:

Rastall, Peter. Mach's principle and scalar theories of gravitation. Canad. J. Phys. **39** (1961), 218–219.

The author presents an argument to show that any generally covariant theory of gravitation derived from a scalar potential is incompatible with Mach's Principle. *W. B. Bonnor* (London)

7869:

Wigner, Eugene P. Measurement of the curvature in a two-dimensional universe. Phys. Rev. (2) **120** (1960), 643.

Correction (with detailed derivation) of formula (5.7) of an earlier paper in Rev. Mod. Phys. **29** (1957), 255–268 [MR **19**, 925]. *F. A. E. Pirani* (London)

7870:

Chambers, Ll. G. The Hund gravitational equations and the expanding universe. Monthly Not. Roy. Astr. Soc. **120** (1960), 263–270.

The author states that his object is "to provide a semi-Newtonian cosmology". He starts from field-equations proposed by F. Hund [Z. Physik **124** (1948), 742–756; MR **11**, 410] which imply that there is creation of mass per unit volume. Approximate solutions of the field equations are obtained under the hypothesis that the "Hubble constant" is independent of the time. The model universes so deduced have ages that run from 0.2 to 13 billion years, according to the assumed mean density of the universe, the reciprocal of the Hubble constant being taken as 13 billion years. *G. C. McVittie* (Urbana, Ill.)

7871:

Swann, W. F. G. Processes involved in electromagnetic acceleration of particles to cosmic-ray energies. J. Franklin Inst. **270** (1960), 343–352.

The author discusses a betatron type mechanism for the acceleration of particles to cosmic ray energies and considers the case of axially symmetrical problems. *H. Messel* (Sydney)

7872:

Arend, S. ★Le problème géométrique du rattachement d'un astre a n étoiles de référence, basé sur le principe de l'homographie linéaire. Actes des colloques de calcul numérique, Caen, 1955; Dijon, 1956, pp. 87–109. Publ. Sci. Tech, Ministère de l'Air, Notes Tech. no. 77, Paris, 1958. vi+144 pp. 2105 francs.

The astrometric problem of determining positions of an unknown object (X_0, Y_0), on a photographic plate containing $n>3$ comparison stars whose positions are known is handled in the following manner: C_3^n triangles are selected and a unique set of dependences is determined for each triangle with respect to the object. The discordances in the positions of the object arising from the linear dependence solutions are minimized in the sense of least squares. This results in a solution of the form

$$X_0 = \sum D_i X_i, \quad Y_0 = \sum D_i Y_i. \tag{1}$$

As a check

$$x_0 = \sum D_i x_i, \quad y_0 = \sum D_i y_i, \quad 1 = \sum D_i. \tag{2}$$

The D_i have the property of dependences in that they

provide a weighted mean, according to (1) and along with the last of (2) for the position of the object.

After developing the formulae for special cases of a stellar quadrilateral, pentagon and hexagon, the author develops a concise solution in terms of cracovians.

Advantages claimed for this approach are (1) the number of arithmetic operations in computing a position is reduced, (2) the equations are simple, (3) comparison of positions determined from the ternary dependences gives an idea concerning the error in X_0, Y_0 and (4) there is a large number of numerical checks that can be applied as controls on the calculation.

Geometrically, the adjustment by least squares of a stellar polygon on one plate $A_1A_2\cdots A_n$ to that of the same polygon on a second plate $A_1'A_2'\cdots A_n'$ amounts to an affine deformation in which the center of mass remains unchanged, i.e., the weights remain unchanged.

The author also shows that if the dependences are computed as corrections to their mean value $D_i=n^{-1}+\delta D_i$ according to the principle of least squares, the optimal dependences occur when the $\delta D_i=0$. This result has, of course, long been known, but other interesting relations follow. Furthermore, a remeasurement of X_i, Y_i does not necessitate a recalculation, for then

$$\delta X_0=\sum D_i\delta X_i,\quad \delta Y_0=\sum D_i\delta Y_i.$$

Another method of solution presented is the weighted mean of positions obtained from two separate triangles. This leads to a solution analogous to the stellar quadrilateral. Finally, several useful, numerical examples, for four, five and six stars are given as well as the six-star, two-triangles case. *M. S. Davis* (New Haven, Conn.)

7873:

Stern, D. On the interpretation of cosmic ray μ-meson data. Nuovo Cimento (10) **18** (1960), 1–8. (Italian summary)

The author investigates the mathematical problem of determining the nature of the primary interaction in extensive cosmic ray airshowers from detailed experimental knowledge of the μ-meson component at sea level or underground. *H. Messel* (Sydney)

7874:

Hart, A. B. Problems in the determination of contours of solar lines near the extreme limb. Monthly Not. Roy. Astr. Soc. **120** (1960), 106–120.

The distorting effects of the earth's atmosphere on the distribution of light at the extreme limb of the sun are considered in this paper.

The redistribution function consists of two independent terms; the time-averaged effects of seeing are represented by a Gaussian term and particle scattering and nonspecular reflection by the telescope optics are included in a power-law term. This redistribution function forms the kernel of the integral equation of Fredholm's first type relating the true and observed intensities at any point.

Methods of solving the equation which is modified by geometrical idealization are investigated and it is shown that the solutions must be regarded as discontinuous.

These methods are applied to observations of K line of CaII at points within $\pm 10''$ arc of the limb.

Y. Kozai (Cambridge, Mass.)

7875:

Whitehead, J. D. Focusing of radio waves reflected from a rough curved ionosphere. Austral. J. Phys. **13** (1960), 621–624.

Author's summary: "The reflection of radio waves from a partially rough, curved ionosphere is considered. The relationship between the amplitude of the echo, A, and phase path P when the ionosphere moves overhead with a horizontal velocity V at a height h is the same as that for a smooth curved ionosphere, i.e., $A^2\propto 1-(h/2V^2)\,d^2P/dt^2$, although because of the different physical conditions the large increases in echo amplitude observed when reflection takes place from a smooth ionosphere are not expected for reflection from a rough ionosphere. A method of testing the relationship experimentally is suggested."

7876:

Cohen, M. H. Magnetoionic mode coupling at high frequencies. Astrophys. J. **131** (1960), 664–680.

This paper discusses the important problem of radio propagation in a magnetoionic medium in which the properties of the medium, particularly of the imposed magnetic field, may vary so fast that the assumption of local homogeneity is not valid. The coupling of the propagating modes may then be so strong that Faraday rotation is absent; again, the polarization of the resultant wave may not then be determined by the local conditions along its path.

The calculations are based on the assumption that the medium is stratified in planes parallel to the wave front and that the standard parameters X, Y, Z remain small. The resulting coupled differential equations are solved approximately, following Budden [Proc. Roy. Soc. London Ser. A **215** (1952), 215–233; MR **14**, 824], by variation of the parameters associated with the first-order *WKB* solutions of the uncoupled equations. The degree of coupling is shown to depend on a ratio which defines a transitional frequency between the extreme cases of strong and weak coupling. This frequency is calculated for the standard cases of quasi-longitudinal (QL) and quasi-transverse (QT) propagation, from which conclusions are drawn on the effects on radiation traversing ionospheric, interplanetary, and interstellar regions. Under the conditions stated, coupling is weak for the QL conditions that will normally be encountered, but in the narrow band of directions where QT conditions will obtain a number of interesting effects may result. *K. C. Westfold* (Clayton)

7877:

Gerks, Irvin H. Scattering of electromagnetic waves in the troposphere and the use of this technique for communications. Proc. Iowa Acad. Sci. **67** (1960), 399–430.

Author's summary: "This article is an introduction to the subject of tropospheric scatter propagation for the nonspecialist. It opens with a review of various modes of propagation which may exist in a non-turbulent atmosphere, such as diffraction and ionospheric reflection. The scattering of energy in a turbulent medium is then examined, and statistical methods are introduced to describe the resultant field. Special characteristics of the signal are discussed, such as variation with distance, climatic effects, frequency dependence, fading, bandwidth, and noise level. The paper concludes with a description of methods and

equipment employed in the design of a communication system operating over a distance in the range of 100 to 500 miles. It is concluded that tropospheric scatter terminals are large and costly, but that under some circumstances such a system has economic advantages over a line-of-sight relay system and can furnish comparable quality and reliability."

7878:

Wheelon, Albert D. Relation of turbulence theory to ionospheric forward scatter propagation experiments. J. Res. Nat. Bur. Standards Sect. D **64D** (1960), 301–309.

This is a short review paper, devoted to a comparison of the more recent results on ionospheric scatter propagation (in the vhf range) with theoretical results based on the picture of turbulent mixing as the origin of electron density fluctuations.

The main conclusion is that despite the scarcity of experimental data, none of the currently used theories of turbulent mixing seems to give a satisfactory account of the data, in particular of the frequency dependence of the observed signal strength. On the other hand, analysis of the fluctuations of received signals supports the view that scattering is the agent responsible for signal transmission.

F. Villars (Cambridge, Mass.)

GEOPHYSICS

See also 7417, 7556, 7872.

7879:

Robinson, A. R. On two-dimensional inertial flow in a rotating stratified fluid. J. Fluid Mech. **9** (1960), 321–332.

The problem is to find the flow pattern of an oceanic current over an uneven ocean bed. Normally the inertia forces and static stability forces are comparable and the problem is the same as that already solved for the atmosphere except that the upper boundary condition is different. But for very slow currents in which the inertia forces are negligible the disturbance of the motion is confined to a shallow layer close to the boundary; higher up the motion is geostrophic. The amplitude is not seriously restricted to being small so that arbitrarily shaped beds of moderate size can be treated.

R. S. Scorer (London)

7880:

Eckart, Carl. ★Hydrodynamics of oceans and atmospheres. Pergamon Press, New York-Oxford-London-Paris, 1960. xi+290 pp. $9.00.

In the introductory part of this text, the author states that much of the book could have been written fifty or sixty years ago. This is indeed true, but at the same time it could not have been written with the clarity and with the simplicity of notation that the author has employed, nor could some other author have integrated the work fifty years ago as well as has been done in this text. Anyone who is a serious advanced student of hydrodynamics as applied to oceans and atmosphere, both those of the earth and of other planets, ought to study this book. The subjects covered are the basic equations, the perturbation equations, steady motions, the field equations, the formulation of the major mathematical problems in this domain of interest, the properties of the isothermal atmosphere, the properties of oceans with constant coefficients, the thermocline, and so on. Most of the material is of very advanced nature. However, all of the material that can be obtained in the areas to which the author has limited himself is available in a well organized presentation. The references, in particular, are most interesting as many of the early works of the mathematical giants in this area are given.

With these comments in mind, a caution to mathematicians must be interjected because someone not familiar with the present areas of dominant interest in meteorology and oceanography might be seriously misled by the contents of this book. The study of the hydrodynamics of oceans and atmospheres has not pursued, in the main, the path followed in this book. For example, in meteorology, the present procedure is to derive more and more realistic models of atmospheric motion by integrating through several layers of the atmosphere, retaining the nonlinear features of the original equations of motion, and coupling the layers together.

Perturbation techniques do not seem to have preserved certain essential features of the atmosphere. Numerical integrations and iterations by finite time steps of the integrated models described immediately above have given more realistic pictures of the atmosphere than the results discussed in this book. Energy sources and sinks, latent heat of condensation, the transformation from mean flow to eddies, the transformation from zonal to meridional flow, and so on, are some of the areas of dominant interest in the atmosphere at the present time. The techniques that are used, although not so dependent on a formal mathematical structure in terms of eigenvalues and eigen solutions, nevertheless have provided great insight into the properties of the atmosphere. In this context this reviewer was somewhat dismayed by statements concerning negative absolute temperatures for an adiabatic atmosphere and the difficulties of treating sloping ground in the equations of motion. Both of these problems are relatively insignificant to the dynamic meteorologist as he would never use an adiabatic atmosphere to such great heights and as sloping terrain and the effects of mountains on the atmosphere have been well understood from a theoretical point of view for quite a few years.

Another point of interest is the omission of the subject of turbulence. The reason given by the author for such an omission may or may not be valid. The point is, however, that the atmosphere and the oceans are turbulent and that almost any reasonable assumption concerning turbulence, even the ancient Goldberg-Mohn hypothesis for friction, leads to more realistic results, at times, than an assumption of a frictionless fluid with no dissipative mechanisms. There is a vast area of work of considerable import in turbulence that is not treated in this reference.

In summary, from the classical mathematical point of view, this book has a thorough coverage of the techniques that have been developed. However, many of the areas of greatest interest in oceanography and meteorology do not seem to be adequately treated. This book can be recommended for anyone thoroughly familiar with the field who may wish to obtain a new perspective, but those not thoroughly familiar with the field may be given a lopsided and distorted picture.

W. J. Pierson, Jr. (New York)

7881:

Goody, R. M. The influence of radiative transfer on the propagation of a temperature wave in a stratified diffusing atmosphere. J. Fluid Mech. **9** (1960), 445–454.

The problem considered is that of the propagation of heat through a horizontal stratified fluid medium in the direction perpendicular to the stratification planes. The transfer of heat is due to the combined effects of radiation and of turbulent "conductivity". A fourth order partial differential equation for the temperature as a function of time and of a space coordinate perpendicular to the stratification planes is set up. The solution obtained corresponds to two waves whose relative amplitudes vary with distance from the lower boundary. Apparent diffusivities computed from phase lag and attenuation coefficients can differ greatly from each other and can vary with height even if the actual diffusivity does not. Calculations using parameters simulating those of the earth's atmosphere indicate that the upward propagation of the diurnal temperature wave from the earth's surface may be misinterpreted if radiative effects are ignored.

G. C. McVittie (Urbana, Ill.)

7882:

Baur, Franz. Die wissenschaftlichen Grundlagen und Probleme der langfristigen Witterungsvorhersage. Naturwissenschaften **48** (1961), 61–66.

7883:

Riehl, Herbert. On the mechanisms of angular momentum transport in hurricanes. J. Meteorol. **18** (1961), 113–115.

7884:

Nemčinov, S. V. Registration of transformation of atmospheric masses in the case of short-range prediction of non-zonal deviations of altitudes of the absolute topography of an isobaric surface. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1959**, 732–738. (Russian)

Dans un mémoire *Vvedenie v gidrodinamičeskie metody kratkosročnogo prognoza pogody* [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1957; MR **20** #3031], Monsieur I. A. Kibel' a donné une solution assez ingénieuse du problème de la transformation de la température dans une couche de l'atmosphère au voisinage du sol. L'auteur utilise cette méthode pour établir une équation de la variation de la pression atmosphérique pour un processus d'un mouvement à grande échelle.

Cette équation sert de base pour la prévision des écarts des altitudes non zonales de la topographie absolue de la surface isobarique. On suppose que le mouvement est quasi géostrophique et on utilise les facteurs principaux de la baroclinie de l'atmosphère et de la transformation des masses d'air.

M. Kiveliovitch (Paris)

7885:

Pichler, Helmut. Zeitliche und räumliche Mittelbildungen bei Rossby-Wellen. Arch. Meteorol. Geophys. Bioklimatol. Ser. A. **12**, 8–16 (1960). (English and French summaries)

Author's summary: "The investigation is based upon a relation found by H. Reuter which correlates space and time means in a geopotential field. Conditions for the validity of the mentioned relation in the case of Rossby waves are presented in diagrams. Therein, the magnitude of the grid distance by which space means were calculated renders to depend essentially on the phase velocity of Rossby waves. It can be demonstrated that the diagrams are of considerable value for the construction of prognostic charts. In the case of a nonstationary planetary wave system it is possible to decide, by means of the zonal wind component, which grid distance has to be used for calculating the space means."

7886:

Jobert, N. Calcul de la dispersion des ondes de Love de grande période à la surface de la Terre. Ann. Géophys. **16** (1960), 393–413. (English and Russian summaries)

Author's summary: "The dispersion of long period Love waves on a spherical surface is obtained by a numerical integration of the differential equation of motion. After continuous models, we study terrestrial models with a superficial crust (continental, oceanic). For the fundamental mode, the group velocities obtained for periods from one to a few minutes are of the same order as the velocities observed for G waves. The effect of a low-velocity layer at a depth of 150 km is a weak inverse dispersion for the fundamental mode when the crust is oceanic. (For the same model the variation of the displacement with depth shows a secondary maximum.) On the contrary, for the first overtone the maximum and minimum of group velocity are more pronounced for the continental model than for the oceanic one."

7887:

Tihonov, A. N.; Skugarevskaya, O. A. Asymptotic behavior of the process of formation of an electromagnetic field. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1959**, 804–814. (Russian)

This investigation is related to electrical sounding measurements on the Earth's crest. A constant field is established in a conducting slab of material by setting up a Hertzian dipole, fed by a constant current, in the plane of the interface of the slab with free space. The problem is to find expressions for the field in this plane at large distances ρ from the source in direction along and perpendicular to the direction of orientation of the dipole.

Time dependent asymptotic series expansions of the field in powers of $1/\rho$ are first obtained for the cases where the other boundary is the interface with (a) a perfect conductor, and (b) an insulator. The latter case is then developed yielding further expansions, valid for large t, from which a series of curves depicting the electric field as a function of ρ and t are derived.

K. C. Westfold (Clayton)

7888:

Tihonov, A. N.; Skugarevskaya, O. A. Asymptotic behavior of the process of establishing an electromagnetic field in stratified media. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1959**, 937–945. (Russian)

This paper continues previous work by the authors [#7887] on the establishment of a dipole field at a large distance from the source in a medium consisting of a number of layers having different electrical conductivities.

Asymptotic expressions for the field vectors for large t are obtained. The stability of these solutions has yet to be investigated. *K. C. Westfold* (Sydney)

7889:

Lebovitz, Norman R. The equilibrium stability of a system of disk dynamos. Proc. Cambridge Philos. Soc. **56** (1960), 154–173.

This paper considers a large number of disk dynamos of the type introduced by Bullard [same Proc. **51** (1955), 744–760] arranged in a circle. The magnetic field coil of each disk is fed from the next. A constant couple is applied to the axle of each disk, and there may or may not be a viscous retarding couple. In the absence of a retarding couple, the system is always unstable when the number of disks is large. In the presence of a retarding couple, there is a stable steady solution with zero currents when the driving couple is small; if the driving couple exceeds a certain critical value, this solution becomes unstable, but there is a solution with steady finite currents which may be stable. *A. Herzenberg* (Manchester)

7890:

Kazinskii, V. A. ★Mathematical tables for the approximation of geophysical anomalies and reductions by interpolation polynomials. Translated by D. E. Brown. Pergamon Press, New York-Oxford-London-Paris, 1960. iv + 94 pp. $5.50.

Detailed instructions and 67 pages of tables of many elementary functions, such as for instance $(x^2+y^2+z^2)^{-1/2}$, for the computation of approximate values of double integrals with the aid of first three or first six terms of Taylor's expansion of their integrands, where the derivatives (coefficients) are replaced by finite differences. Can be useful in the approximate evaluations of gravitational and magnetic anomalies caused by bodies of any prescribed and known shape, of gravitational effects of irregularities in mine workings, of plumb line deviations, etc.

E. Kogbetliantz (New York)

7891:

Vajk, Raoul. Computational problems in exploration geophysics. Appl. Mech. Rev. **12** (1959), 511–513.

Survey article.

7892:

Morrison, L. S.; Watson, Robert. The electronic computer and geophysics. Geophysics **26** (1961), 40–44.

From the authors' summary: "Electronic computers have been used in geophysical exploration to compute and contour derivative maps of gravity and magnetic data. They have been used to reduce gravity data to datum, compute interval and average velocities from velocity profile data and have been used to solve many non-recurring problems."

7893:

Gan'šin, V. N. Allgemeine Lösung der geodätischen Hauptaufgaben. Bul. Inst. Politehn. Iaşi (N.S.) **5 (9)** (1959), no. 1–2, 377–380. (Russian. German and Romanian summaries)

7894:

Ansermet, A. Le calcul de la courbure des méridiens dans les réseaux géodésiques à coordonnées conformes. Schweiz. Z. Vermessg. Kulturtech. Photogr. **59** (1961), 47–53.

7895:

Hallert, B. Investigations into the accuracy of various methods of photogrammetric triangulation. Kungl. Tekn. Högsk. Handl. Stockholm No. 162 (1960), 118 pp.

Für eine rationelle Herstellung moderner Kartenwerke spielt die Photogrammetrie eine sehr bedeutende Rolle. Hier handelt es sich um die Frage der Genauigkeit von streifenweise angeordneten Luftbildern, wobei genäherte Senkrechtaufnahmen und nicht allzugroße Höhenunterschiede des Geländes (etwa bis 15% der Flughöhe) vorausgesetzt werden. Paßpunkte sollen nur am Anfang und am Ende des Streifens existieren. Auch sollen alle systematischen Fehlereinflüsse bereits weitestgehend eliminiert sein. Unter Zugrundelegung des bekannten Fehlerfortpflanzungsgesetzes für zufällige Fehler werden sodann die Gewichtskoeffizienten (=Kofaktoren), bzw. die mittleren Fehler für die Längs- und Querkoordinaten des Streifens berechnet, die beide noch zum mittleren radialen Fehler zusammengefaßt werden. Hinzu kommen noch die Fehler für die Höhenübertragung. Nach der Art der Auswertung werden 3 verschiedene Typen unterschieden (stereoradial triangulation, independent model triangulation, ordinary aerial strip triangulation), sowie ein- und zweiseitig angeschlossene Streifen. Die günstigsten Fehlerübertragungen werden gefunden, wenn (a) der Streifen nur 5–6 Modelle enthält, (b) wenn zwischen der Streifenlänge und der Flughöhe ein bestimmtes Verhältnis besteht. Betrachtungen für den Fall von Konvergentaufnahmen schließen die Arbeit ab. *H. Wolf* (Bonn)

7896:

Köchle, Richard. Untersuchung über die Konvergenz eines Näherungsverfahrens zum Ausgleichen von eingeschnittenen Punkten. Schweiz. Z. Vermessg. Kulturtech. Photogr. **59** (1961), 3–17, 34–46.

Author's introduction: "In diesem Aufsatz wird ein bekanntes halbgraphisches Verfahren zum Ausgleichen von eingeschnittenen Punkten auf seine Konvergenz hin untersucht. Es wird ein Konvergenzfaktor hergeleitet und in Beziehung zu den Fehlerellipsen gebracht. Der Konvergenzfaktor wird für zwei spezielle Konfigurationen in geschlossener Form angegeben und in einem Graphikon dargestellt. In einem besondern Abschnitt sind einige allgemeine Eigenschaften der Fehlerellipsen eines aus denselben Strahlen vorwärts und rückwärts eingeschnittenen Punktes aufgeführt."

7897:

Hirvonen, R. A. New theory of the gravimetric geodesy. Ann. Acad. Sci. Fenn. Ser. A III. No. 56 (1960), 50 pp.

In der klassischen Theorie der Erdmessung (Helmert) wird die physische Erdoberfläche mittels der orthometrischen Höhen auf das Geoid bezogen, dessen Gestalt astronomisch-gravimetrisch ermittelt wird, was nicht ohne Zuhilfenahme von Hypothesen bewerkstelligt werden kann. Im Gegensatz hierzu wird in Hirvonens "Neuer Theorie der gravimetrischen Geodäsie" die wirkliche Erde

durch einen Vergleichskörper ersetzt, dessen Gestalt und Dimensionen als übereinstimmend mit denen des Internationalen Rotations-Ellipsoides angenommen werden. Zu diesem Normal-Körper gehört ein normales Schwerefeld, das in seinen Eigenschaften, wie z.B. in seinem Potential, in seinen Schwerkraftswerten und in seinen Kraftlinien völlig hypothesenfrei und mit unbeschränkter Genauigkeit berechnet werden kann. Man bezeichnet diese dann als Sphäropotential, normale Schwere- und normale Lotlinien. Die Zuordnung des wirklichen Erdkörpers zu diesem Vergleichskörper wird so vorgenommen, daß zu jedem Punkt an der physischen Erdoberfläche ein zweiter definiert wird, der auf der normalen Lotlinie (durch den ersten) liegt und dessen Sphäropotentialwert mit dem wirklichen Potentialwert (=Geopotential) des erstgenannten übereinstimmt. Die Gesamtheit aller dieser zugeordneten Punkte bildet das Telluroid. Der Abstand Telluroid-Ellipsoid ist die normale Höhe, der Abstand der physischen Erdoberfläche vom Telluroid ist die Höhenstörung, und der Unterschied zwischen der (an der Erdoberfläche) gemessenen Schwere und der normalen Schwere (am Telluroid) ist die normale Freiluftanomalie. Sie liefert, in die Stokessche Integralformel eingesetzt, die Höhenstörungen. Unter Benutzung eines elliptischen Koordinatensystems werden die erforderlichen Zusammenhänge aufgestellt. Verf. nennt als Vorteile seiner Theorie: die hypothesenfreie Berechnung der Schwereanomalien, der normalen Höhen und der Lotabweichungen, sowie den Verzicht auf alle Massenverlagerungen bei den Reduktionsrechnungen. Werden die Normalhöhen und die Höhenstörungen dem Bezugsellipsoid aufgesetzt, so erhält man die Gestalt des physikalischen Erdkörpers, womit die Erdmessungs-Aufgabe gelöst ist.

H. Wolf (Bonn)

7898:

Hazay, I. Die mechanischen Prinzipien der Ausgleichung. Acta Tech. Acad. Sci. Hungar. **30** (1960), 133–168. (English, French and Russian summaries)

Es wird das bekannte Prinzip (von Castigliano) von der kleinsten Formänderungsarbeit in statistischen Systemen in Parallele gesetzt zum Gaußschen Ausgleichungsprinzip der kleinsten Verbesserungsquadratsumme in der Ausgleichungsrechnung. Die statischen Gleichgewichtsbedingung, daß die Summe der angreifenden Kräfte Null ist, wird an die Stelle von $[pav]=0$, $[pbv]=0$, usw. gesetzt, wodurch sich die Normalgleichungen ergeben. Der Reihe nach werden die Fälle der direkten, der vermittelnden und der bedingten Beobachtungen, sowie der bedingten Beobachtungen mit Unbekannten und der vermittelnden Beobachtungen mit Bedingungsgleichungen behandelt und ihre Anwendung auf die trigonometrische Höhenbestimmung und die trigonometrische Bestimmung von Lagekoordinaten gezeigt. Hierbei wird vorgeschlagen, Richtungsgewichte proportional zur Zielweite einzuführen. Auf die Anschaulichkeit der Formelableitungen wird besonders hingewiesen.

H. Wolf (Bonn)

OPERATIONS RESEARCH, ECONOMETRICS, GAMES

See also A6727, 7231.

7899:

Hitch, Charles. Uncertainties in operations research. Operations Res. **8** (1960), 437–445.

Der vorliegende Aufsatz ist sehr lesenswert für alle, die mit operations research zu tun haben. Der Verfasser unterzieht die bekannten Methoden einer strengen Kritik, die im wesentlichen darauf hinaus läuft, daß diese Methoden und Theorien (aus der Spieltheorie) nach seiner Meinung zu einfach sind. Auch das Bilden von Modellen und das modellmäßige Nachspielen von Konflikten führt nicht zu sicheren Erfolgen, da in die Modelle zu viele Unsicherheitsfaktoren eingehen. Nach Ansicht des Verfassers sind für denjenigen, der auf dem Gebiet des operations research tätig ist, Phantasie und Einfallsreichtum besonders wichtig. Dies drückt der Verfasser etwa so aus: Wenn Methode A im Falle 1 sehr gut ist und im Falle 2 sehr schlecht, und wenn Methode B sich in etwa umgekehrt verhält, dann solle man nicht versuchen, durch raffinierte Verfahren herauszubekommen, welche von beiden die Bessere ist, sondern eine bessere Method C ersinnen.

H. Kiesow (Oberhausen)

7900:

Yablonskiĭ, S. V.; Gil′man, A. M.; Kotel′nikov, I. V.; Potylicyn, P. M. A device for the investigation of street-traffic control algorithms. Dokl. Akad. Nauk SSSR **132** (1960), 78–81 (Russian); translated as Soviet Physics. Dokl. **5**, 474–476.

7901:

Fuchs, Siegmund. Die Variationen der Rechnungsgrundlagen in der Lebensversicherung, besonders im Hinblick auf Reserveberechnung und Technik erhöhter Risiken. Bl. Deutsch. Ges. Versicherungsmath. **4** (1959/60), 347–382. (English summary)

Der Verf. untersucht den Einfluss der Veränderungen des Zinsfusses und der Sterblichkeit auf das Deckungskapital. Er bedient sich dabei der Theorie von Schärf. Ganz besonders wird die Frage erhöhter Risiken diskutiert. Der Einfluss der Sterbegesetze von Moivre, Dormoy und Makeham auf die Reserven einer gewählten Versicherung als Funktion von q_x wird geprüft. Praktische Beispiele.

W. Saxer (Zürich)

7902:

Berger, Gottfried. Über die wechselseitige Abhängigkeit von Prämien, Reserven und Rechnungsgrundlagen. Bl. Deutsch. Ges. Versicherungsmath. **4** (1959/60), 383–406. (English summary)

Der Verf. interpretiert die Aequivalenzgl. als eine lineare Abbildung. Gegeben ist der Rechnungsgrundlagen-Vektor $\mathbf{r}=(v, q_{m-1}^{(k)}, T_m^{(k)}, S_m, S, \pi_n)$; gesucht wird der Vektor $\mathbf{p}(P, V_1, V_2, \cdots, V_{n-1})$. Mit Hilfe dieser Interpretation werden das Invarianzproblem, die Variation von $\mathbf{r}$ und $\mathbf{p}$ systematisch behandelt und bekannte Sätze in neue Form gebracht und bewiesen, beispielsweise Zinsfussproblem, Vorzeichensätze, Zusammenhang mit der Theorie von Schärf.

W. Saxer (Zürich)

7903:

Hansen, Chr. Über Lebensversicherungen mit Optierungsrecht. Skand. Aktuarietidskr. **1959**, 125–131 (1960).

Der Verfasser zeigt, dass bei einer retrospektiven Lebensversicherung das Optierungsrisiko eingeschlossen

werden kann, d.h. dass ungeachtet des Gesundheitszustandes der Versicherte zu jeder Zeit die Risikosumme der Versicherung bis zur vertraglich vereinbarten Optierungssumme erhöhen kann. *W. Saxer* (Zürich)

7904:

Bernholtz, B. Optimum allocation of discharge to units in a hydro-electric generating station. SIAM Rev. **2** (1960), 247–258.

The optimal distribution of a given flow of water among the turbines of a hydro-electric generating station can be determined by nonlinear programming. Let $p_i(q_i)$ be the rate of power generation of the ith turbine when a flow of q_i is allocated to it (these functions are assumed to be continuous, increasing, and concave), and let M_i be the maximum rate of flow permitted through the ith turbine. Then the optimal distribution is the numbers q_i that maximize $P(Q) \equiv \sum p_i(q_i)$, where Q is the total available flow, subject to $0 \leq q_i \leq M_i$ and $\sum q_i = Q$. This problem is solved directly by using the Kuhn-Tucker conditions [Proc. 2nd Berkeley Symposium Math. Statist. and Probability, 1950, pp. 481–492, Univ. of California Press, Berkeley, Calif., 1951; MR **13**, 855]. If the turbines are all identical, a frequent special case, the resultant symmetries make the problem easily solvable. If the turbines are not identical the author recommends classifying them into groups of identical turbines and then solving by an iterative procedure in which the flow is first divided between two of the groups, then a third group is added and the water reallocated, etc. It is not clear to the reviewer why this iterative procedure is needed since the Kuhn-Tucker theorem applies directly to the more general case. Indeed, since the objective function is separable the elegant method developed by Charnes and Lemke [Naval Res. Logist. Quart. **1** (1954), 301–312; MR **17**, 537] should apply.
R. Dorfman (Stanford, Calif.)

BIOLOGY AND SOCIOLOGY

See also 7185.

7905:

Laborit, H.; Weber, B. The value of the application of cybernetic principles to physiological regulatory mechanisms. Cybernetica **3** (1960), 216–323.

7906:

Estes, W. K. A random-walk model for choice behavior. Mathematical methods in the social sciences, 1959, pp. 265–276. Stanford Univ. Press, Stanford, Calif., 1960.

The random walk studied involves $N(m+1)$ states, O_i (for orienting) and A_{ij} (for approach); $i = 1, 2, \cdots, N$; $j = 1, 2, \cdots, m$. The process begins in O_i with probability o_i, $\sum_{i=1}^{N} o_i = 1$. The non-zero transition probabilities are:

$$\Pr(A_{ij}|O_i) = p_i, \quad \Pr(O_k|O_i) = (1-p_i)/(N-1), \quad k \neq i;$$

$$\Pr(A_{i,j+1}|A_{ij}) = p_i, \quad \Pr(O_k|A_{ij}) = (1-p_i)/(N-1), \quad j < m, \quad k \neq i; \quad \Pr(A_{im}|A_{im}) = 1.$$

The probability that a particle is absorbed in A_{im} when $o_i = 1/N$ is shown by standard methods to be

$$p_i^m \prod_{j \neq i} (1-p_j^m/N) / \sum_k p_k^m \prod_{j \neq k} (1-p_j^m/N).$$

For $N = 2$, the probability of absorption in A_{im} is

$$p_1^m[o_1 + o_2(1-p_2^m)]/(p_1^m + p_2^m - p_1^m p_2^m).$$

Assuming p_i and o_i are subject to a simple linear learning process, asymptotic absorbtion probabilities for $N = 2$ are obtained and compared with a conventional stochastic learning model. Comparisons are also made with several pair comparison models. *R. D. Luce* (Philadelphia, Pa.)

INFORMATION AND COMMUNICATION THEORY

See also 7224.

7907:

Levine, A.; McGhee, R. B. Cumulative distribution functions for a sinusoid plus Gaussian noise. Trans. IRE **IT-5** (1959), 90–91.

A tabulation of values of

$$\pi^{-1} \int_0^{\pi} [\operatorname{erf}\{(1+r)^{1/2}y - (2r)^{1/2}\cos\theta\} - \operatorname{erf}\{-(2r)^{1/2}\cos\theta\}]\, d\theta$$

for y: 0.00(.10)4.00, $r = 0$, .20, .50, 1, 2, 3, 4, 6, 8, 10, 12, 15, 20, ∞. *S. Kullback* (Washington, D.C.)

7908:

Fryer, R. G. Note on "On upper bounds for error detecting and error correcting codes of finite length". Trans. IRE **IT-6** (1960), 502.

[The paper cited in the title is by N. Wax, same Trans. IT-5 (1959), 168–174; MR **22** #1472.] This paper shows that the best known code of length 12 with minimum distance 5 has 28 members, an improvement on the formerly best known of 24. *R. W. Hamming* (Stanford, Calif.)

7909:

Leipnik, Roy. First and second order distributions of a sine wave of random phase plus Gaussian noise. Z. Angew. Math. Phys. **11** (1960), 117–126. (German summary)

The author obtains the distributions described in the title in terms of rapidly convergent series of Bessel functions. He derives these results from the characteristic functions previously obtained by S. O. Rice [Bell System Tech. J. **27** (1948), 109–157; MR **9**, 362].
I. J. Good (Teddington)

7910:

Locke, William N.; Booth, A. Donald (Editors). ★Machine translation of languages. The Technology Press of The Massachusetts Institute of Technology and John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London; 1958. xii + 243 pp. $6.00.

Fourteen papers by seventeen authors discuss the application of computers to language translation. The contributors represent most of the early workers in the field. Papers are concerned with problems of multiple meaning, idioms, language structure, etc., and also with the question of the adequacy of the computers of the early '50's for the proposed search and symbol-manipulation processes. With the rapid growth of this field, the collection probably retains mainly historical interest; in that respect it contains important reviews and bibliographies of the early history.
M. L. Minsky (Cambridge, Mass.)

7911:

Delavenay, E.; Delavenay, K. ★Bibliographie de la traduction automatique. Bibliography of mechanical translation. Janua Linguarum, Nr. XI. Mouton & Co., The Hague, 1960. 69 pp. f 10.00.

A selected bibliography of all types of publications—monographs, journal articles, technical reports, internal papers, etc.—compiled for the International Study Group on Machine Translation (now developed into ATALA). Works recommended for the non-initiated are indicated, and the whole is classified under the following headings: Bibliographical sources; information theory; cybernetics; mechanical retrieval of information; mechanical abstracting and analysis; linguistics; computers; mechanical translation; bilateral programmes.

SERVOMECHANISMS AND CONTROL

7912:

Desoer, C. A.; Wing, J. An optimal strategy for a saturating sampled-data system. IRE Trans. AC-6 (1961), 5–15.

Authors' summary: "Consider the usual sampled-data control system in which the sampler is followed by a zero-order hold and the transfer function is $G(s)=1/s(s+a)$. Saturation is represented by the fact that the forcing function applied to $G(s)$ may not be larger than 1 in absolute value. The problem is to determine a saturating zero-order hold forcing function which forces the system from an arbitrary initial state to equilibrium in the least number of sampling periods. Such a forcing function is defined as an optimal strategy.

"The state plane is divided into boundary states and interior states. To each boundary state corresponds a unique optimal strategy. To each interior state correspond infinitely many optimal strategies.

"From the system parameters a polygonal curve, called the critical curve, is defined in the state plane. An optimal strategy is then proposed in which the required forcing function is simply obtained by computing the distance of the representative point in state plane to the critical curve. A simple computer is proposed to implement this optimal strategy. Finally, the proposed optimal strategy is shown to reduce in the limit as $T \to 0$ to that of the corresponding continuous system."

AUTHOR INDEX

PART B